MODERN TRANSMISSION LINE THEORY AND APPLICATIONS

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Preface

The study of transmission line theory in electrical engineering curricula is usually limited to considerations of the circuit properties of lengths of transmission line with given properties. Unfortunately, this is no longer adequate. There is a continuing effort in the electronics industry to build smaller circuits that operate at a higher frequency, or faster, than last year's version. Microwave integrated circuit technology design techniques are tending more and more toward stripline or microstrip designs on high dielectric constant ceramic substrates. The result of this trend, insofar as the circuit designer is concerned, is that almost every interconnection in a circuit will exhibit transmission line properties. An immediate corollary is that if circuits are to be well designed, the transmission lines in the circuit must be appropriately treated as part of the circuit. It is no longer possible to separate the transmission line user from the transmission line designer–not only are the lines present, but their properties are functions of the circuit layout itself.

The purpose of this book is to extend the initial treatment of transmission line theory received by most electrical engineers to the point where transmission line effects can be properly considered, and transmission line properties can be calculated as a function of materials and geometries. Properties of stripline and microstrip circuits are emphasized, since these are the two line types that emerge naturally in the microwave integrated circuit-printed circuit layout.

This book is intended for students and engineers who have had some exposure to transmission line theory. Although the properties of transmission lines are derived from basic considerations, these derivations are brief–being intended as a review–and the underlying justifications are assumed to be understood. That is, it is assumed that the reader is familiar with the concepts of distributed circuits, wave propagation, and the constant interplay between field variables and circuit variables that takes place in descriptions of distributed circuits.

Once the circuit designer has learned to treat the transmission lines in the circuit properly, it becomes possible to take advantage of the different circuit functions that are realizable in single or coupled transmission line form. These functions include directional couplers, coupled line filters, and tapered line impedance transformers. Since microwave integrated circuits are fabricated using some form of printed circuit technology, circuit functions obtained by using the transmission lines that are printed along with the connecting lines are very economical and repeatable, hence desirable.

An important tool of the transmission line designer is the digital computer. The only practical way to design or analyze arbitrary transmission line geometries is by means of some numerical procedure using a computer. Therefore numerical approaches to transmission line analysis are treated in great detail, with many examples. On the other hand, no attempt has been made to treat exhaustively the number of different numerical approaches available. Instead, several approaches were followed through many examples so that the reader can see these approaches being applied to varying types of problems. In this way there is a reasonable chance that some example resembles an actual problem at hand and that a solution procedure, though possibly not the optimum one, can be found for most problems.

At the end of each chapter is an annotated reading list. The comments should help the interested reader to locate quickly a reference providing more detail on a topic of interest than is contained on these pages.

I thank my wife, Tamara, and my colleague, Dr. Melvyn Slater, for their help in structuring and reviewing the pages that follow. Without this help I could not have completed the job.

LAWRENCE N. DWORSKY

Fort Lauderdale
August 1979

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The Transmission Line Equations

The electrical transmission line is an example of a one-dimensional propagating electromagnetic wave system. The equations governing this system may be derived from Maxwell's equations, either directly or from a circuit theory point of view. Although both these derivations lead to the same result it is instructive to examine them consecutively so that the two approaches can be compared.

1.1 THE TRANSMISSION LINE: A DEFINITION

A transmission line can be rigorously defined as any structure that guides a propagating electromagnetic wave from point *a* to point *b*. In other words, we can regard a transmission line as a set of boundary conditions to Maxwell's equations that allow the description of a one-dimensional propagating wave between two points.

The common use of the term "transmission line" is far more restrictive. It is usually required that the electrical length of the (transmission) line be at least several percent of a wavelength at the highest frequency of interest. Also, wave guides are excluded. That is, we require that the line propagate a signal at all frequencies from the frequency of interest down to *and including* dc, with the line characteristics varying in a smooth and continuous manner over this frequency range.

