Fundamentals of Vibrations

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Leonard Meirovitch

College of Engineering

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McGraw-Hill Higher Education
A Division of The McGraw-Hill Companies

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Published by McGraw-Hill, an imprint of The McGraw-Hill Companies, Inc. 1221 Avenue of the Americas, New York, NY, 10020.

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 0 FGR/FGR 0 9 8 7 6 5 4 3 2 1 0

ISBN 0070413452

Publisher: *Thomas Casson*
Senior sponsoring editor: *Jonathan Plant*
Editorial assistant: *Kristen Druffner*
Senior marketing manager: *John T. Wannemacher*
Senior project manager: *Mary Conzachi*
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Freelance design coordinator: *Pam Verros*
Supplement coordinator: *Nate Perry*
Compositor: *Techsetters, Inc.*
Typeface: *10/12 Times Roman*
Printer: *Quebecor Printing Book Group/Fairfield*

Library of Congress Cataloging-in-Publication Data

Meirovitch, Leonard.
Fundamentals of vibrations / Leonard Meirovitch.
p. cm.
Includes bibliographical references and index.
ISBN 0-07-041345-2 (alk. paper)
1. Vibration I. Title
TA355 .M43 2001
620.3–dc21

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Leonard Meirovitch, a well-known researcher and educator, is a University Distinguished Professor Emeritus at Virginia Polytechnic Institute and State University (VPI&SU). He is the author of a very large number of journal publications in the areas of analytical vibrations, computational structural dynamics and control of structures and of the books *Analytical Methods in Vibrations* (Macmillan, 1967), *Methods of Analytical Dynamics* (McGraw-Hill, 1970), *Elements of Vibration Analysis*, first edition (McGraw-Hill, 1975), *Computational Methods in Structural Dynamics* (Sijthoff & Noordhoff, 1980), *Introduction to Dynamics and Control* (Wiley, 1985), *Elements of Vibration Analysis*, second edition (McGraw-Hill, 1986), *Dynamics and Control of Structures* (Wiley-Interscience, 1990) and *Principles and Techniques of Vibrations* (Prentice-Hall, 1997). Dr. Meirovitch is a Fellow of the American Institute of Aeronautics and Astronautics (AIAA) and the recipient of the VPI&SU Alumni Award for Research Excellence (1981), the AIAA Structures, Structural Dynamics, and Materials Award (1983), the AIAA Pendray Aerospace Literature Award (1984), the AIAA Mechanics and Control of Flight Award (1987), the Japan Society of Mechanical Engineers Award (1989), an Alexander von Humboldt Award for Senior U.S. Scientists (Germany, 1991) and the American Society of Mechanical Engineers J. P. Den Hartog Award (1999).

PREFACE

This book presents material fundamental to a modern treatment of vibrations, placing the emphasis on analytical developments and computational solutions. It is intended as a textbook for a number of courses on vibrations ranging from the junior level to the second-year graduate level; the book can also serve as a reference for practicing engineers. Certain material from pertinent disciplines was included to render the book self-contained, and hence suitable for self-study. Consistent with this, the book begins with very elementary material and raises the level gradually. A large number of examples and homework problems, as well as computer programs written in MATLAB[1], are provided.

The following review is designed to help the reader decide how best to use the book:

Chapter 1. Concepts from Vibrations—Sections 1.1–1.6 are devoted to a review of basic concepts from Newtonian mechanics. Issues concerning the modeling of mechanical systems, from components to assembled systems, are discussed in Secs. 1.7 to 1.9, and the differential equations of motion for such systems are derived in Sec. 1.10. Sections 1.11 and 1.12 are concerned with the nature of the excitations, the system characteristics and the nature of the response; the concept of linearity and the closely related principle of superposition are discussed. Finally, in Sec. 1.13, the concepts of equilibrium points and motions about equilibrium points are introduced.

The whole chapter is suitable for a first course on vibrations at the undergraduate level, but Secs. 1.1–1.6 may be omitted from a first course at the graduate level.

