ELECTROMAGNETISM

Second Edition

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Editors' Preface to the Manchester Physics Series

The first book in the Manchester Physics Series was published in 1970, and other titles have been added since, with total sales world-wide of more than a quarter of a million copies in English language editions and in translation. We have been extremely encouraged by the response of readers, both colleagues and students. The books have been reprinted many times, and some of our titles have been rewritten as new editions in order to take into account feedback received from readers and to reflect the changing style and needs of undergraduate courses.

The Manchester Physics Series is a series of textbooks at undergraduate level. It grew out of our experience at Manchester University Physics Department, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course and that this material is only rarely so arranged as to allow the definition of a shorter self-contained course. In planning these books, we have had two objectives. One was to produce short books: so that lecturers should find them attractive for undergraduate courses: so that students should not be frightened off by their encyclopaedic size or their price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second aim was to produce books which allow courses of different length and difficulty to be selected, with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of later parts of each volume. Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each author's preface gives details about the level, prerequisites, etc., of his volume.

We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, whose helpful criticisms and stimulating comments have led to many improvements. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, our many suggestions and requests. We would also like to thank the publishers, John Wiley & Sons, who have been most helpful.

F. MANDL
R. J. ELLISON
D. J. SANDIFORD

January, 1987

Preface to the Second Edition

The basic content of undergraduate courses in electromagnetism does not change rapidly, and the range of topics covered in the second edition of this book is almost the same as in the first edition. We have made a few additions, for example by giving a fuller treatment of circuit analysis and by discussing the dispersion of electromagnetic waves. Some material which now seems outdated has been removed, and illustrative examples have been modernized.

We have made many small changes in presentation which we hope will make the argument clearer to readers. We gratefully acknowledge the help of all those who have suggested ways of improving the text. We are particularly indebted to Dr. R. Mackintosh and his colleagues at the Open University for a host of detailed suggestions. The adoption of this book as the text for the new 'third-level' Open University course on electricity and magnetism led to a careful scrutiny of the first edition by the course team. This has resulted, we believe, in changes which make the book more useful for students. Any errors or obscurities which remain are our responsibility.

Manchester
January, 1990

I. S. GRANT
W. R. PHILLIPS

Preface to the First Edition

This book is based on lectures on classical electromagnetism given at Manchester University. The level of difficulty is suitable for honours physics students at a British University or physics majors at an American University. A-level or high school physics and calculus are assumed, and the reader is expected to have some elementary knowledge of vectors. Electromagnetism is often one of the first branches of physics in which students find that they really need to make use of vector calculus. Until one is used to them, vectors are difficult, and we have accordingly treated them rather cautiously to begin with. Brief descriptions of the properties of the differential vector operators are given at their first appearance. These descriptions are not intended to be a substitute for a proper mathematical text, but to remind the reader what div, grad and curl are all about, and to set them in the context of electromagnetism. The distinction between macroscopic and microscopic electric and magnetic fields is fully discussed at an early stage in the book. It is our experience that students do get confused about the fields **E** and **D**, or **B** and **H**. We think that the best way to help them overcome their difficulties is to give a proper explanation of the origin of these fields in terms of microscopic charge distributions or circulating currents.

The logical arrangement of the chapters is summarized in a flow diagram on the inside of the front cover. Provided that one is prepared to accept Kirchhoff's rules and the expressions for the e.m.f.s across components before discussing the laws on which they are based, the A.C. theory in Chapters 7 and 8 does not require any prior knowledge of the earlier chapters. Chapters 7 and 8 can therefore be used at the beginning of a course on electromagnetism. Sections of the book which are starred may be omitted at a first reading, since they do not contain material needed in order to understand later chapters.

We should like to thank the many colleagues and students who have helped with suggestions and criticisms during the preparation of this book; any errors which remain are our own responsibility. It is also a pleasure to thank Mrs Margaret King and Miss Elizabeth Rich for their rapid and accurate typing of the manuscript.

May, 1974
Manchester, England.

I. S. Grant
W. R. Phillips

Contents

★Starred sections may be omitted as they are not required later in the book.

CHAPTER 1

Force and energy in electrostatics

The only laws of force which are known with great precision are the two laws describing the gravitational forces between different masses and the electrical forces between different charges. When two masses or two charges are stationary, then in either case the force between them is inversely proportional to the square of their separation. These inverse square laws were discovered long ago: Newton's law of gravitation was proposed in 1665, and Coulomb's law of electrostatics in 1785. This chapter is concerned with the application of Coulomb's law to systems containing any number of stationary charges. Before studying this topic in detail, it is worth pausing for a moment to consider the consequences of the law in the whole of physics.