The statement above requires further discussion. At dc, an ideal (lossless) transmission line is surrounded by electric and magnetic fields that are normal both to each other and to the direction of energy propagation. This is the common transverse electromagnetic (TEM) mode of propagation. This is not to say, however, that a transmission line must propagate a signal in the TEM mode. As is shown below, there are several types of transmission line that cannot support TEM waves at frequencies other than zero. This situation arises

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Symbol	Common Name	TEM Type?	Transmission Line?
 (a)	Coaxial cable	Yes	Yes
 (b)	Stripline	Yes	Yes
 (c)	Balanced two-wire line	Yes	Yes
 (d)	Microstrip	No	Yes
 (e)	Slotline	No	Yes
 (f)	Rectangular wave guide	No	No

Figure 1 Six common structures for one-dimensional wave propagation.

when there is an inhomogeneous dielectric cross section of the line normal to the direction of propagation. The transmission line definition given above is specifically constructed to include lines of these types while excluding conventional wave guides.

Figure 1 shows cross-sectional views of six types of one-dimensional wave guiding structures. The names usually associated with these (and many other) wave guiding structures came about through the first applications of these structures, and unfortunately often bear very little descriptive relation to the structures themselves.

1.2 THE TRANSMISSION LINE EQUATIONS FROM MAXWELL'S EQUATIONS

To find a solution to Maxwell's equations specific enough to be compared with a set of circuit equations, a specific example must be chosen. This example may be any TEM system, or with certain approximations, any transmission line. As a simple example, consider the coaxial cable shown in Figure 2. In most practical cases the only propagating mode in coaxial cable is the TEM mode, and only this mode is considered here.

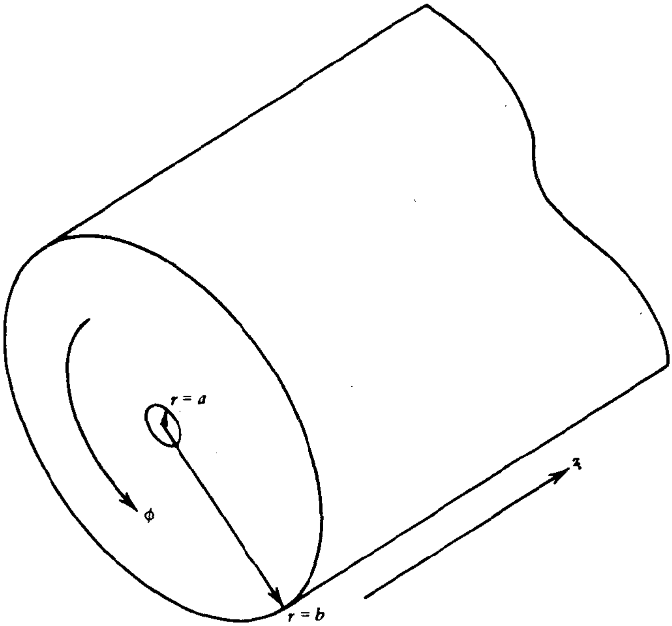

Figure 2 Coaxial cable cross section.

Assume that all of space is linear, isotropic, and homogeneous. Maxwell's equations are then

$$\nabla \cdot \mathbf{D} = \rho \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.4}$$

where **D** = electric displacement vector
B = magnetic flux vector
E = electric field intensity vector
H = magnetic field intensity vector
J = electric current density vector
ρ = space charge density

Since the wave is propagating through a dielectric,

$$\rho = 0 \tag{1.5}$$

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Also,

$$\mathbf{B} = \mu \mathbf{H} \tag{1.6}$$

$$\mathbf{D} = \epsilon \mathbf{E} \tag{1.7}$$

where μ and ϵ are the (scalar) permeability and permittivity, respectively, of the material.

Substituting the relationships above into Maxwell's equations, we have

$$\nabla \cdot \mathbf{E} = 0 \tag{1.8}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{1.9}$$

Since in the notation of Figure 2 the wave is propagating in the z direction, both **E** and **H** must be in the plane normal to z. Therefore, from the described geometry

$$\mathbf{E} = \mathbf{a}_r E_r(r, t) \tag{1.10}$$

$$\mathbf{H} = \mathbf{a}_\varphi H_\varphi(r, t) \tag{1.11}$$

where $\mathbf{a}_r$ and $\mathbf{a}_\varphi$ are unit vectors.

Substituting (1.10) and (1.11) into (1.4), in cylindrical coordinates,

$$\nabla \times \mathbf{a}_\varphi H_\varphi(r, t) = -\mathbf{a}_r \frac{\partial H_\varphi}{\partial z} + \mathbf{a}_z \frac{1}{r} \frac{\partial}{\partial r}(rH_\varphi) = \mathbf{a}_r \epsilon \frac{\partial E_r}{\partial t} \tag{1.12}$$

For a TEM mode, the z-directed term in (1.12) must vanish. This is because $H_\varphi \sim 1/r$, therefore $\partial(rH_\varphi)/\partial r = 0$.

