QUANTUM MECHANICS

Volume II

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Translated from the French by **Susan Reid Hemley, Nicole Ostrowsky, Dan Ostrowsky**

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Definition of some units

Angström	$1\ \text{Å} = 10^{-10}$ m	(order of magnitude of the atomic dimensions)
Fermi	$1\ \text{F} = 10^{-15}$ m	(order of magnitude of the nuclear dimensions)
Barn	$1\ \text{b} = 10^{-28}\ \text{m}^2 = (10^{-4}\ \text{Å})^2 = (10\ \text{F})^2$	
Electron Volt	$1\ \text{eV} = 1.602\,189(5) \times 10^{-19}$ joule	

Useful orders of magnitude

Electron rest energy : $m_e c^2 \simeq 0.5$ MeV	$[0.511\,003(1) \times 10^6\ \text{eV}]$
Proton rest energy : $M_p c^2 \simeq 1\,000$ MeV	$[938.280(3) \times 10^6\ \text{eV}]$
Neutron rest energy : $M_n c^2 \simeq 1\,000$ MeV	$[939.573(3) \times 10^6\ \text{eV}]$

One electron volt corresponds to :

- a frequency $\nu \simeq 2.4 \times 10^{14}$ Hz through the relation $E = h\nu$ $[2.417\,970(7) \times 10^{14}\ \text{Hz}]$
- a wavelength $\lambda \simeq 12\,000$ Å through the relation $\lambda = c/\nu$ $[12\,398.52(4)\ \text{Å}]$
- a wave number $\dfrac{1}{\lambda} \simeq 8\,000\ \text{cm}^{-1}$ $[8\,065.48(2)\ \text{cm}^{-1}]$
- a temperature $T \simeq 12\,000$ K through the relation $E = k_B T$ $[11\,604.5(4)\ \text{K}]$

In a 1 gauss magnetic field (10^{-4} Tesla) :

- the electron cyclotron frequency $\nu_c = \omega_c/2\pi = -\,qB/2\pi m_e$ is $\nu_c \simeq 2.8$ MHz $[2.799\,225(8) \times 10^6\ \text{Hz}]$
- the orbital Larmor frequency $\nu_L = \omega_L/2\pi = -\,\mu_B B/h = \nu_c/2$ is $\nu_L \simeq 1.4$ MHz $[1.399\,612(4) \times 10^6\ \text{Hz}]$

  (this corresponds by definition to a $g = 1$ Landé factor)

Some general physical constants

Planck's constant	$h = 6.626\ 18(4) \times 10^{-34}$ joule second $\hbar = \dfrac{h}{2\pi} = 1.054\ 589(6) \times 10^{-34}$ joule second
Speed of light (in vacuum)	$c = 2.997\ 924\ 58(1) \times 10^{8}$ m/s
Electron charge	$q = -1.602\ 189(5) \times 10^{-19}$ coulomb
Electron mass	$m_e = 9.109\ 53(5) \times 10^{-31}$ kg
Proton mass	$M_p = 1.672\ 65(1) \times 10^{-27}$ kg
Neutron mass	$M_n = 1.674\ 95(1) \times 10^{-27}$ kg
	$\dfrac{M_p}{m_e} = 1\ 836.1515(7)$
Electron Compton wavelength	$\lambda_c = h/m_e c = 2.426\ 309(4) \times 10^{-2}$ Å $\bar{\lambda}_c = \hbar/m_e c = 3.861\ 591(7) \times 10^{-3}$ Å
Fine structure constant (dimensionless)	$\alpha = \dfrac{q^2}{4\pi\varepsilon_0 \hbar c} = \dfrac{e^2}{\hbar c} = \dfrac{1}{137.036\ 0(1)}$
Bohr radius	$a_0 = \dfrac{\bar{\lambda}_c}{\alpha} = 0.529\ 177\ 1(5)$ Å
Hydrogen atom ionization energy (without proton recoil effect)	$-E_{I\infty} = \alpha^2 m_e c^2/2 = 13.605\ 80(5)$ eV
Rydberg's constant	$R_\infty = -E_{I\infty}/hc = 1.097\ 373\ 18(8) \times 10^{5}$ cm^{-1}
"Classical" electron radius	$r_e = \dfrac{q^2}{4\pi\varepsilon_0 m_e c^2} = 2.817\ 938(7)$ fermi
Bohr magneton	$\mu_B = q\hbar/2m_e = -9.274\ 08(4) \times 10^{-24}$ joule/tesla
Electron spin g factor	$g_e = 2 \times 1.001\ 159\ 657(4)$
Nuclear magneton	$\mu_n = -q\hbar/2M_p = 5.050\ 82(2) \times 10^{-27}$ joule/tesla
Boltzmann's constant	$k_B = 1.380\ 66(4) \times 10^{-23}$ joule/K
Avogadro's number	$N_A = 6.022\ 05(3) \times 10^{23}$

