Vibration of solids and structures under moving loads

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Vibration of solids and structures under moving loads

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Preface

Transport engineering structures are subjected to loads that vary in both time and space. In general mechanics parlance such loads are called moving loads. It is the aim of the book to analyze the effects of this type of load on various elements, components, structures and media of engineering mechanics.

In recent years all branches of transport have experienced great advances characterized by increasingly higher speeds and weights of vehicles. As a result, structures and media over or in which the vehicles move have been subjected to vibrations and dynamic stresses far larger than ever before.

The author has studied vibrations of elastic and inelastic bodies and structures under the action of moving loads for many years. In the course of his career he has published a number of papers dealing with various aspects of the problem. On the strength of his studies he has arrived at the conclusion that the topic has so grown in scope and importance as to merit a comprehensive treatment. The book is the outcome of his attempt to do so in a single monograph.

The subject matter of the book is arranged in 27 chapters under six main Parts. The Introduction — a review of the history and the present state of the art — is followed by Part II, the most extensive of all, devoted to the discussion of dynamic loading of one-dimensional solids. The latter term refers to all kinds of beams, continuous beams, frames, arches, strings, etc. with a predominant length dimension — a typical feature of transport engineering structures. The exposition covers beams with several types of support and various alternatives of moving load, and presents the method of computing their deflections and stresses at different speeds of the moving objects.

Part III deals with two-dimensional solids such as rectangular plates and infinite plates on elastic foundation. Part IV is focused on stresses in three-dimensional space produced by moving forces. There, too, consideration is given to all types of speed, i.e. subsonic, transonic and supersonic.

Part V is a lengthy treatise devoted to special problems. It deals, for example, with the effect of variable speed of the load, the action of an axial force and the longitudinal vibration of beams. Study is also made of three-dimensional vibrations of thin-walled beams under moving loads, and of the effect of shear and rotatory inertia on beam stresses. Considerable attention is accorded to the inelastic properties of materials, i.e. to viscoelasticity and plasticity in connection with beam analyses. The concluding part examines the effects of random loads, a very topical problem at present.

The Appendix contains comprehensive tables of integral transformations frequently used throughout the book, and of practical importance in general.

The methodological approach adopted in the book is as follows: the exposition starts with a theoretical analysis of the problem at hand and solves it for all possible cases likely to be met with. The most important results established in this phase are expressed by formulas, represented by diagrams, etc. Many of the theoretical findings are verified experimentally and the test values compared with the computed ones. The conclusion of each chapter outlines the possible applications of the theory explained, and gives a list of recommended reading.

The broad range of problems discussed in the book makes the author hopeful that his work will be found equally useful in civil as in mechanical, transport, marine, aviation and astronautical engineering, for moving loads are present in all these fields. The publication may serve research scientists as an incentive to further development of an interesting and very modern branch of science, project engineers and designers as a guide to safer and more economic design of structures, and students as an advanced text in engineering mechanics and dynamics.

As the results presented in the book are deduced in detail, all that is necessary on the part of the reader is a knowledge of the fundamentals of mechanics, dynamics, vibration and elasticity theories, analysis, theory of differential and integral equations, functions of the complex variable and integral transformations.

In conclusion grateful acknowledgment is due to all those who in any way have contributed towards the successful termination of the book. In the first place the author wishes to thank his wife, Mrs. Dagmar Frýbová, for her rare understanding and support of his scientific work, as well as for her effort of typing most of the manuscript. He is indebted to Mr. Petr Chaloupka for his careful work of drawing all the illustrations and computing some of the diagrams, and is deeply grateful to his colleagues in the Research Institute of Transport in Prague where the book originated in the course of his theoretical and experimental research.

Very special thanks are extended to Professor Vladimír Koloušek, DrSc., corresponding member of the Czechoslovak Academy of Sciences, the scientific editor of the book, to Josef Henrych, DrSc., the referee, both from the Technical University in Prague, and to Professor A.D. de Pater, the reviewer, from the Technological University in Delft, for their thorough study of the manuscript and many valuable suggestions which the author has gratefully incorporated in the last revision.

