
QUANTUM MECHANICS II

A Second Course in Quantum Theory

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A Second Course in Quantum Theory

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PREFACE

*Though a seeker since my birth,
Here is all I've learned on earth,
This is the gist of what I know:
Give advice and buy a foe.*

Phyllis McGinley

This book is a labor of love aimed at helping students see the beauty and unity of physics. It evolved from a second-year graduate course in quantum mechanics (QMII), in which I joked to my students that in one year we covered three or four books on different topics, each of which normally takes a year's course! While at times it seemed difficult to strike a satisfying balance between depth and breath, at some point the material began to adhere into a collection of topics that I believed all graduate students in physical sciences wanted to study before taking research-oriented specialty courses. Whereas much space in the book is given to developing the needed tools, much is also given to developing new ideas about physics and nature.

Officially the purpose of this book is to help experimental and theoretical students—in all disciplines—learn intermediate, graduate-level physics. As such it is not the ultimate research tool or the last word on any subject but rather a vehicle to move a student from pupil to researcher. To achieve this, I have tried to emphasize physical understanding and intuition, starting with the concrete, progressing to the formal, and eventually facing the uncertain.

Accordingly, Part I moves slowly and systematically through the material, while Part III and the optional material (marked with an asterisk) have a sketchiness similar to that found in the research literature.

Scattered through all parts of the text are over 200 exercises to ensure students contribute to their own education (and stay awake) and over 200 multi-part problems that assist in the understanding of the text material, test the reader's understanding and recall, or extend the coverage to fascinating examples (which I had to restrain myself from covering in the text). In addition, some variety is provided by a number of "tutorials" within the chapters that lead the reader through programmed learning on special topics. I do not believe material of this difficulty is understood without the student's commitment to work through these exercises and problems.

This book differs from others in its scope and its design to be accessible to a wide range of students. Whereas most effective when used in conjunction with regular lectures, I tried to keep in mind students throughout the world who need a physics book they can "read." I assumed a familiarity with quantum mechanics at the level of Merzbacher (up to scattering) and familiarity with special relativity and classical electromagnetism at the level of Jackson. There are, however, appendices on momentum states, representations, special relativity, and Dirac equation that can be consulted. As such, a good deal of the material should also be accessible for perusal by advanced undergraduates and practicing scientists.

The material contained in this text is divided into three parts. It is more than enough for a one-year or one-semester course (it all depends on whether the instructor starts at Part I or II and how much of the optional material is included). The emphasis in all parts is toward developing understanding and intuition, which is tested in the world of experimental physics; I have not tried to lay a mathematical foundation through rigorous proofs and definitions. There is nonetheless a slant toward integral formulations, momentum-space techniques, and computational physics since I see this as the modern use of quantum mechanics.

Part I, "Integral Quantum Mechanics and Scattering," is the most concrete and detailed—and most like first-year quantum mechanics. Part II, "Relativistic Quantum Mechanics," while quite captivating, is new, different, and often confusing for students; consequently, I here adopted a more operational point of view. After vigorous exercises burn off the haze of Dirac accounting from Part II, the physics should emerge. Part III, "Quantum Fields," is difficult material rarely mastered in a first-time encounter. I emphasized the physics and connections with familiar quantum mechanics by using the (old-fashioned) perturbation theory, avoiding proofs, "deriving" the Coulomb potential, and examining actual physical processes as proof that field theory works. To me this was the most effective introduction to the subject. Feynman diagrams and the now-standard metric and Dirac matrices of Bjorken and Drell are used.

A detailed description of the chapter-by-chapter contents is found in the

Table of Contents and will not be repeated. I shall, however, give some suggestions regarding rearrangements and omissions.

If often fits in better with the specialty courses of second- and third-year graduate studies to start with the second quantization techniques of Chapter 19, possibly supplemented by its applications in Chapter 20. The course can then begin Part I and afterward pick up the rest of Part II. If the previous QMI course included an adequate study of scattering and its applications, the first four chapters can be passed through quickly or only reviewed. Chapters 5–7 are probably new for most students, and with their emphasis on Green's function techniques in quantum mechanics, are useful for much of modern physics. The description in Chapter 8 of the $|klm\rangle$ basis is valuable for practical problem solving but contributes little to the logical development of the theory. It can be scanned on first reading and then used for reference. Chapters 11 and 12 give some basics and a survey of many-body physics. They fit in well with the preceding formal development, yet are not needed for the logical development (they should be covered somewhere in a graduate curriculum). The instructor may care to enlarge upon it, particularly if second quantization had been covered.

Part II has optional chapters on integral forms of the Dirac equation (17) and on solving even relativistic integral equations (18). This material is usually not found in texts and gives students an understanding of how the Dirac equation and computers are used in modern physics, and it would be a shame if it were never read.

Part III covers the basic subject in Chapters 19–22. The optional Chapter 23 is mind expanding; the reader sees how the Coulomb potential arises as an approximation to the real interaction between two electrons and how similar it is to the nuclear force. I end the book looking at the practical world, the elegant, and the unknown. The first part of Chapter 24 surveys phonons, the second parts surveys the weak interaction, and Chapter 25 examines attempts at relativistic, 2-body wave equations. The use in Chapter 25 of so many of the theoretical tools in this book, along with the beauty, simplicity, and remaining mysteries of these equations, is the climax of the book.

As indicated by the quoted references, hardly any of this book describes my own contributions to physics. I am consequently deeply indebted to those who have put me in contact with this material and have tried to help me to understand it. It is my pleasure to acknowledge my teachers (who have no doubt been plagiarized by having their lectures and problems absorbed beyond recognition into "my" physics), my colleagues, in particular V. Madsen and A. Wasserman for their perceptive and caring comments, F. Tabakin for proselytizing momentum-space techniques, and H. Jansen, J. Milana, D. Griffiths, M. Sagen, J. Schnick, G. He, and P. Fink for their input and assistance, and my students, who have helped me learn. Thanks also goes to the staff of John Wiley & Sons for their encouragement and assistance, in particular, Beatrice Shube, Maria Taylor, and Bob Hilbert.

Essentially all the writing of this text was undertaken at Oregon State University and my home starting in the summer of 1986 and ending in the fall of 1988. Accordingly, I wish to acknowledge the support of the U.S. Department of Energy at Oregon State University and that of my family.

RUBIN H. LANDAU

Corvallis, Oregon

*Your comedy I've read, my friend,
And like the half you pilfer'd best;
But sure the piece you yet may mend:
Take courage, man! and steal the rest.*

Anonymous

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