Physics and Applications of the Josephson Effect

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Preface

This book surveys all aspects of the Josephson effect—from the underlying physical theory to actual and proposed engineering applications. Both ends of this spectrum are interesting. The physical theory is novel and important for many macroscopic quantum effects, which have a rich and yet untapped potential for technical development. We attempt here to present more than a survey and less than an exhaustive exposition of this wide field. Rather than to cover everything, we have tried to *uncover* those aspects of theory, fabrication technology, and device application that will be of lasting value.

Chapter 1 briefly surveys Josephson junction phenomenology. Although the reader is assumed to have a basic knowledge of superconductivity, we begin with the simplest possible description of Josephson structures and their dynamic behavior. Chapter 2 presents microscopic theory in simple terms. We discuss salient features of the underlying theory that are most useful in appreciating experimental results. To some extent the chapters are self-contained; for example, the reader could skip the microscopic theory (at least on a first reading) without seriously impairing continuity. In Chapter 3 we discuss the dependence of critical current on temperature and on junction parameters. The static (i.e., zero voltage) behavior of "small" and "large" junctions is considered in Chapters 4 and 5. Chapter 6 presents several important results on the current voltage behavior of small weak links, and a variety of weak link structures is described in Chapter 7

What we discuss in Chapter 8 are those basic technological considerations and certain advanced techniques that have been found useful in several laboratories throughout the world over the past decade. We expect such techniques to continue to be of value, especially for those who are beginning to experiment with Josephson junctions.

Chapters 9 and 10 discuss self-resonant modes in small junctions and the dynamical behavior of extended junctions from the perspective of modern "soliton" theory.

The last three chapters are directed toward applications of Josephson junctions. Chapter 11 discusses the various features of junction interactions with periodic signals and considers such applications as mixing, parametric amplification, and the voltage standard. Chapters 12 and 13 deal with quantum interference loops and their application to measurement of very small magnetic fields. Finally, in Chapter 14, we describe the potential of the Josephson junction as the basic logic and memory element in a very large digital computer system.

Throughout the book our choice of theoretical material has been guided by our later discussions of device applications. In this way we have tried to achieve a large scale coherence in a range of subject matter that at first glance might appear to be rather diffuse. We hope that the book will be useful in graduate courses in the theory and applications of superconductive devices, as well as for research scientists and engineers. Although extensive, the bibliography is not exhaustive, and we apologize to those whose work may have been overlooked.

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A. B.
G. P.

Contents

CHAPTER 1

Weak Superconductivity— Phenomenological Aspects

In this chapter we briefly review the phenomenology of the Josephson effect, outlining the basic experimental results and providing a qualitative interpretation on the basis of very simple models. However, first let us say a few words about its history.

The discovery of what is usually referred to as the Josephson effect dates back about 20 years (1961–1962). At that time Brian Josephson was a research student at the Royal Society Mond Laboratory in Cambridge under the supervision of Brian Pippard. There is no doubt, as reported by Josephson in his Nobel lecture, that the stimulating oven of the Mond Laboratory, the presence at that time of Phil Anderson, the development of new researches, both on experiments (Giaever, 1960a, b; Nicol, Shapiro, and Smith 1960) and on theory (Cohen, Falicov, and Phillips 1962) of superconductive tunneling, provided an ideal ground for Josephson's intuition and outstanding conclusions. Josephson's prediction and the following experimental confirmation (Anderson and Rowell 1963) opened not only a new important chapter of physics but also new horizons for a wide variety of stimulating applications.

We shall not dwell further on the history of the discovery of the Josephson effect, though it certainly deserves adequate space and a deep analysis. Instead, we prefer to refer the reader to the historical surveys given by Josephson himself (1974) and other protagonists (Anderson 1970; Pippard 1976), thus avoiding any possible deformation of the fascinating atmosphere in which those events took place.