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Symbols

This is an alphabetical list of the basic symbols used throughout the book. Symbols specific to the matter at hand will be explained as they first come up in the text.

a	a constant
a	parameter expressing the depth of track uneveness
a	parameter expressing the variable stiffness of roadway
a	acceleration or deceleration of motion
a	distance
a	real coordinate in the complex plane
$a_i^2 =$	$\pm 1 \pm \alpha_i^2$
a_n	coefficients in a power series
a_1	parameter expressing a harmonic force
b	a constant
b	parameter expressing the length of track uneveness
b	parameter expressing the length of the variable stiffness of roadway
b	parameter of non-uniform motion
b	imaginary coordinate in the complex plane
b_0	track gauge
b_1	parameter expressing the frequency of a harmonic force
c	speed of motion
$c_i(t)$	speed of displacement
c_{cr}	critical speed
c_1, c_2	velocity of propagation of longitudinal or bending, and transverse or shear waves, respectively
d	a constant
d	wheel base parameter
f	frequency
f_i	frequency of sprung or unsprung parts of vehicle

$f_{(j)}$	natural frequency
$\bar{f}_{(j)}$	natural frequency of a loaded beam
$f(x)$	a function
$\mathring{f}(x, t)$	centred value of function $f(x, t)$
$f(t)$	equation of motion at non-uniform speed
$g =$	9·81 m/s² acceleration of gravity
$g_1(x)$	initial deflection of a beam
$g_2(x)$	initial speed of a beam
$g_{1,2}$	self-weight load of a beam, or of a cable, respectively
h	integration step
h	height difference
h	plate thickness
$h(x)$	beam depth
$h(x, t)$	influence function
$h_{(j)}(t)$	impulse function
$i =$	1, 2, 3, ...
$\mathrm{i}^2 =$	−1 imaginary unit
$j =$	1, 2, 3, ...
k	a constant
k	coefficient of Winkler elastic foundation
$k =$	1, 2, 3, ...
$\mathrm{kei}(x)$	Thomson function of the zero order
l	span
l	length
m	mass of load P
m_i	mass of sprung or unsprung parts of vehicle
m^2	expression (23.29) dependent on beam and foundation properties
$n =$	1, 2, 3
n	roots of the characteristic equation
n	half the number of vehicle axles
p	external load
p	complex variable in the Laplace-Carson integral transformation
$p_{(j)}(t)$	expansion of load in normal modes
q	complex variable in the Fourier integral transformation
q	continuous load
$q_{(j)}(t)$	generalized deflection
r	radius of a wheel, of an arch, of gyration, respectively

r	auxiliary variable
r	radius in polar coordinates
$r_{1,2} =$	$\Omega \pm \omega$
$r(x)$	ordinate of track uneveness
s	auxiliary variable
s	coordinate of point of load application
s	number of equations
t	time
u	displacement in the x-direction
$u(\varphi, t)$	tangential displacement
$u(t)$	auxiliary function
v	displacement in the y-direction
$v(\varphi, t)$	radial displacement
v_i	displacement of joints
$v_i(t)$	vertical displacement of a moving load, of sprung or unsprung parts of vehicle, respectively
$v(x, t)$	beam deflection
$v_{(j)}(x)$	natural mode of beam vibration
v_0	static deflection produced by load P
$v_n(x, t)$	n-th approximate solution of function $v(x, t)$
$v(s)$	dimensionless deflection of an infinite beam on elastic foundation
w	displacement in the z-direction
x	coordinate
x_i	point of contact between vehicle and beam
x_0	fixed point
x_p	coordinate of plastic hinge
y	coordinate
$y, \tilde{y}$	exact, approximate solution
y_h	solution at integration step h
$y(\xi, \tau)$	dimensionless beam deflection
$y_i(\tau)$	dimensionless vertical displacement of load, sprung or unsprung parts of vehicle, respectively
$y(x, t)$	cable sag
z	coordinate
z	auxiliary variable or complex variable
$z(x, t)$	cable deflection
$z(0, l, t)$	function dependent on boundary conditions