Directions for Use

This book is made up of chapters and their complements :

– *The chapters* contain the fundamental concepts. Except for a few additions and variations, they correspond to a course given in the last year of a typical undergraduate physics program.

These fourteen chapters are *complete in themselves* and can be studied independently of the complements.

– *The complements* follow the appropriate chapter. They are listed at the end of each chapter in a "*reader's guide*" which discusses the difficulty and importance of every one of them. Each is labelled by a letter followed by a subscript which gives the number of the corresponding chapter (for example, the complements of chapter V are, in order, A_V, B_V, C_V...). They can be recognized immediately by the symbol ● which appears at the top of each of their pages.

The complements vary : some are intended to expand the treatment of the corresponding chapter or to provide more detailed discussion of certain points; others describe concrete examples or introduce various physical concepts. One of the complements (usually the last one) is a collection of exercises.

The *difficulty* of the complements varies. Some are very simple examples or extensions of the chapter, while others are more difficult (some are at graduate level); in any case, the reader should have studied the material in the chapter before using the complements.

The student should not try to study all the complements of a chapter at once. In accordance with his aims and interests, he should choose a small number of them (two or three, for example), plus a few exercises. The other complements can be left for later study.

Some passages within the book have been set in small type and these can be omitted on a first reading.

VOLUME II

Complements of chapter XI

Chapter XII An application of perturbation theory : the fine and hyperfine structure of the hydrogen atom . . . 1209

Complements of chapter XII

Complements of chapter XIII

Complements of chapter XIV

CHAPTER VIII

An elementary approach to the quantum theory of scattering by a potential

OUTLINE OF CHAPTER VIII

A. INTRODUCTION

1. Importance of collision phenomena

Many experiments in physics, especially in high energy physics, consist of directing a beam of particles (1) (produced for example, by an accelerator) onto a target composed of particles (2), and studying the resulting collisions : the various particles* constituting the final state of the system – that is, the state after the collision (*cf.* fig. 1) – are detected and their characteristics (direction of emission, energy, etc.) are measured. Obviously, the aim of such a study is to determine the interactions that occur between the various particles entering into the collision.

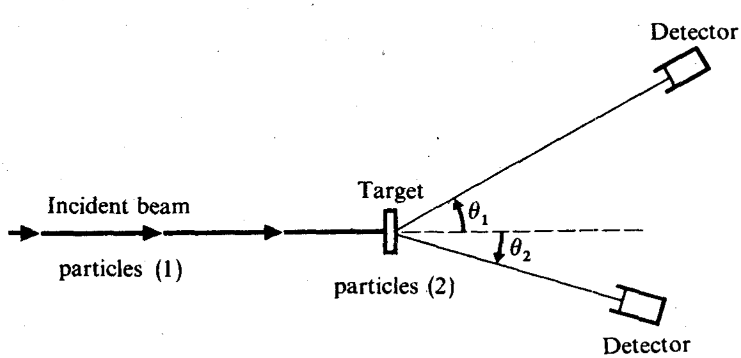

FIGURE 1

Diagram of a collision experiment involving the particles (1) of an incident beam and the particles (2) of a target. The two detectors represented in the figure measure the number of particles scattered through angles θ_1 and θ_2 with respect to the incident beam.

The phenomena observed are sometimes very complex. For example, if particles (1) and (2) are in fact composed of more elementary components (protons and neutrons in the case of nuclei), the latter can, during the collision, redistribute themselves amongst two or several final composite particles which are different from the initial particles; in this case, one speaks of "rearrangement collisions".

* In practice, it is not always possible to detect all the particles emitted, and one must often be satisfied with partial information about the final system.