Equation 1.12 is now reduced to the scalar equation

$$\frac{\partial H_\varphi}{\partial z} = -\epsilon \frac{\partial E_r}{\partial t} \tag{1.13}$$

Integrating both sides of (1.13) about a circular path of radius r, where $a < r < b$, we have

$$\frac{\partial}{\partial z} \int_0^{2\pi} H_\varphi r \, d\varphi = -\frac{\partial}{\partial t} \int_0^{2\pi} \epsilon E_r r \, d\varphi \tag{1.14}$$

By Ampere's law the above then can be written as

$$\frac{\partial I}{\partial z} = -\frac{\partial}{\partial t} \int_0^{2\pi} \epsilon E_r r \, d\varphi \tag{1.15}$$

where I is the total current enclosed by the integration path—that is, the current flowing in the center conductor of the coaxial cable.

Multiplying and dividing the right-hand side of (1.15) by a length h, and using Gauss' law, yields

$$\frac{\partial I}{\partial z} = \frac{-1}{h}\frac{\partial}{\partial t}\int_0^{2\pi} \epsilon E_r rh \, d\varphi = -\frac{\partial}{\partial t} q_h \tag{1.16}$$

where q_h is the charge per unit length enclosed by the cylindrical volume of length h and radius r. Since there is no free charge in the dielectric, this charge must reside on the center conductor.

Let us define the capacitance per unit length of the coaxial cable as C, where $C = q_h/V$. Since the electric field lines originating on the center conductor terminate on the outer conductor, V is the voltage between the inner and outer conductors, measured at any given z.

Rewriting (1.16) in terms of I, C, and V, we have

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \tag{1.17}$$

Equation 1.17 is the first of the two transmission line equations. It couples the two variables I and V. A second equation is necessary to solve for these variables. This second transmission line equation is found by substituting (1.10) and (1.11) into (1.3). As above, in cylindrical coordinates,

$$\nabla \times \mathbf{a}_r E_r = \mathbf{a}_\varphi \frac{\partial E_r}{\partial z} - \mathbf{a}_z \frac{1}{r}\frac{\partial E_r}{\partial \varphi} = -\mathbf{a}_\varphi \mu \frac{\partial H_\varphi}{\partial t} = -\mathbf{a}_\varphi \frac{\partial B_\varphi}{\partial t} \tag{1.18}$$

The z-directed term in (1.18) must be zero from symmetry considerations. This leaves the scalar equation

$$\frac{\partial E_r}{\partial z} = -\frac{\partial B_\varphi}{\partial t} \tag{1.19}$$

Integrating (1.19) along a radial path from $r = a$ to $r = b$, and at the same time multiplying and dividing the right-hand side by the length h, yields

$$\frac{\partial}{\partial z}\int_a^b E_r \, dr = \frac{-1}{h}\frac{\partial}{\partial t}\int_a^b hB_\varphi \, dr \tag{1.20}$$

The left-hand side of (1.20) can be identified as $\partial V/\partial z$, while the right-hand side is $\partial \Phi_h/\partial t$, where Φ_h is the total magnetic flux per unit length passing through the rectangle of length h and width $b - a$. Defining the inductance per unit length of the cable as L, where $L = \Phi_h/I$, (1.20) becomes

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \tag{1.21}$$

Equation 1.21, the second required transmission line equation, also couples the variables I and V. Equations 1.21 and 1.17, together with the appropriate boundary conditions, provide a complete description of the voltage and current waves as they propagate along the transmission line.

Before examining the same transmission line from a circuit theory viewpoint, let us consider some of the implications of the derivation above. If the inner and outer conductors of the coaxial cable are perfect conductors, no electric fields may exist within them. Therefore the transverse electric field E_r must exist only between the outer surface of the inner conductor and the inner surface of the outer conductor. In other words, E_r exists only in the dielectric region separating the two conductors.

Similarly, since the current flowing in the conductors exists only on the outer surface of the inner conductor and the inner surface of the outer conductor, the magnetic field H_φ must exist only in the dielectric region between the conductors.

