# The Earth and Its Gravity Field

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#### PREFACE

Knowledge of the gravity field of the earth is important in the study of our globe. If we know the gravity anomalies, as we now do in many places, not only can we investigate their causes and the phenomena connected with them, but also we can determine the figure of the earth and the shape of the geoid. If the earth were in equilibrium, its figure would be close to an ellipsoid of revolution, and the gravity field around it would be regular. There are, however, deviations from equilibrium and, consequently, irregularities in the shape of the geoid and gravity anomalies. The latter can be measured, and if we know them over a sufficiently large part of the globe, we can, by means of Stokes' theorem, derive the deviations between geoid and ellipsoid over a corresponding though smaller part. These deviations are relatively small, probably not exceeding 50 meters.

The gravity survey over a great part of the earth's surface thus makes possible a new and important branch of geodesy, which we may call physical geodesy. Besides the deviation of geoid from ellipsoid, we can also derive the deflection of the vertical from the gravity anomalies; this fact has consequences of vast importance in geodesy as it allows us to unify the geodetic nets over the earth, regardless of their separation by oceans, into one world geodetic system.

The great number of gravity data already available allow us, in addition, to make important conclusions about the way the earth tends toward equilibrium and about the character and size of the deviation from it. Gravity data are thus of great value in many geophysical problems. Needless to say, these problems are often intimately related to geological data and interpretations and to the results of seismology, geomagnetism, and geomorphology.

In recent years only short papers have been published on the earth's gravity field, the possibilities provided by the large number of gravity data, and the conclusions which can be drawn from them. The authors therefore feel that a book dealing with these problems and giving the most important lines of advance and results may be useful. Although concerned principally with the earth's gravity field and related subjects, the book also touches on the results of other branches of geodesy and geophysics, e.g., mathe-

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matical and astronomical geodesy and the inner constitution of the earth as indicated by seismology and geomagnetism.

With a few minor exceptions, Chaps. 4 to 9 have been written by Heiskanen and Chaps. 1 to 3 and 10 to 12 by Vening Meinesz. The authors agree entirely on the content of their respective contributions but, preferring to preserve their personal styles, have not attempted to harmonize the details.

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#### THE EARTH AND ITS GRAVITY FIELD

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#### CHAPTER 1

## INTRODUCTION AND SUMMARY

This introductory chapter explains how the purpose of the book, broadly outlined in the preface, is carried out in the ensuing chapters, and thus it also serves as a summary of the contents.

Before dealing with the main subject, the earth's gravity field and the geodetic and geophysical implications and conclusions which can be drawn from it, we give a summary of the significant data relating to the earth provided by seismology, geomagnetism, and other geophysical means. Chapter 2 treats of the three major discontinuities, or interfaces, dividing the earth into four parts. Above the Mohorovičić, or M, interface is the M crust, so called to distinguish it from the rigid crust. It normally reacts elastically to stresses, and only tectonic forces are strong enough to cause plastic deformation in it. Between the M interface and the boundary of the core, at a depth of 2900 km, is the mantle. Since the mantle can be expected to react plastically to stresses, it can further be supposed that currents are possible there, though only slow ones of a few centimeters per year velocity. Between the interfaces at 2900 and about 5100 km depth is the outer, liquid core, in which currents of 200,000 to 1,000,000 times greater velocity can be assumed, i.e., of 5 to 25 km/year. Their presence seems to be indicated by geomagnetic data and by the irregularities astronomers have found in the earth's rotation, both of which are briefly dealt with in Sec. 2-5. Below the interface, at a depth of about 5100 km, is the inner core, about which little is known. The supposition has been made that it consists of the same matter as the outer core, i.e., of iron and nickel, but that it reacts more or less as a solid.

In Chap. 3 we deal with the foundation, by means of potential theory, of the treatment of the earth's gravity field and its relations to the geoid and to other equipotential surfaces outside the earth, and for this purpose spherical harmonics are introduced. In the course of our deductions we imagine the earth's mass to be divided into two parts: (1) a regular group of masses which together equal the total earth mass, have axial symmetry around the earth's axis, and are symmetric with respect to the equator plane and (2) the remainder of the earth's masses, of positive as well as negative sign, of which the total mass is zero.