Moreover, at high energies, the relativistic possibility of the "materialization" of part of the energy appears: new particles are then created and the final state can include a great number of them (the higher the energy of the incident beam, the greater the number). Broadly speaking, one says that collisions give rise to *reactions*, which are described most often as in chemistry:

$$(1) + (2) \longrightarrow (3) + (4) + (5) + \ldots \qquad \text{(A-1)}$$

Amongst all the reactions possible* under given conditions, *scattering* reactions are defined as those in which the final state and the initial state are composed of the same particles (1) and (2). In addition, a scattering reaction is said to be elastic when none of the particles' internal states change during the collision.

2. Scattering by a potential

We shall confine ourselves in this chapter to the study of the elastic scattering of the incident particles (1) by the target particles (2). If the laws of classical mechanics were applicable, solving this problem would involve determining the deviations in the incident particles' trajectories due to the forces exerted by particles (2). For processes occurring on an atomic or nuclear scale, it is clearly out of the question to use classical mechanics to resolve the problem; we must study the evolution of the wave function associated with the incident particles under the influence of their interactions with the target particles [which is why we speak of the "scattering" of particles (1) by particles (2)]. Rather than attack this question in its most general form, we shall introduce the following simplifying hypotheses:

(*i*) We shall suppose that particles (1) and (2) have no spin. This simplifies the theory considerably but should not be taken to imply that the spin of particles is unimportant in scattering phenomena.

(*ii*) We shall not take into account the possible internal structure of particles (1) and (2). The following arguments are therefore not applicable to "inelastic" scattering phenomena, where part of the kinetic energy of (1) is absorbed in the final state by the internal degrees of freedom of (1) and (2) (*cf.* for example, the experiment of Franck and Hertz). We shall confine ourselves to the case of *elastic scattering*, which does not affect the internal structure of the particles.

(*iii*) We shall assume that the target is thin enough to enable us to neglect multiple scattering processes; that is, processes during which a particular incident particle is scattered several times before leaving the target.

(*iv*) We shall neglect any possibility of coherence between the waves scattered by the different particles which make up the target. This simplification is justified when the spread of the wave packets associated with particles (1) is small compared to the average distance between particles (2). Therefore we shall concern ourselves only with the elementary process of the scattering of a particle (1) of the beam by a particle (2) of the target. This excludes a certain number of phenomena which

* Since the processes studied occur on a quantum level, it is not generally possible to predict with certainty what final state will result from a given collision; one merely attempts to predict the probabilities of the various possible states.

are nevertheless very interesting, such as coherent scattering by a crystal (Bragg diffraction) or scattering of slow neutrons by the phonons of a solid, which provide valuable information about the structure and dynamics of crystal lattices. When these coherence effects can be neglected, the flux of particles detected is simply the sum of the fluxes scattered by each of the $\mathcal{N}$ target particles, that is, $\mathcal{N}$ times the flux scattered by any one of them (the exact position of the scattering particle inside the target is unimportant since the target dimensions are much smaller than the distance between the target and the detector).

(*v*) We shall assume that the interactions between particles (1) and (2) can be described by a potential energy $V(\mathbf{r}_1 - \mathbf{r}_2)$, which depends only on the relative position $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ of the particles. If we follow the reasoning of § **B**, chapter VII, then, in the center-of-mass reference frame* of the two particles (1) and (2), the problem reduces to the study of *the scattering of a single particle by the potential* $V(\mathbf{r})$. The mass μ of this "relative particle" is related to the masses m_1 and m_2 of (1) and (2) by the formula :

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{A-2}$$

3. Definition of the scattering cross section

Let Oz be the direction of the incident particles of mass μ (fig. 2). The potential $V(\mathbf{r})$ is localized around the origin O of the coordinate system [which is in fact