Thus it has been said that when dealing with perfect conductors, current flows only along the surfaces of the conductors at which a magnetic field is present, and an electric field will exist only between the charged surfaces at which this electric field terminates. In the case of real metals, which are usually very good but not perfect conductors, the electric and magnetic fields will be shown to penetrate the surfaces slightly. Similarly, the currents will be shown to flow in a thin "skin" at and just below the same surfaces.

Poynting's vector, $\mathbf{E} \times \mathbf{H}$, predicts the power flow in the dielectric. Integrating Poynting's vector over the cross-sectional area of the dielectric yields the total power flow across any cross section of the line. This result must, of course, agree with the power flow calculation that is obtained from the voltage and current. The voltage-current calculation, however, does not bring out the point that the power is flowing in the dielectric–not in the conductors. Restating this observation, electric power does not flow in "wires"; rather, it flows in the fields surrounding the wires. As the coaxial cable example above demonstrated, the wires provide the boundary conditions for establishing a one-dimensional TEM wave solution to Maxwell's equations–that is, they guide the power flow.

1.3 THE TRANSMISSION LINE EQUATIONS FROM KIRCHHOFF'S EQUATIONS

The laws of circuit theory, Kirchhoff's voltage and current laws, can be used to derive the transmission line equations. To accomplish this, it is necessary to visualize a transmission line as a chain of discrete inductors and capacitors very closely spaced in a lattice structure. Figure 3 presents two such structures.

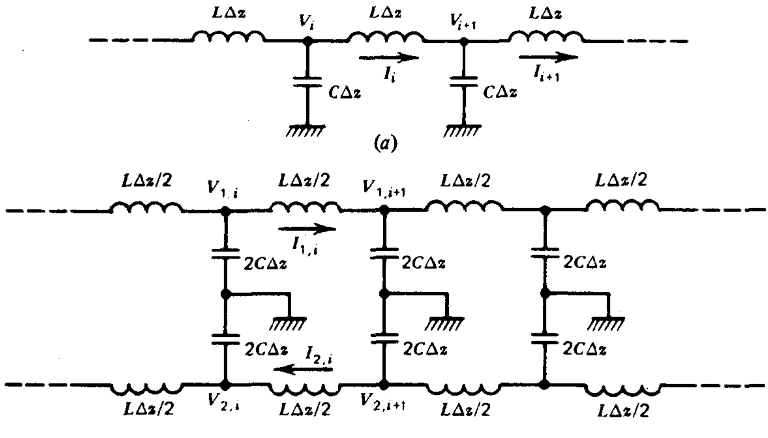

Figure 3 Balanced (*b*) and unbalanced (*a*) circuit models of a transmission line.

When a transmission line consists of two symmetrical conductors and a separate ground, or voltage reference, the transmission line is said to be balanced (see, e.g., Figure 1*c*). In this case if the pair of conductors has a total inductance per unit length of L, the inductance per unit length of each conductor must be $L/2$. If the capacitance per unit length between the conductors is C, to establish a ground reference symmetrically between the conductors, the capacitance per unit length to ground of each conductor is $2C$. Note that the separate ground often is not shown explicitly, as is the case in Figure 1*c*.

If the transmission line is represented by a chain of series inductors of value $L\Delta z/2$ and a chain of shunt capacitors of value $2C\Delta z$, the line can be approximated by the circuit of Figure 3*b*. The nodes are separated by the small distance Δz, and the node subscript i locates the node on the line according to $z = i(\Delta z)$.

In many cases it is convenient to consider one of the conductors of a transmission line as the voltage reference. This is particularly true when the geometry of the line causes the inductance per unit length to be much greater in one conductor than in the other. The conductor with the relatively small inductance is then taken as the voltage reference. When this is done, the total series inductance per unit length and the total shunt capacitance per unit length must be ascribed to the nonreference, or "ungrounded" conductor. Figure 3*a* depicts a transmission line, again as a chain of incremental elements. The most common example of this type of line is the coaxial cable. The outer conductor's inductance is much smaller than the inner conductor's inductance. The outer conductor is therefore chosen as the ground or reference line.

The two choices described above and shown in Figure 3 are often referred to as "balanced" and "unbalanced" transmission lines. This is because the voltages on each wire in the balanced line (Figure 3*b*) are equal in magnitude and opposite in sign when referred to the ground (reference) line, whereas the voltage on the nonreference wire of the unbalanced line (Figure 3*a*) is the only nonzero voltage present.

