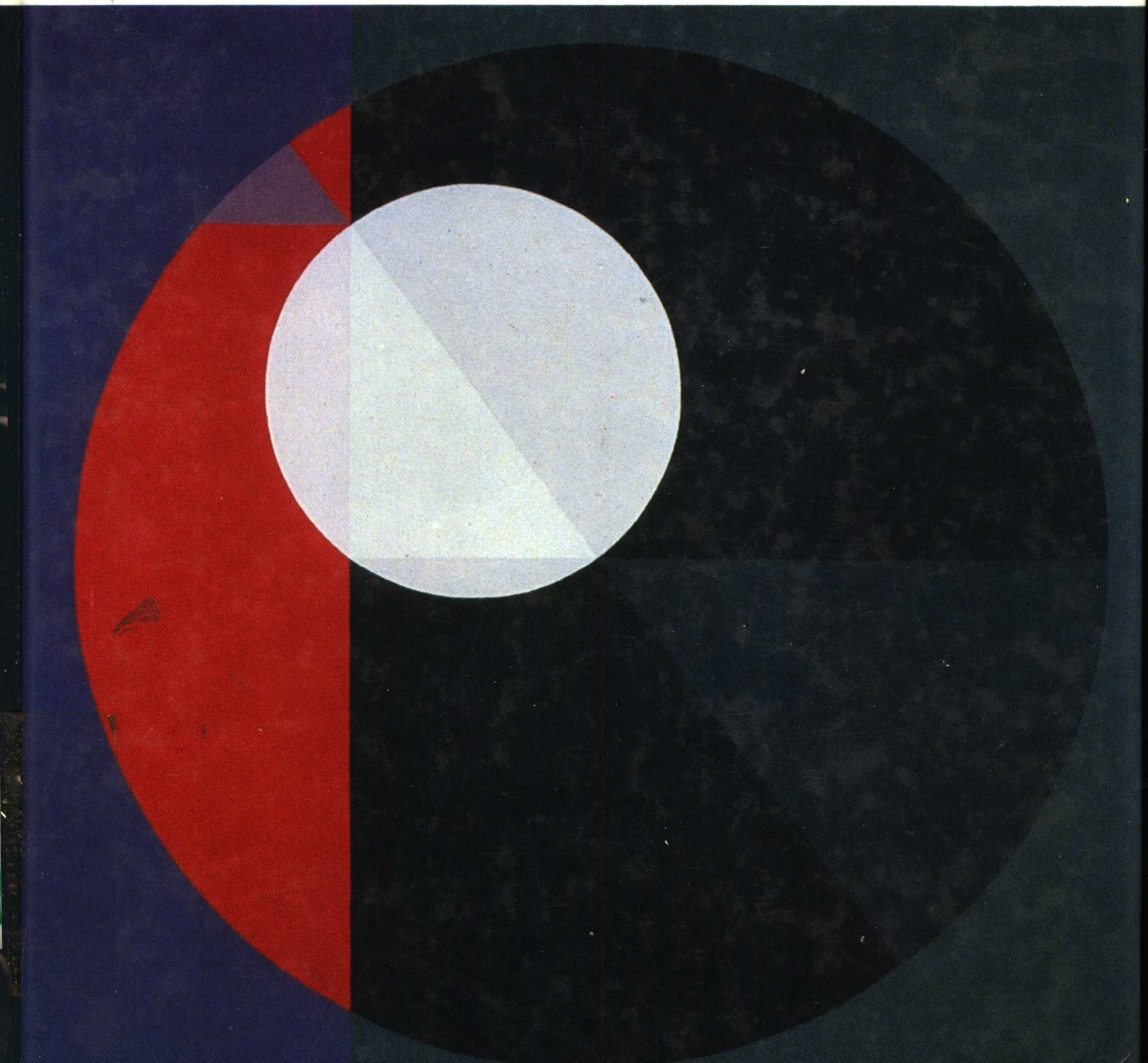
MIZRAHI • SULLIVAN

FINITE MATHEMATICS
WITH APPLICATIONS FOR BUSINESS AND SOCIAL SCIENCES

FIFTH EDITION



FINITE MATHEMATICS WITH APPLICATIONS

FOR BUSINESS AND SOCIAL SCIENCES
FIFTH EDITION

ABE MIZRAHI
INDIANA UNIVERSITY NORTHWEST

MICHAEL SULLIVAN
CHICAGO STATE UNIVERSITY

JOHN WILEY & SONS
NEW YORK • CHICHESTER • BRISBANE • TORONTO • SINGAPORE

Cover painting by: Crockett Johnson, Smithsonian Institute

Library of Congress Cataloging in Publication Data:

Mizrahi, Abe.
Mathematics for business and social sciences: an applied approach / Abe Mizrahi, Michael Sullivan.—4th ed.
p. cm.
Includes index.
ISBN 0-471-85291-0
1. Business mathematics 2. Business Mathematics—Problems, exercises, etc. 3. Social sciences—Mathematics. I. Sullivan, Michael, 1942- II. Title.
HF5691.M59 1988
513′.93—dc19 87-31756
CIP

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

CHANGES TO THE FIFTH EDITION

Chapter 1: The section on linear inequalities now appears in Chapter 3, where it is needed for the geometric approach to linear programming.

Chapter 2: This chapter has been completely reorganized. Systems of linear equations appear early and are solved using the reduced row-echelon method. An application of the method of least squares has been added.

Chapter 3: The chapter now begins with a section on linear inequalities.

Chapter 4: Rewritten to improve readability, this chapter now has a new section on mixed constraints, using the Phase I/Phase II method of solution.

Chapter 5: New examples and exercises have been added and previous ones updated to reflect current market conditions.

Chapter 6: The Multiplication Principle is given more emphasis. More demanding counting techniques are now found in Chapter 8.

Chapter 7: This chapter now concludes with Bayes' Theorem. The Binomial Probability Model is found in Chapter 8.

Chapter 8: This chapter contains more advanced counting techniques, the Binomial Theorem, and the Binomial Probability Model. New models include error correcting codes and mortgage selection.

Chapter 9: A new result, Chebychev's theorem, is included.

Chapter 10: No significant changes.

Chapter 11: A new model on the stock market is included.