We first derive the geoid and the gravity field of the regular group of max is. The geoid is a spheroid which we adopt as a reference surface for all geodetic measurements; the gravity on this surface represents our formula for normal gravity. By the proper choice of constants we can make the reference spheroid coincide with an ellipsoid of revolution or with the equilibrium figure of a fluid earth of the same total mass, the same total volume, and the same internal density distribution as the real earth. It is fortunate that both cases can be brought to practical coincidence and that therefore the appropriate ellipsoid of revolution and the gravity field belonging to it can be considered so nearly representing equilibrium of the earth that the difference can be neglected.

As a consequence, the gravity anomalies of the real earth, with respect to the formula for normal gravity corresponding to the ellipsoid of revolution, can be considered as representing deviations from equilibrium actually present in the earth, while the deviations of the real geoid from the equilibrium ellipsoid showing the warping and tilting of the geoid, and the deflections of the vertical correspond to these deviations from equilibrium. We can derive the geoid deviations and deflections of the vertical as functions of the gravity anomalies, and we thus arrive at the theorem first stated by Stokes in the middle of the nineteenth century.

Stokes' theorem is valid only if all the irregular masses of the second group mentioned above, which cause the anomalies, are inside the geoid. In discussing this point we shall come to the conclusion that this problem—like all other problems connected with the application of Stokes' theorem and of the formulas derived from it, which express the deflections of the vertical in the gravity anomalies—can best be solved by applying the appropriate topographic and isostatic reductions to the observed gravity anomalies. This means, however, that we shall have to correct the geoid and the deflections of the vertical derived from the reduced gravity anomalies to bring them back to the real geoid of the earth as it was before these masses were taken away.

Chapter 4, "Gravity Measurements," deals with the theory of the mathematical and physical pendulum, the pendulum gravity observations on continents and at sea, the most important types of ordinary and underwater gravimeters and their accuracy.

Chapter 5, on isostasy, contains the following sections: Definition, First Ideas concerning Isostatic Equilibrium, Isostatic Assumptions of Pratt and Airy, Some Isostatic Studies at the Turn of the Century, Pratt-Hayford Isostatic System, Airy-Heiskanen Isostatic System, Vening Meinesz Regional Isostatic System, and Some Other Isostatic Assumptions.

Chapter 6, "Reduction of Gravity Measurements," explains the theory and practice of the free-air reduction, Bouguer reduction, Helmert's condensation reductions, Rudski's inversion reduction, isostatic reductions

according to the Pratt-Hayford, Airy-Heiskanen, and Vening Meinesz systems, the indirect effect, or Bowie reduction, geological corrections, and the cartographic and mass-line methods for isostatic reduction.

Chapter 7 deals with the different types of gravity anomalies from the point of view of isostatic equilibrium. It shows by tables, diagrams, and maps that the Bouguer gravity anomalies are strongly negative in the mountains and still more strongly positive at sea—evidence that isostatic equilibrium exists. It likewise shows why and to what extent the Bouguer anomalies can be misleading in geophysical interpretations of the gravity anomalies. Further considered are the manner in which isostatic anomalies depend on the type of isostatic system in regard to the mode of isostatic equilibrium, the depths of compensation (Pratt-Hayford theory), normal thickness T of the earth's crust, and the density difference  $\Delta \rho$  of the substratum and earth's crust (floating theories). Finally, it is proved by several examples that the isostatic gravity anomalies according to different T values yield good values for the normal thickness of 'he earth's crust and for the thickness of the mountain roots and of the ocean antiroot. The "magical" figure of 30 km for T is shown to result from many studies carried out on the basis of material from different parts of the world.

For mathematical reasons we have used only one discontinuity in the lso static studies. We thus get T values which correspond very closely, physically as well as numerically, to the discontinuity given by seismology. The assumption that T=30 km and  $\Delta\rho=0.6$  agrees closely with the results of seismologists when they use two discontinuities and with the evidence of recent seismological surveys of the oceans.

From the gravity results found in areas where the topography appears to be regionally compensated, e.g., volcanic islands, we can also derive an approximate figure for the thickness of the rigid crust. It checks well with the depth in the continents of the M discontinuity.