The circuit equations that follow apply to both the balanced and the unbalanced lines. Consequently, there is no need to designate either case in the discussion.

Referring to Figure 3, the voltage drop between nodes i and $i + 1$ is

$$V_{i+1} - V_i = -L\Delta z \frac{\partial I_{i+1}}{\partial t} \tag{1.22}$$

The current through the capacitor at node i is

$$I_i - I_{i+1} = C\Delta z \frac{\partial V_i}{\partial t} \tag{1.23}$$

Rearranging the two equations above into a more convenient form, we write

$$\frac{V_{i+1} - V_i}{\Delta z} = - L \frac{\partial I_{i+1}}{\partial t} \tag{1.24}$$

$$\frac{I_{i+1} - I_i}{\Delta z} = - C \frac{\partial V_i}{\partial t} \tag{1.25}$$

Assume that the increment Δz approaches zero. The left-hand side of both (1.24) and (1.25) would approach partial derivatives with respect to z because as Δz gets smaller, the $(i + 1)$th node is getting closer to the ith node. In the limit, (1.24) and (1.25) are identical to (1.17) and (1.21), namely,

$$\frac{\partial I}{\partial z} = - C \frac{\partial V}{\partial t} \tag{1.17}$$

$$\frac{\partial V}{\partial z} = - L \frac{\partial I}{\partial t} \tag{1.21}$$

When the ground return (voltage reference) path completely encloses the other wire(s) of a transmission line, the line is referred to as a shielded transmission line. In this case all electric and magnetic field lines terminate within the confines of the line cross section. Conversely, a balanced line can depend on the outside world for ground return paths. In this case, the line is unshielded and the field lines may extend an undeterminable distance from the line.

A distinction should be made at this point between an unshielded balanced line—even with ground returns only at infinity—and an antenna. Whereas a balanced, unshielded line has electric and magnetic fields that extend indefinitely, all power flow is along the line with no component normal to the line. This one-dimensional TEM wave is the proper transmission line wave. If, on the other hand, there were to exist a discontinuity or irregularity along the line, the fields may be disturbed in a manner that would cause radiation normal to the line. In this case the line would be acting, at least partially, as an antenna. It must be emphasized that this is a flaw condition, not the nature of the system.

In the case of the balanced line, a disturbance that might cause radiation could be either on the line or somewhere nearby—remember that the fields from an unshielded balanced line extend indefinitely. Since, in general, it is not possible to control the "neighborhood" through which a line must pass, a shielded line is usually preferable to avoid spurious radiation. On the other hand, since no dielectric is totally loss-free, when the "neighborhood" of a line can be controlled and line loss is intolerable, an open air, two-wire, balanced line is usually the optimum choice. As an example of this situation, consider the feed line to a transmitter on the top of a mountain, with the feed wires strung above the tree tops between towers erected specifically for this purpose.

1.4 LOW PASS FILTERS AND SIMULATED TRANSMISSION LINES

The cascade of series inductors and shunt capacitors treated in Section 1.3 can be redrawn as a cascade of identical T sections (Figure 4). For simplicity's sake, only the unbalanced line is shown. Again, the arguments pertain to both balanced and unbalanced lines. Each of the T sections (in Figure 4) can be recognized as a low pass filter section—that is, a two-port network that attenuates signals according to some monotonically increasing function of frequency. At first glance, it would seem natural to assume that a transmission line should have some sort of low pass filter characteristic.

This question can be investigated by finding the image impedance of one of the T sections. Image impedance is defined as that impedance which will appear at the input of a symmetric two-port network when the same impedance is used as the load to that network. Considering one of the T sections of Figure 4, the

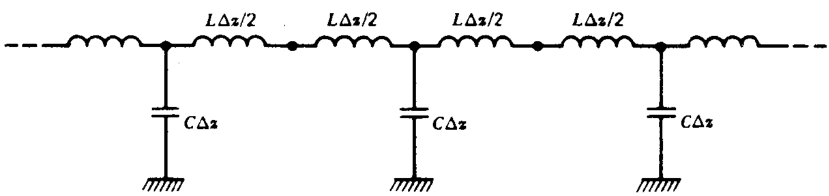

Figure 4 Cascade network of identical low pass T sections

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