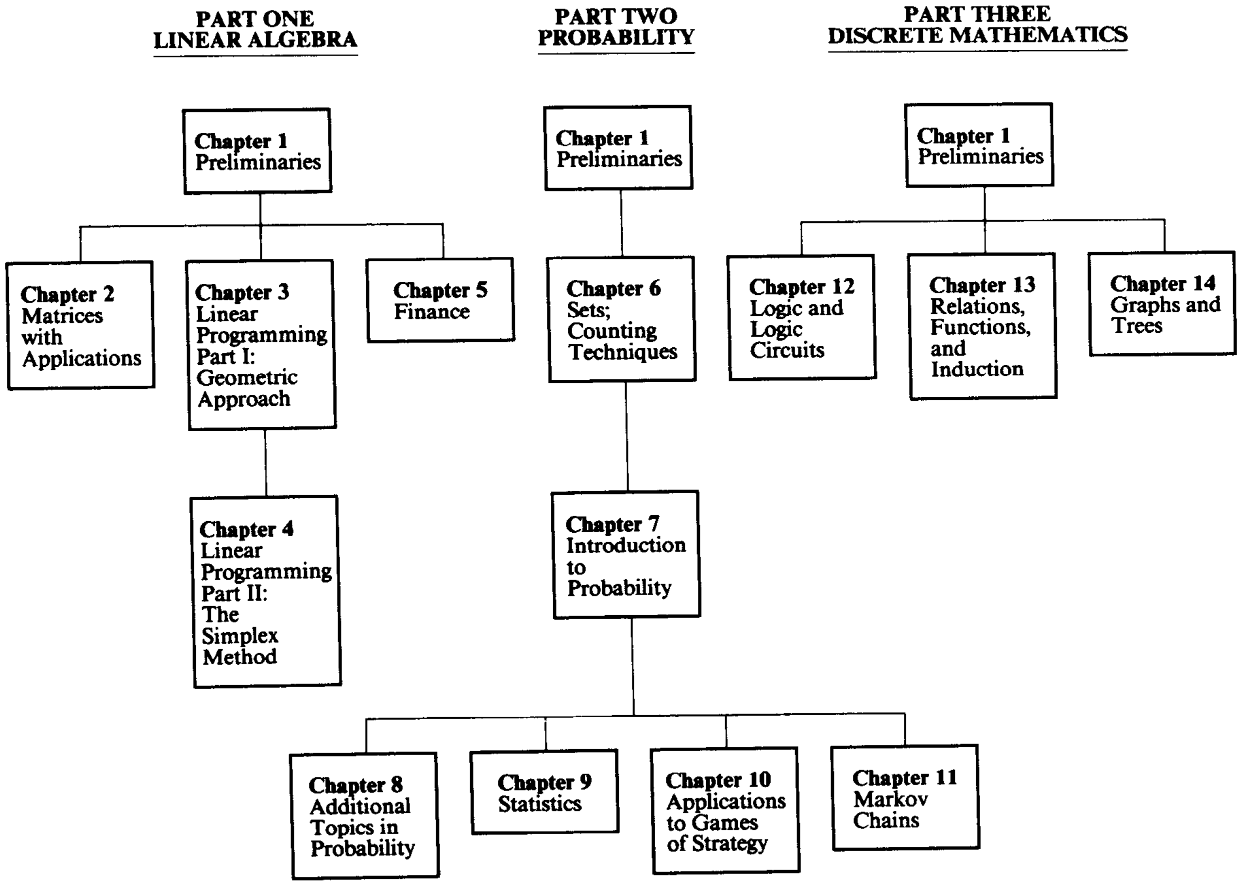
Chapter 12: Rewritten to improve readability, this chapter now contains a full section on circuit design.

Chapters 13 and 14 are completely new to this edition.

The chart on the opposite page illustrates the interaction of the chapters.

FEATURES

* Over 350 illustrative examples.
* Procedures and processes for solving problems are given as a series of steps and are prominantly displayed in boxes. See, for example, page 130 and page 314.
* Over 2000 exercises, ranging from drill to challenging. Many of these are applied-type problems.
* Each chapter contains a review featuring a list of vocabulary from the chapter, True-False and Fill-in-the-blank questions, a collection of Review Exercises, and, when appropriate, Mathematical Questions taken from CPA, CMA, and actuary exams.
* Important Definitions are presented in bold face; formulas and theorems are placed in a box, screened in color.
* Answers to the Odd-Numbered Problems appear in the back of the book.
* Flowcharts are utilized whenever possible to outline procedures. See, for example, page 169.


PART ONE
LINEAR ALGEBRA
PART TWO
PROBABILITY
PART THREE
DISCRETE MATHEMATICS
Chapter 1 Preliminaries
Chapter 2 Matrices with Applications
Chapter 3 Linear Programming Part I: Geometric Approach
Chapter 4 Linear Programming Part II: The Simplex Method
Chapter 5 Finance
Chapter 1 Preliminaries
Chapter 6 Sets; Counting Techniques
Chapter 7 Introduction to Probability
Chapter 8 Additional Topics in Probability
Chapter 9 Statistics
Chapter 10 Applications to Games of Strategy
Chapter 11 Markov Chains
Chapter 1 Preliminaries
Chapter 12 Logic and Logic Circuits
Chapter 13 Relations, Functions, and Induction
Chapter 14 Graphs and Trees

SUPPLEMENTS

* An Instructor's Manual containing worked-out solutions to both the even and the odd-numbered problems, plus a list of books and articles of interest, is available.
* A Student Solutions Manual contains worked-out solutions to the odd-numbered problems.
* Software to Accompany Mizrahi & Sullivan Finite Mathematics with Applications to Business & Life Science. Computer explorations in Finite/Discrete Math is a series of 12 Software activities that enable the user to perform calculations and present graphical representation of mathematical concepts. These activities serve as an interactive tool for teacher demonstrations, reinforce concepts and promote learning by discovery. Available for IBM PC. Free to adopters.
* Computer-Generated Test Bank. This microcomputer testing system contains questions prepared by the authors and is available for Apple, IBM-PC, and most compatibles.