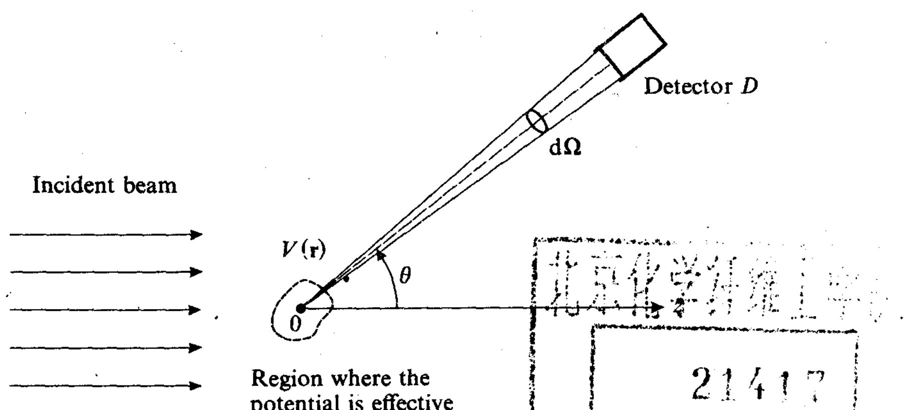

FIGURE 2

The incident beam, whose flux of particles is F_i, is parallel to the axis Oz; it is assumed to be much wider than the zone of influence of the potential $V(\mathbf{r})$, which is centered at O. Far from this zone of influence, a detector D measures the number $\mathrm{d}n$ of particles scattered per unit time into the solid angle $\mathrm{d}\Omega$, centered around the direction defined by the polar angles θ and φ. The number $\mathrm{d}n$ is proportional to F_i and to $\mathrm{d}\Omega$; the coefficient of proportionality $\sigma(\theta, \varphi)$ is, by definition, the scattering "cross section" in the direction (θ, φ).

* In order to interpret the results obtained in scattering experiments, it is clearly necessary to return to the laboratory reference frame. Going from one frame of reference to another is a simple kinematic problem that we will not consider here. See for example Messiah (1.17), vol. I, chap. X, § 7.

the center of mass of the two real particles (1) and (2)]. We shall designate by F_i the flux of particles in the incident beam, that is, the number of particles per unit time which traverse a unit surface perpendicular to Oz in the region where z takes on very large negative values. (The flux F_i is assumed to be weak enough to allow us to neglect interactions between different particles of the incident beam.)

We place a detector far from the region under the influence of the potential and in the direction fixed by the polar angles θ and φ, with an opening facing O and subtending the solid angle $d\Omega$ (the detector is situated at a distance from O which is large compared to the linear dimensions of the potential's zone of influence). We can thus count the number dn of particles scattered per unit time into the solid angle $d\Omega$ about the direction (θ, φ).

dn is obviously proportional to $d\Omega$ and to the incident flux F_i. We shall define $\sigma(\theta, \varphi)$ to be the coefficient of proportionality between dn and $F_i \, d\Omega$:

$$\boxed{dn = F_i \, \sigma(\theta, \varphi) \, d\Omega} \tag{A-3}$$

The dimensions of dn and F_i are, respectively, T^{-1} and $(L^2T)^{-1}$. $\sigma(\theta, \varphi)$ therefore has the dimensions of a surface; it is called the *differential scattering cross section* in the direction (θ, φ). Cross sections are frequently measured in barns and submultiples of barns :

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 \tag{A-4}$$

The definition (A-3) can be interpreted in the following way: the number of particles per unit time which reach the detector is equal to the number of particles which would cross a surface $\sigma(\theta, \varphi) \, d\Omega$ placed perpendicular to Oz in the incident beam.

Similarly, the *total scattering cross section* σ is defined by the formula:

$$\sigma = \int \sigma(\theta, \varphi) \, d\Omega \tag{A-5}$$

COMMENTS:

(*i*) Definition (A-3), in which dn is proportional to $d\Omega$, implies that only the scattered particles are taken into consideration. The flux of these particles reaching a given detector D [of fixed surface and placed in the direction (θ, φ)] is inversely proportional to the square of the distance between D and O (this property is characteristic of a scattered flux). In practice, the incident beam is laterally bounded [although its width remains much larger than the extent of the zone of influence of $V(\mathbf{r})$], and the detector is placed outside its trajectory so that it receives only the scattered particles. Of course, such an arrangement does not permit the measurement of the cross section in the direction $\theta = 0$ (the forward direction), which can only be obtained by extrapolation from the values of $\sigma(\theta, \varphi)$ for small θ.

(*ii*) The concept of a cross section is not limited to the case of elastic scattering: reaction cross sections are defined in an analogous manner.