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A	a constant
A	centre of flexure
$A^2 =$	$(1 - \alpha_1^2)(1 - \alpha_2^2)$
A_j	poles of the function of a complex variable
A, A_j	integration constant dependent on boundary conditions
A, A_i	distance between track irregularities
$A(t)$	reaction at beam left-hand end
B	a constant
B	parameter of acceleration or deceleration of motion
$B =$	$\alpha^2 - m^2(1 - \alpha_1^2)$
B, B_j	integration constant dependent on boundary conditions
$B(t)$	reaction at beam right-hand end
C	a constant
C	spring constant
C, C_j	integration constant dependent on boundary conditions
C_b	coefficient of viscous damping in vehicle springs
$\mathrm{C}(x)$	Fresnel integral
$C_f(x, t)$	coefficient of variation of function $f(x, t)$
D	a constant
D	vehicle base
D	bending stiffness of plate
D	operation of partial or ordinary differentiation
E	Young's modulus
$E^*(p)$	Laplace-Carson integral transformation of a time variable Young's modulus
$E[f(x, t)]$	mean value of function $f(x, t)$
F	cross-sectional area
$F(q)$	Fourier integral transformation of function $f(x)$
$F_i(\lambda)$	functions tabulated in [130]
$\mathrm{F}(a, b, c, x)$	hypergeometric series
G	beam weight
G	modulus of elasticity in shear
$G(x, s)$	influence or Green's function
$G_{1,2}(j)$	Fourier transformation of initial functions $g_{1,2}(x)$
H	vehicle height
H	horizontal force of a string
$H(x)$	Heaviside function
$H_1(x)$	impulse function of the second order

$H(q, \omega)$	transfer function
I	impulse
I	mass moment of vehicle inertia
Im	imaginary part of the function of a complex variable
J	moment of inertia
$J_n(x)$	Bessel function of the first kind of index n
K	spring constant
K	material constant
K	beam curvature
$K_{ff}(x_1, x_2, t_1, t_2)$	correlation function (covariance)
L	differential operator
L	vehicle length
L	auxiliary datum
M	bending moment
M	torsion moment
M_0	static bending moment produced by load P
M_p	limit bending moment
N	horizontal force
N	normal force
N	printing after N steps
N	number of impulses
O	wheel circumference
O	vehicle centre of gravity
P	concentrated constant force
$P(t)$	concentrated force generally varying in time
$P^*(p)$	Laplace-Carson integral transformation of force $P(t)$
P_i	weight of sprung and unsprung parts of vehicle
$P_i(D)$	linear differential operator
Q	amplitude of harmonic force
Q_i	dimensionless static axle pressure
$Q(t)$	harmonic force
$Q_{(j)}(t)$	generalized force
$R_i(t)$	force acting between vehicle and beam
R	radius
Re	real part of the function of a complex variable
S	axial force
$S(x)$	Fresnel integral
$S_{ff}(q_1, q_2, \omega_1, \omega_2)$	spectral density of function $f(x, t)$

T	shear force
T	vertical force of a string
T	time of load traverse over beam
T_0	time interval
$T_{(j)}$	period of free vibration
U	Fourier integral transformation of function u
V	Fourier integral transformation of function v
V_j	expression (6.6)
$V(j, t)$	Fourier sine finite integral transformation of function $v(x, t)$
$V^*(j, p)$	Laplace-Carson integral transformation of function $V(j, t)$
W	Fourier integral transformation of function w
W	the Wronskian
X	coordinate axis of the centre of gravity
$X_{gh}(t)$	longitudinal force acting on bar gh at point g
X_i	force per unit volume along axis x_i
Y	coordinate axis of the centre of gravity
$Y_{gh}(t)$	transverse force acting on bar gh at point g
Z	coordinate axis of the centre of gravity
$Z(t)$	force acting along axis Z
$Z^*(p)$	Laplace-Carson integral transformation of function $z(0, l, t)$
$Z_i(t)$	force acting in spring C_i
$Z_{bi}(t)$	damping force acting in spring C_i
α	speed parameter
α'	parameter inversely proportional to speed
$\alpha_i =$	c/c_i
β	damping parameter
γ_i	frequency parameter of sprung and unsprung parts of vehicle
γ_{ij}	shear strain in plane $x_i x_j$
δ	dynamic coefficient (impact factor)
$\delta(x)$	Dirac delta function
δ_{ij}	Kronecker delta symbol
$\varepsilon_i =$	0 or 1
ε_i	relative elongation (strain)
ζ	cross-section rotation
η	error