ACKNOWLEDGMENTS

We thank the many students at Indiana University Northwest and Chicago State University for their comments and criticisms.

We also thank these reviewers for their suggestions and contributions to this and earlier editions:

Professor Judy Barclay	Cuesto College
Professor Charles Barker	DeAnza College
Professor Carole Bernett	William Rainey Harper College
Professor William Blair	Northern Illinois University
Professor W. J. Bonini	Western Wyoming College
Professor Gary G. Cochell	Calver-Stockton College
Professor Portia Cornell	University of Redlands
Professor Milton D. Cox	Miami University
Professor Neale Fadden	Belleville Area College
Professor Eugene Franks	Dyke College
Professor James C. Fraventhal	SUNY at Stony Brook
Professor Dennis Freeman	Montgomery College
Professor Samuel Graff	John Jay College of Criminal Justice
Professor Gerald Hahn	College of St. Thomas
Professor Joseph Hansen	Northeastern University
Professor Herbert Hethcote	University of Iowa
Professor Brian Hickey	First Central College
Professor Wayne Hiles	Kankakee Community College

Professor Roy Honde	West Valley College
Professor Martin Kotler	Pace University
Professor James E. Lightner	Western Maryland College
Professor Rick Lottmann	Sonoma State College
Professor Aima McKinney	North Florida Junior College
Professor Jeffrey McLean	College of St Thomas
Professor Kishore Marathe	Brooklyn College
Professor Charles J. Miller	Foothill College
Professor Robert Muksian	Bryant College
Professor Henry J. Osner	Modesto Junior College
Professor Willard A. Parker	Kansas State University
Professor Elaine Pavelka	Morton College
Professor Carlos A. Pereira	The King's College
Professor Dana Piens	Rochester Community College
Professor George H. Potter	Schenectady Community College
Professor Thomas Radin	San Juaquin Delta Community College
Professor Eric E. Robinson	Ithaca College
Professor Gerald Root	American International College
Professor Arthur Schlissel	John Jay College of Criminal Justice
Professor Charles Sheumaker	University of S.W. Louisiana
Professor Carol Slinger	Marian College
Professor Richard Stansfield	Jefferson College
Professor R. Staum	Kings Borough Community College
Professor Olive D. Taylor	Meredith College
Professor Elizabeth E. Taylor	University of Richmond
Professor Luther Truell	Victoria College
Professor Ernest True	Norwich College
Professor Jack Wadhaus	Golden West College
Professor Hubert Walezak	College of St. Thomas
Professor Robert Wheeler	Northern Illinois University
Professor John L. Whitcomb	University of North Dakota
Professor John A. Winn	Hofstra University
Professor M. Yablon	John Jay College/CUNY

We are deeply grateful to Professors Leroy Peterson and George Zazi for their major contributions to Chapters 12, 13, and 14.

We especially thank William Hosch, Marsha Vihon, and Henry Wyzinsky, who read the manuscript in its entirety and made valuable suggestions.

We also appreciate the skill and patience of Lorraine Bierdz and Mary Jane Wilcox, who typed the revision.

We are grateful to the following organizations who permitted the reproduction of actual questions from previous professional examinations.

American Institute of Certified Public Accountants (CPA Exams)

Educational Testing Service (Actuary Exams)

Institute of Management Accounting of the National Association of Accountants (CMA Exams)

Again we are indebted to the people at John Wiley and at Hudson River Studios, whose talent played a significant part in the publication of this book. We particularly want to single out Ed and Lorraine Burke, Carolyn Moore, and Melissa Van Hise for their assistance.