Chapter 8, "Physical Geodesy," contains, among others, the following sections: Meaning of Geodetic Systems, Historical Development of Geodesy, Most Important Dimensions of Reference Ellipsoids, Significance of the Initial Point of Geodetic Systems, Principles of Physical Geodesy Gravity Material Necessary and Currently Available, Reduction Method to Be Used, Deflections of the Vertical, Dimensions of the Reference Ellips oid, and Shape of the Geoid.

Chapter 9, "World Geodetic System," shows in what way and with what accuracy the geodetic system can be converted to the world geodetic system and explains the significance of astronomic-gravimetric supercontrol points, the control of small-scale maps without triangulation, and several other applications of the gravimetric method.

Chapters 8 and 9 contain the essential portions of the main problems of modern geodesy, those which can be solved solely by the gravimetric method and those in which the gravimetric method leads to simpler yet more accurate solutions than are possible by other existing methods.

Chapter 10 discusses the geophysical meaning of isostasy and of different types of deviations from isostatic equilibrium. In Part 10A, formulas for elastic, plastic, and shear deformations of the crust arc developed, and the manner in which geosynclines are formed is dealt with. Geosyncimes are probably a consequence of horizontal compression causing plastic thickening and downbulging of the crust.

In Part 10B the way in which the isostaticequilibrium of the crust is readjusted is dealt with by discussing the readjustment phenomena in Fennoscandia and elsewhere, where at the end of the glacial period the relatively quick removal of the ice load left a disturbance of equilibrium. We shall find that this phenomenon is mainly a function of plastic flow in the substratum.

In Part 10C we use the conclusions reached in Part 10A to discuss the geophysical phenomena in the Indonesian Archipelago and other island-arc areas, especially the belts of strong negative gravity anomalies found there. The phenomena may be interpreted as being the result of great crustal blocks' moving with respect to each other and thus causing, in the zones between them, crustal downbulging and geosyncline development. On the sides of the arcs, crustal shear is predominant, and the shapes of the island arcs can thus be explained.

Part 10D discusses the effects of stress release in the crust and the resulting development of grabens. Such crustal deformations, caused mainly by the formation of tilted fault planes, lead to gravity anomalies, although the system as a whole is in isostatic equilibrium.

Chapter 11 deals with the hypothesis of convection currents in the mantle. Section 11-1 contains a short summary of the chapter and gives the arguments for adopting this hypothesis, one of the most important of which is that the relative block movement in the Indonesian Archipelago, dealt with in Chap. 10A, points over a great area, to uniaxial horizontal stress in the crust; such a stress can best be explained by the drag exerted by undercurrents.

Section 11-2 treats of the density-transition layer between 200 and 900 km depth, which, in this connection, we interpret as a layer where two modifications of the mantle material (probably olivine) are coexistent. The following sections develop formulas for convection in a plane layer, in a spherical shell, and in a sphere.

Section 11-6 begins the discussion of Prey's development, in spherical harmonics, of the earth's topography, which shows that its main lines, the distribution of continents and oceans, present regular features pointing to their connection with current systems in the mantle. These current systems have probably been preceded by a current system in the whole earth.

This conclusion leads to a theory of the origin, at the beginning of the earth's history, of the core of the earth and of an "ur" continent at the surface, both caused by a current system in the whole earth, which was probably followed by a current system in the mantle that drew the ur continent apart into the continents now existing. The elaboration of this hypothesis brings us further to the assumption of subsequent current systems in the mantle caused by the earth's cooling and repeating themselves during the whole of the earth's history. To these systems of convection currents we may attribute not only the relative movement of crustal blocks, and between them the development of geosynclines with accompanying orogeny, but also the subsidence in the latter zones of deep basins, e.g., the Banda Sea and Celebes Sea in the Indonesian Archipelago, the Gulf of Mexico and the Caribbean Sea in the West Indies, the Mediterranean basins, and the West Pacific basins.

The further probability that these episodic current systems brought about movements of the entire crust of the earth over the mantle explains the wanderings of the poles with respect to the crust, to which geomagnetic evidence seems to point.

In conclusion Chap. 12 investigates the shear pattern in the crust to which the stresses caused by crustal migrations over a flattened earth must have led.

#### CHAPTER 2

## INTERNAL CONSTITUTION OF THE EARTH

Seismology has made major contributions to the study of the earth's interior. It has revealed three principal discontinuities within the earth, viz., the boundaries between the crust and the mantle, the mantle and the core, and inside the core. In addition, seismic-wave velocities have provided data for the densities of the different layers.