η	imaginary coordinate in the complex plane
η	straight line along which a force moves
η	viscosity coefficient
η	auxiliary variable
ϑ	logarithmic decrement of damping
$\varkappa$	weight parameter
$\varkappa_i$	weight parameter of unsprung and sprung parts of vehicle
λ	coefficient expressing elastic foundation and stiffness of a beam or plate
λ	coefficient expressing rotation of sprung parts of vehicle
λ	Lamé's constant
λ, λ_j	value dependent on natural frequency
μ	mass per unit length of beam or unit area of plate
μ_p	mass appertaining to external load p
ν	Poisson's ratio $(\nu < 1)$
$\xi =$	x/l dimensionless length coordinate
ξ_i	dimensionless coordinate of the point of contact
$\xi =$	$x - ct$ length coordinate in the moving coordinate system
ξ	auxiliary variable
ξ	real coordinate in the complex plane
ϱ	mass per unit volume
ϱ	radius in polar coordinates
$\sigma_{i,j}$	stress component
$\bar{\sigma}$	Fourier integral transformation of stress σ
$\sigma_f(x, t)$	standard deviation of function $f(x, t)$
$\sigma_f^2(x, t)$	variance of function $f(x, t)$
τ	dimensionless time
τ	auxiliary time variable
τ_{ij}	tangential stress
$\bar{\tau}$	Fourier integral transformation of stress τ
φ	polar angle
$\varphi(t)$	rotation of sprung parts of vehicle
$\varphi(x)$	function
$\varphi_j(x)$	linearly independent functions satisfying boundary conditions
$\psi(x, t)$	beam section rotation
$\psi(x)$	Euler's psi-function
ω	circular frequency

ω	circular frequency expressing load motion
ω_b	circular frequency of damping
$\omega_{(j)}$	natural circular frequency
Δ	expression (2.13)
Δ	dynamic increment of deflection or stress
∇	Laplace's operator
Θ	relative volume change
$\Theta(x, t)$	rotation of transverse section about the centre of flexure
Θ	frequency
Σ	summation sign
Φ	central angle of an arc
Φ	angle between a straight line and axis x
Φ	polar angle
$\Psi(j, t)$	Fourier integral transformation of function $\psi(x, t)$
$\Psi^*(j, p)$	Laplace-Carson integral transformation of function $\Psi(j, t)$
$\Psi(q)$	Fourier integral transformation of function $\psi(s)$
Ω	circular frequency of a harmonic force
Ω	circular frequency of non-uniform motion

Subscripts

b	damping
cr	critical or limit value of a quantity
g	left-hand end of a beam
h	homogeneous
h	right-hand end of a beam
i =	1, 2, 3
j =	1, 2, 3, ...
k =	1, 2, 3, ...
p	particular
p	plastic
0	initial conditions
1	unsprung
1	left-hand end
1	first general linearly independent solution of the homogeneous differential equation
2	sprung or unsprung

2	right-hand end
2	second general linearly independent solution of the homogeneous differential equation
3	sprung

Superscripts

$'$ $''$ $'''$ ^{IV}	derivatives with respect to the length coordinate
$'$	damped vibration
$'$	in the oblique direction
$\cdot$ $\cdot\cdot$	dots over the letter denote derivatives with respect to the time coordinate
(n)	n-th derivative
—	bar over the letter denotes a loaded quantity
—	bar over the letter denotes a quantity with a dimension

Units

	International System SI
length:	metre [m], centimetre [cm], kilometre [km]
force:	Newton [N], kilo Newton [kN], mega Newton [MN] 1 kilopond = 1 kilogram force = 9·806 65 N $\doteq$ $\doteq$ 10 N
mass:	kilogram [kg], metric ton [t]
time:	second [s], hour [h]
frequency:	cycle per second = Hertz [Hz]
circular frequency:	[s^{-1}]
speed:	metre per second [m/s], kilometre per hour [km/h]
stress:	Newton per square centimetre [N/cm^2], kilo Newton per square centimetre [kN/cm^2]
coordinate system:	right-handed.

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