1.1 Macroscopic Quantum System

The interpretation of superconductivity as a quantum phenomenon on a macroscopic scale was introduced by F. London (1935). The theory of Ginzburg and Landau (1950) provided an enormous insight into the nature of superconductivity. They developed a modification of the London theory (F. London and H. London 1935a, b) by introducing a position dependent parameter, ψ, which gives a measure of the order in the superconducting phase. Unlike the earlier two fluid models proposed by Gorter and Casimir (1934), such an order parameter is complex and can be regarded as a wave function for superconducting electrons. As shown by Gor'kov (1959), ψ is proportional to the local value of the energy gap function Δ. In this framework a single wave function is associated with a macroscopic number of electrons which are assumed to "condense" in the same quantum state. In this sense, the superconductive state can be regarded as a "macroscopic quantum state." Therefore we are dealing with particles, having effective mass and charge m^* and e^* respectively, which can be described as a "whole" by a macroscopic wave function of the form

$$\psi=\rho^{1/2}e^{j\varphi} \tag{1.1.1}$$

where φ is the phase common to all the particles and ρ represents, in this macroscopic picture, their actual density in the macrostate $|s\rangle$:

$$\langle s|\psi^*\psi|s\rangle=|\psi|^2=\rho$$

The electric current density can be written, in the presence of a ve~tor potential **A**:

$$\mathbf{J}=\frac{e^*}{m^*}\left[\frac{j\hbar}{2}(\psi\nabla\psi^*-\psi^*\nabla\psi)-\frac{e^*}{c}\mathbf{A}|\psi|^2\right]$$

where c is the velocity of the light.

As follows from flux quantization, the charge e^* is twice the electronic charge e, since the "particles" we are dealing with are in fact pairs of coupled electrons. This is contained within the framework of the microscopic theory of superconductivity first derived by Bardeen Cooper and Schrieffer (1957) and usually referred as B.C.S. theory. It is assumed that $m^*=2m$ (m=electronic mass), but, it is easy to see that the choice of m is arbitrary, since it depends essentially on the normalization assumed for the pair wave function ψ.†

Thus with the ψ given by (1.1.1) the expression for **J** becomes

$$\mathbf{J}=\rho\frac{e}{m}\left(\hbar\nabla\varphi-\frac{2e}{c}\mathbf{A}\right) \tag{1.1.2}$$

Gauge invariance requires that under the transformations of the vector potential **A** and scalar potential U

$$\mathbf{A}\to\mathbf{A}+\nabla\chi \qquad U\to U-\frac{\partial\chi}{\partial t}$$

the observable physical quantities remain unchanged. This implies the phase transformation

$$\varphi\to\varphi+\frac{2e}{\hbar c}\chi \tag{1.1.3}$$

†See also the microscopic derivation of the Ginzburg Landau theory by Gor'kov (1959).

as can be readily verified for the current density **J** by (1.1.2). The choice of constant values for the scalar quantity χ does not affect potentials but just implies different values of the phase factor. This corresponds to the unobservability of ψ.

We can arbitrarily assign a phase value at a given point; however, because of the occurrence of the so-called long range order the value of the phase is fixed in all points. Obviously, as is evident from (1.1.2), spatial variations of the phase φ describe carrying current states of the superconductor.

For a system in equilibrium the required gauge invariance leads necessarily to a time dependent ψ. It is clear in fact that, even assuming a constant ψ in one gauge, any transformation to another gauge would imply a change of φ as in (1.1.3) in which χ is time dependent. The time evolution of ψ in stationary conditions obeys the usual quantum mechanical equation of the form

$$j\hbar\frac{\partial\psi}{\partial t}=E\psi$$

As can be seen from the microscopic theory (Gor'kov 1959) the quantity E is equal to twice the electrochemical potential μ. This value represents the minimum energy required to add a Cooper pair to the system. Thus $\psi(\mathbf{r},t)=\psi(\mathbf{r})e^{-2j\mu t/\hbar}$ (See also Anderson 1963, 1966).