#### 2-1. The Crust

We shall call the first of the discontinuities the M interface and the crust above it the M crust, after Mohorovičić, who first mentioned this discontinuity in a paper <sup>46\*</sup> published in 1909, in which he assumed a single crustal layer 60 km thick to explain the two phases found in the Kulpa Valley earthquake. The reason for giving a special name to the crust above the M interface is to distinguish it from the rigid crust, which is based on a different definition related to its physical reaction to stress.

Since the publication of Mohorovičić's paper many investigations have been made of possible interfaces between surface layers of the earth in the continents. Discontinuities have been found between sedimental layers and the granite layer below them and between the granite layer and the lowest layer of the M crust, which is usually called the intermediate layer, but no clear picture has been obtained. Seismic velocities in the intermediate layer seem to indicate that it is basaltic, but its thickness and the thickness of the granite layer appear to vary considerably over the earth's Jeffreys's results 34 (1926) calling for 12 km of granite and 25 km of basalt have been contradicted by other figures derived elsewhere. Lee 42 obtained for Northern Europe a sedimentary layer of 1 km, a granitic layer of 14 km, and a basaltic layer of 15 km. In 1937 Jeffreys 35 critically examined all the European data then available and, finding much inconsistency, assumed 17 km of granite and 9 km of basalt as fitting best. For southern California Gutenberg 30 found evidence for four layers of 14, 11, 6, and 8 km thickness, respectively, and for northern California Byerly and Wilson <sup>16</sup> obtained three layers of 13, 12, and 6 km, respectively. Leet <sup>43</sup> found, in 1933, a surface layer 23 km thick for New England; Robertson's

<sup>\*</sup> These numbers refer to references/at the end of each chapter.

results <sup>50</sup> for Missouri showed two layers of 16 and 13 km, respectively. The great diversity of these figures is apparent; Macelwane <sup>44\*</sup> appears justified in pointing out "the futility of trying to determine these quantities by combining heterogeneous groups of observations of inadequate accuracy from many earthquakes."

We shall not discuss the matter further here, and henceforth we shall suppose a mean constitution of the continental crust of 17.5 km of granite of density 2.67 and of 17.5 km of basalt of density 3.00. We must keep in mind, however, while using these figures, the uncertainties resulting from the variability of constitution.

For the crust under the oceans present data are much fewer than for the continents, but they agree well. Seismic work at sea was started by Ewing 24, 25 and Bullard 9 and continued by their collaborators and by Hill 32 and Raitt. 48 The results indicate that the crust below the oceans consists of a thin layer of usually less than 1 km of unconsolidated sediments (small P-wave velocity  $V_P$ ), of 1 to 4 km of sediments ( $V_P = 4.5$  to 5.3 km/sec), of a layer probably basaltic ( $V_P = 6.5$  to 7.0 km/sec) possibly also containing old or dolomitized sediments, and lastly ultrabasic rocks (according to Raitt:  $V_P = 8.0$  to 8.6 km/sec). So, as far as our present knowledge goes, we are probably near the truth if we assume that the rigid oceanic crust consists on the average of 0.8 km of unconsolidated sediments of density 2.0, of 2.5 km of sediments of density 2.7, of 5 km of basalt of density 3.0, and below that of ultrabasic rocks of density 3.27. These views are also consistent with Kuenen's 39 estimate of about 3 km of sediments for the whole geological past, and (see Chap. 11) with petrographic ideas. For a more detailed picture we may have to change the thickness of the sediments and perhaps also of the basalt. Raitt 48 stresses the variability of these thicknesses.

Since it must be assumed that the M discontinuity is the interface between the basalt and the ultrabasic rock, the M crust would be only 8.3 km thick. Thus there can hardly be any doubt that, as far as the oceans are concerned, there is a great difference between the two crustal conceptions, the M crust and the rigid crust; in Chap. 7 we find evidence that the thickness of the rigid crust must be from 30 to 40 km.

We find the weights P of the continental and oceanic columns of unit cross section to be in equilibrium if the sea has a depth of 5100 m, which seems an acceptable figure for the mean depth of the deeper ocean basins. Figure 2-1 represents these columns, and Table 2-1 shows the balance.