This is truly a collaborative effort, and the order of authorship signifies alphabetical precedence. We assume equal responsibility for the book's strengths and weaknesses, and welcome comments and suggestions for its improvement.

Abe Mizrahi
Michael Sullivan

ABOUT THE AUTHORS

Abe Mizrahi, received his doctorate in mathematics from the Illinois Institute of Technology in 1986. He is currently Professor of Mathematics at Indiana University Northwest. Professor Mizrahi is the author of other mathematic books and is a member of the Mathematical Association of America. Articles of Professor Mizrahi dealt with topics in math education and the applications of mathematics to economics. Professor Mizrahi served on many CUPM committees, was a panel member on CUPM committee on applied mathematics in the undergraduate curriculum. Professor Mizrahi is a recipient of many NSF grants and served as a consultant to a number of businesses and Federal agencies.

Michael Sullivan, received his Ph.D. degree in mathematics from the Illinois Institute of Technology in 1967. Since 1965 he has been teaching at Chicago State University in the Department of Mathematics and Computer Science, holding the rank of Professor. Professor Sullivan and Professor Mizrahi are co-authors of *Mathematics for Business and Social Science,* 4th Edition, John Wiley and Sons, 1988 as well as *Calculus with Analytic Geometry,* 2nd Edition, Wadsworth Publishing Company, 1986. Professor Sullivan has also written *College Algebra* and *College Algebra and Trigonometry,* Dellen/Macmillan, 1987. Dr. Sullivan is a member of the American Mathematical Society, the Mathematics Association of America, and Sigma Xi. He has served on CUPM Curriculum Committees and is a member of the Illinois Section MAA High School Lecture Committee.

CONTENTS

PART ONE: LINEAR ALGEBRA 1

1 PRELIMINARIES 2

1.1 Real Numbers 3
1.2 Rectangular Coordinates 13
1.3 The Straight Line 16
1.4 Parallel and Intersecting Lines 24
1.5 Applications 32
Chapter Review 39
MATHEMATICAL QUESTIONS FROM CPA AND CMA EXAMS 42

2 MATRICES WITH APPLICATIONS 45

2.1 Systems of Linear Equations 46
2.2 Solving Systems of *m* Linear Equations in *n* Unknowns 56
2.3 Matrix Algebra 65
2.4 Multiplication of Matrices 75
2.5 Inverse of a Matrix 82
2.6 Applications 91
Chapter Review 112

3 LINEAR PROGRAMMING PART I: GEOMETRIC APPROACH 115

3.1 Introduction 116
3.2 Linear Inequalities 116
3.3 A Geometric Approach to Linear Programming Problems 125
Chapter Review 139
MATHEMATICAL QUESTIONS FROM CPA AND CMA EXAMS 141

4 LINEAR PROGRAMMING PART II: THE SIMPLEX METHOD 148

4.1 The Simplex Tableau: Pivoting 149
4.2 The Simplex Method: The Maximum Problem 160
4.3 The Simplex Method: The Minimum Problem 175
4.4 The Simplex Method with Mixed Constraints: Phase I/Phase II 187
Chapter Review 201
MATHEMATICAL QUESTIONS FROM CPA EXAMS 202

5 FINANCE 207

5.1 Simple Interest and Simple Discount 208
5.2 Compound Interest 211
5.3 Annuity; Sinking Fund 217
5.4 Present Value of an Annuity; Amortization 221
5.5 Leasing; Capital Expenditure; Bonds 227
Chapter Review 230
MATHEMATICAL QUESTIONS FROM CPA EXAMS 232

PART TWO: PROBABILITY 235

6 SETS; COUNTING TECHNIQUES 236

6.1 Sets 237
6.2 Multiplication Principle 252
6.3 Permutations 257
6.4 Combinations 262
Chapter Review 267

7 INTRODUCTION TO PROBABILITY 271

7.1 Introduction 272
7.2 Sample Spaces and Assignment of Probabilities 275
7.3 Properties of the Probability of an Event 286
7.4 Probability for the Case of Equally Likely Events 294
7.5 Conditional Probability 301
7.6 Independent Events 310
7.7 Bayes' Formula 316
Chapter Review 326
MATHEMATICAL QUESTIONS FROM ACTUARY EXAMS 329