In order to make full use of our knowledge of a law of force, we must have a theory of mechanics, that is to say, a theory which describes the behaviour of an object under the action of a known force. Large objects which are moving at speeds small compared to the speed of light obey very closely the laws of classical Newtonian mechanics. For example, these laws and the gravitational force law together lead to accurate predictions of planetary motion. But classical mechanics does not apply at all to observations made on particles of atomic scale or on very fast-moving objects. Their behaviour can only be understood in terms of the ideas of quantum theory and of the special theory of relativity. These two theories have changed the framework of discussion in physics, and have made possible the spectacular advances of the twentieth century.

It is remarkable that while mechanics has undergone drastic amendment, Coulomb's law has stood unchanged. Although the behaviour of atoms does not fit the framework of the old mechanics. when the Coulomb force is used with the theories of relativity and quantum mechanics, atomic interactions are explained with great precision in every instance when an accurate comparison has been made between experiment and theory. In principle, atomic physics and solid state physics, and for that matter the whole of chemistry, can be derived from Coulomb's law. It is not feasible to derive everything in this way, but it should be borne in mind that atoms make up the world around us, and that its rich variety and complexity are governed by electrical forces.

1.1 ELECTRIC CHARGE

Most of this book applies electromagnetism to large-scale objects, where the atomic origin of the electrical forces is not immediately apparent. However, to emphasize this origin, we shall begin by consideration of atomic systems. The simplest atom of all is the hydrogen atom, which consists of a single proton with a single electron moving around it. The hydrogen atom is stable because the proton and the electron attract one another. In contrast, two electrons repel one another, and tend to fly apart, and similarly the force between two protons is repulsive*. These phenomena are described by saying that there are two different kinds of *electric charge*, and that like charges repel one another, whereas unlike charges are attracted together. The charge carried by the proton is called *positive*, and the charge carried by the electron *negative*.

The magnitude and direction of the force between two stationary particles, each carrying electric charge, is given by Coulomb's law. The law summarizes four facts:

(i) Like charges repel, unlike charges attract.
(ii) The force acts along the line joining the two particles.
(iii) The force is proportional to the magnitude of each charge.
(iv) The force is inversely proportional to the square of the distance between the particles.

The mathematical statement of Coulomb's law is:

$$\mathbf{F}_{21} \propto \frac{q_2 q_1}{r_{21}^3}\mathbf{r}_{21}. \tag{1.1}$$

The vector $\mathbf{F}_{21}$ in Figure 1.1 represents the force on particle 1 (carrying a charge q_1) exerted by the particle 2 (carrying charge q_2). The line from q_2 to q_1 is represented by the vector $\mathbf{r}_{21}$, of length r_{21}: since the unit vector along the direction $\mathbf{r}_{21}$ can be written $\mathbf{r}_{21}/r_{21}$, Equation (1.1) is an ***inverse square law*** **of force**,

*If the protons are separated by a distance less than 10^{-14} m, they are affected by the very short range nuclear forces. Unlike gravitational and electrical forces, nuclear forces are not known precisely. However, in the study of atoms one does not need to know anything about nuclear forces beyond the fact that they are strong enough to bind together the constituent parts of the atomic nucleus.

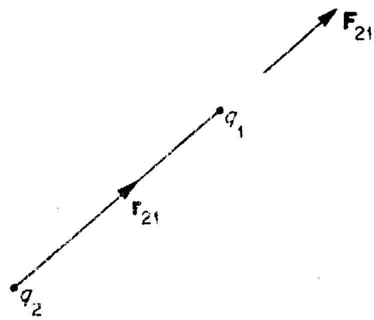

Figure 1.1. The force between two charges.

although r_{21}^3 appears in the denominator. Notice that the equation automatically accounts for the attractive or repulsive character of the force if q_1 and q_2 include the sign of the charge. When the charges q_1 and q_2 are both positive or both negative, the force on q_1 is along $\mathbf{r}_{21}$, i.e. it is repulsive. On the other hand, when one charge is positive and the other negative, the force is in the direction opposite to $\mathbf{r}_{21}$, i.e. it is attractive.

To complete the statement of the force law, we must decide what units to use, and hence determine the constant of proportionality in Equation (1.1). We shall use SI (Système International) units, which are favoured by most physicists and engineers applying electromagnetism to problems involving large-scale objects. A different system of units, called the Gaussian system, is frequently used in atomic physics and solid state physics, and it is an unfortunate necessity for students to become reasonably familiar with both systems. (The two systems of units are discussed in Appendix A.) In SI units, Coulomb's law is written as

$$\boxed{\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r_{21}^3}\mathbf{r}_{21}} \tag{1.2}$$

where

q_1 and q_2 are measured in coulombs,
$\mathbf{r}_{21}$ is measured in metres

and

$\mathbf{F}_{21}$ is measured in newtons.

The magnitude of the unit of charge, which is called the *coulomb*, is actually defined in terms of magnetic forces, and we shall leave discussion of the definition until Chapter 4. The factor 4π in the constant of proportionality in Coulomb's law is introduced in order to simplify some important equations which we shall