The schematic picture used in this book for the application of the Airy hypothesis of isostasy assumes a homogeneous crust in continents and oceans of density 2.67, floating on a subcrustal plastic layer of density 3.27.

\* Page 248 of his contribution to Gutenberg's "The internal constitution of the earth," 2nd ed., from which all the above data have been taken.

$$0.8 = \frac{5.1 \times 1.028}{2.5 \times 2.7} = 2.0$$

$$17.5 \times 3.00$$

$$21.6 \times 3.27$$

Fig. 2-1. Continental and oceanic crust.

TABLE 2-1

<b>Thickness</b>	Density	$P_c$	Thickness		Density	,	$P_o$
17.5 km >	2.67 =	46.73	5.1 km	×	1.028	=	5.24
17.5 km >	3.00 =	52.50	0.8 km	×	2.0	=	1.60
			2.5 km	×	2.7	=	6.75
			5.0 km	X	3.00	=	15.00
			21.6 km	X	3.27	=	70.63
35 km		99.23	35 km				99.22

For its thickness T, in case of zero topography, values of 20, 30, 40, and 60 km have been introduced, and tables for isostatic reduction of gravity have been based on them. From the 37 observed gravity profiles over continental margins, reduced according to these tables, the value of T in those areas seems to be between 20 and 30 km. It is interesting to see that for the mean of these two values the crust is just in equilibrium with the crustal columns mentioned above. Figure 2-2 represents this case, and Table 2-2 gives the

	Table 2-2
25 × 2.67	Thickness Density $P_c$
20 2.07	$25 \text{ km} \times 2.67 = 66.75$
	$10 \text{ km} \times 3.27 = 32.7$
	99.45

Fig. 2-2. Schematic crust, used for isostatic reductions.

weight  $P_c$  of the corresponding crustal column. It nearly approaches the previous figures  $P_c$  and  $P_o$ , and so the gravity results check well with the picture obtained seismically.

We may ask how great the sea depth would be if there were no sediments covering the basaltic layer. Comparing the continental column with the column corresponding to this condition, we find that a depth of 6200 m would result in equilibrium. The weight  $P_o$  of a column of unit cross section would be

$$P_o = 6.2 \times 1.028 + 5 \times 3.00 + 23.8 \times 3.27$$
  
=  $6.37 + 15.00 + 77.83 = 99.20$ 

and this checks with the value of  $P_c$  of Table 2-1. If we assume that the basalt is also absent and that the ocean floor is therefore formed by ultrabasic matter of density 3.27, we find an ocean depth of 6800 m. The column weight would then be

$$P = 6.8 \times 1.028 + 28.2 \times 3.27 = 6.99 + 92.21 = 99.20$$

Since seismology shows that, when the effect of increasing pressure and temperature is neglected, the density of 3.27 probably continues for far over 100 km, we come to the interesting conclusion that the depth obtained would be about the maximum depth for an ocean floor in isostatic equilibrium. This result seems to be in fair agreement with the depth values of broad ocean basins, which seldom are much greater than 6500 m.

The last two discussions have already involved problems of isostasy. We shall deal with this subject in Chaps. 6 to 8, where it is discussed in detail.

#### 2-2. The Mantle

The greater part of the earth, between the M interface and the discontinuity at 2900 km depth, is the mantle of the earth. There is no evidence of a discontinuity in the mantle itself, but that does not mean its density is constant. The density curve (Fig. 2-4) and Table 2-3 were derived by Bullen, <sup>12, 13</sup> using certain assumptions, from Jeffreys's <sup>36</sup> velocity curve for seismic waves (Fig. 2-3) combined with figures of Lambert and Darling.<sup>41</sup>

Birch <sup>6</sup> has derived the density curve after reducing each value to surface temperature and pressure conditions, with highly interesting results. He finds an upper layer of the mantle, below the crust, reaching to a depth of 200 km, which probably has a constant density of about 3.3. Then there follows a layer from 200 to 900 km depth, where a gradual transition of density from 3.3 to 4.0 occurs, and finally a layer from 900 to 2900 km depth with a constant density of about 4.0 throughout (Fig. 2-5). The fact that the major part of the mantle, reaching from 900 to 2900 km depth, seems to be homogeneous points to the existence of current systems to keep it that way. The upper layer of the mantle, of 200 km, also appears to be