Since the number of pairs N and the phase φ are conjugate variables (Anderson 1963) there is an uncertainty relation, $\Delta N\Delta\varphi\simeq 2\pi$, which corresponds to the circumstance that within an isolated superconductor N will be fixed and, consequently, the phase φ undefined.

1.2 Coupled Superconductors

Let us now consider two superconductors S_L and S_R separated by a macroscopic distance. In this situation, the phase of the two superconductors can change independently. As the two superconductors are moved closer, so that their separation is reduced to about 30 Å, quasiparticles can flow from one superconductor to the other by means of tunneling (single electron tunneling). If we reduce further the distance between S_L and S_R down to say 10 Å, then, as we shall see, also Cooper pairs can flow from one superconductor to the other (Josephson tunneling). In this situation if we assign a given phase in S_L is the possibility of altering independently the phase in S_R still allowed? The answer is no! This degree of freedom is removed, since phase correlation is realized between the two superconductors; that is, the long range order is "transmitted" across the boundary. Therefore we expect that the whole system of the two superconductors separated by a thin ($\sim$10 Å) dielectric barrier will behave, to some extent, as a single superconductor. Unlike ordinary superconductivity, this phenomenon is often called "weak superconductivity" (Anderson 1963) because of the much lower values of the critical parameters involved. The above-quoted work by Anderson should be considered a milestone in the development of the field.

Josephson theory (1962a, b, 1964, 1965, 1969, 1974) deals with such systems of weakly coupled superconductors. We devote our attention mostly to tunneling structures although Josephson effects take place in various types of superconducting "weak links" (Dayem bridge, point contacts, etc.; see Section 1.8). To begin, we recall the basic concepts of single electron tunneling within a simple phenomenological approach. An account of both single electron tunneling and Josephson phenomenology can be found in Solymar (1972).

1.3 Single Electron Tunneling

The history of superconductive tunneling began with the experiments performed by Giaever (1960a, b) and by Nicol, Shapiro, and Smith (1960). A tunneling structure consists essentially of two metal films separated by a thin (~30 Å) dielectric barrier as sketched in Fig. 1.1. The behavior of such a structure can be investigated by studying the dependence of the tunneling current I on the voltage V across the junction.

To "visualize" the tunneling process, we adopt a simple representation in terms of the energy (E)–momentum (**k**) diagrams. The normal metal is represented in the E-**k** plane by the curve of Fig. 1.2*a*. The dashed line corresponds to the portion of the parabola below the Fermi energy E_F (hole states) which has been reflected across the Fermi level. In this picture the electron hole pair creation is regarded as excitations of two states of energy $E_l = |\varepsilon_l|$ and $E_h = |\varepsilon_h|$ respectively. That is, all excited states have positive energy measured with respect to E_F. In the case of the superconductor, all the condensed pairs are at the Fermi level and a minimum threshold energy Δ (energy gap) is required by an excitation as shown in Fig. 1.2*b*. In this case there exists a particle which is "partially" in the hole state and "partially" in the electron state. These are the quasiparticle excitations which have energy

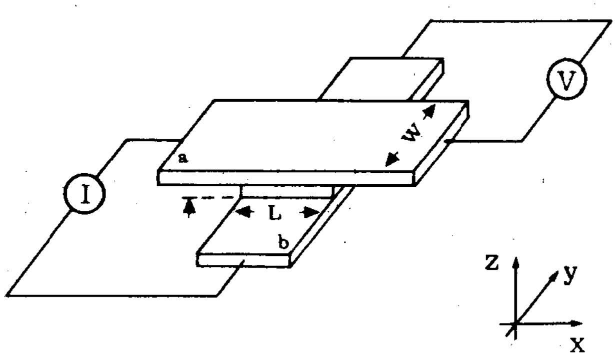

Figure 1.1 Tunneling junction of cross-type geometry. The dimensions are L and W; *a* and *b* are the two superconducting films.