8 ADDITIONAL TOPICS IN PROBABILITY 331

8.1 More Counting Techniques 332
8.2 The Binomial Theorem 341

8.3 Binomial Probability Model 347
8.4 Expectation 357
8.5 Applications 366
Chapter Review 372
MATHEMATICAL QUESTIONS FROM CPA AND ACTUARY EXAMS 375

9 STATISTICS 378

9.1 Introductory Remarks 379
9.2 Organization of Data 381
9.3 Measures of Central Tendency 388
9.4 Measures of Dispersion 394
9.5 Normal Distribution 400
Chapter Review 410
MATHEMATICAL QUESTIONS FROM ACTUARY EXAMS 412

10 APPLICATIONS TO GAMES OF STRATEGY 413

10.1 Introduction 414
10.2 Mixed Strategies 418
10.3 Optimal Strategy in Two-Person Zero-Sum Games with 2 × 2 Matrices 421
10.4 Optimal Strategy in Other Two-Person Zero-Sum Games Using Geometric Methods 428
10.5 Two-Person Nonzero-Sum Games 438
Chapter Review 441

11 MARKOV CHAINS 443

11.1 An Introduction to Markov Chains 444
11.2 Regular Markov Chains 452
11.3 Absorbing Markov Chains 462
11.4 An Application to Genetics 471
Chapter Review 475

PART THREE: DISCRETE MATHEMATICS 478

12 LOGIC AND LOGIC CIRCUITS 479

12.1 Propositions 480
12.2 Truth Tables 485
12.3 Implications; Biconditional Propositions; Tautologies 493
12.4 Arguments 501
12.5 Logic Circuits 507
Chapter Review 512

13 RELATIONS, FUNCTIONS, AND INDUCTION 515

13.1 Relations 516
13.2 Functions 523
13.3 Sequences 531
13.4 Algorithms 535
13.5 Mathematical Induction 538
13.6 Recurrence Relations 546
Chapter Review 552

14 GRAPHS AND TREES 555

14.1 Graphs 556
14.2 Paths and Connectedness 561
14.3 Eulerian and Hamiltonian Circuits 566
14.4 Trees 573
14.5 Directed Graphs 581
Chapter Review 589

TABLES 592

ANSWERS TO ODD-NUMBERED PROBLEMS 599

INDEX 649

PART ONE

LINEAR ALGEBRA

1. PRELIMINARIES
2. MATRICES WITH APPLICATIONS
3. LINEAR PROGRAMMING PART I: GEOMETRIC APPROACH
4. LINEAR PROGRAMMING PART II: THE SIMPLEX METHOD
5. FINANCE

1

PRELIMINARIES

1.1 REAL NUMBERS
1.2 RECTANGULAR COORDINATES
1.3 THE STRAIGHT LINE
1.4 PARALLEL AND INTERSECTING LINES
*1.5 APPLICATIONS
CHAPTER REVIEW
MATHEMATICAL QUESTIONS FROM CPA AND CMA EXAMS

* This section may be omitted without loss of continuity.

1.1 REAL NUMBERS

SETS □ SETS OF NUMBERS □ DECIMALS AND PERCENTS □ PROPERTIES OF REAL NUMBERS □ POSITIVE AND NEGATIVE NUMBERS □ COORDINATES □ VARIABLES □ INEQUALITIES

SETS

Set

Empty Set

We begin with the idea of a *set*. A *set* is a collection of objects considered as a whole. The objects of a set S are called *elements* of S, or *members* of S. The set that has no elements, called the *empty set* or *null set*, is denoted by the symbol $\varnothing$.

If a is an element of the set S, we write $a \in S$, which is read "a is an element of S" or "a is in S." To indicate that a is not an element of S, we write $a \notin S$, which is read "a is not an element of S" or "a is not in S."

Ordinarily, a set S can be written in either of two ways. These two methods are illustrated by the following example: Consider the set D that has the elements

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

In this case, we write

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

This expression is read "D is the set consisting of the elements 0, 1, 2, 3, 4, 5, 6, 7, 8, 9." Here, we list or display the elements of the set D.

Another way of writing this same set D is to write

$$D = \{x | x \text{ is a nonnegative integer less than } 10\}$$

This is read "D is the set of all x such that x is a nonnegative integer less than 10." Here, we have described the set D by giving a property that every element of D has and that no element not in D can have.

SETS OF NUMBERS

Counting Numbers

One of the most frequently used set of numbers in finite mathematics is the set of *counting numbers*, namely

$$\{1, 2, 3, 4, \ldots\}$$

These are sometimes referred to as *positive integers*. As the name *counting numbers* implies, they are used to count things. For example, there are 26 letters in our alphabet, and there are 100 cents in a dollar.

Integers

Another important collection of numbers is the set of *integers*,

$$\{0, 1, -1, 2, -2, \ldots\}$$

which consists of the *nonnegative integers*,

$$\{0, 1, 2, 3, \ldots\}$$

and the *negative integers*,

$$\{-1, -2, -3, \ldots\}$$

Integers enable us to handle certain types of situations. For example, if a company shows a loss of \$3 per share, we might decide to denote this loss as a gain of $-\$3$. Then, if this same company shows a profit of \$4 per share next year, the profit over the 2 year period can be obtained by adding -3 to 4, getting a profit of \$1.

However, integers do not enable us to solve *all* problems. For example, can we use an integer to answer the question, "What part of a dollar is 49¢?" Or, can we use an integer to represent the length of a city lot (in feet) if we end up with a length more than 125 feet and less than 126 feet?

The answer to both these questions is "No"! We need new or different numbers to handle such situations. These new numbers are called *rational numbers.*

Rational Numbers

Rational numbers are ratios of integers. More specifically, for a rational number $\frac{a}{b}$, the integer a is called the *numerator,* and the integer b, which cannot be zero, is called the *denominator.*

Thus to answer the first question, "What part of a dollar is 49¢?" we can say $\frac{49}{100}$.

Examples of rational numbers are $\frac{3}{4}, \frac{5}{3}, -\frac{2}{7}, \frac{100}{3}, -\frac{8}{3}$. Since the ratio of any integer to 1 is that integer, the integers are also rational numbers. Thus, $\frac{3}{1} = 3, -\frac{2}{1} = -2, \frac{0}{1} = 0$.

In this book, we will always write rational numbers in *lowest terms;* that is, the numerator and the denominator will contain no common factors. Thus, $\frac{4}{6}$, which is not in lowest terms, will be written as $\frac{2}{3}$, which is in lowest terms.

In some situations, even a rational number will not accurately describe the situation. For example, if you have an isosceles right triangle in which the two equal sides are 1 foot long, can the length of the third side be expressed as a rational number? See Figure 1.

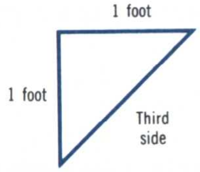


Figure 1

And how about the value of π (Greek letter "pi")? The Greeks first learned that no matter what two circles are used, the ratio of the circumference to the diameter of the first circle is always the same as the ratio of the circumference to the diameter of the second one. Can this common value, which we call π, be represented by a rational number? See Figure 2.

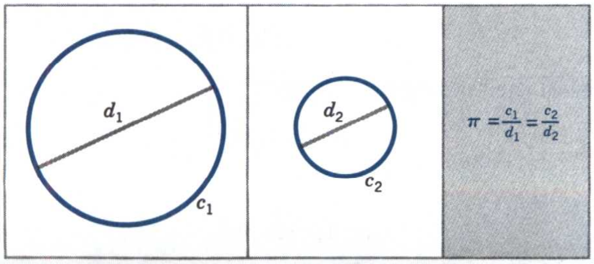


Figure 2