Chapter 2. Response of Single-Degree-of-Freedom Systems to Initial Excitations—This chapter is concerned with the free vibration of undamped, viscously damped and Coulomb damped systems to initial displacements and velocities. It includes a MATLAB program for plotting the response of viscously damped systems.

This chapter is essential to a first course on vibrations at any level.

Chapter 3. Response of Single-Degree-of-Freedom Systems to Harmonic and Periodic Excitations—In Secs. 3.1 and 3.2, the response to harmonic excitations is represented in the frequency domain, through magnitude and phase angle frequency response plots. Sections 3.3–3.7 discuss applications such as systems with rotating eccentric masses, systems with harmonically moving support, vibration isolation and vibration measuring instruments. In Sec. 3.8, structural damping is treated by means of an analogy with viscous damping. Finally, in Sec. 3.9, the approach to the response of systems to harmonic excitations is extended to periodic excitations through the use of Fourier series. A MATLAB program generating frequency response plots is provided in Sec. 3.10.

The material in Secs. 3.1–3.6 is to be included in a first course on vibrations, but the material in Secs. 3.7–3.9 is optional.

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[1] MATLAB ® is a registered trademark of The MathWorks, Inc.

Chapter 4. Response of Single-Degree-of-Freedom Systems to Nonperiodic Excitations—Sections 4.1–4.3 introduce the unit impulse, unit step function and unit ramp function and the respective response. Then, regarding arbitrary excitations as a superposition of impulses of varying magnitude, the system response is represented in Sec. 4.4 as a corresponding superposition of impulse responses, becoming the convolution integral in the limit. Section 4.5 discusses the concept of shock spectrum. Sections 4.6 and 4.7 are devoted to the system response by the Laplace transformation; the concept of transfer function is introduced. Next, in Sec. 4.8, the response is obtained by the state transition matrix. Numerical solutions for the response are carried out in discrete time by the convolution sum in Sec. 4.9 and by the discrete-time transition matrix in Sec. 4.10. A MATLAB program for the response using the convolution sum is given in Sec. 4.11 and another program using the discrete-time transition matrix is given in Sec. 4.12.

Sections 4.1–4.4 are to be included in a first course on vibrations at all levels. Section 4.5 is optional, but recommended for a design-oriented course. Sections 4.6–4.10 are optional for a junior course, recommended for a senior course and to be included in a first course at the graduate level.

Chapter 5. Two-Degree-of-Freedom Systems—Sections 5.1–5.6 present in a simple fashion such topics as the eigenvalue problem, natural modes, response to initial excitations, coupling, orthogonality of modes and modal analysis. Section 5.7 is concerned with the beat phenomenon, Sec. 5.8 derives the response to harmonic excitations and Sec. 5.9 discusses vibration absorbers. The response to nonperiodic excitations is carried out in continuous time in Sec. 5.10 and in discrete time in Sec. 5.11. Three MATLAB programs are included, the first in Sec. 5.12 for the response to initial excitations, the second in Sec. 5.13 for producing frequency response plots and the third in Sec. 5.14 for the response to a rectangular pulse by the convolution sum.

The material belongs in an undergraduate course on vibrations, but is not essential to a graduate course, unless a gradual transition to multi-degree-of-freedom systems is deemed desirable.

Chapter 6. Elements of Analytical Dynamics—Sections 6.1–6.3 provide the prerequisite material for the development in Sec. 6.4 of the extended Hamilton principle, which permits the derivation of all the equations of motion. In Sec. 6.5, the principle is used to produce a generic form of the equations of motion, namely, Lagrange's equations.

This chapter is suitable for a senior course on vibrations and is a virtual necessity for a first-year graduate course.

Chapter 7. Multi-Degree-of-Freedom System—Sections 7.1–7.4 are concerned with the formulation of the equations of motion for linear and linearized systems, as well as with some basic properties of such systems. In Secs. 7.5–7.7, some of the concepts discussed in Ch. 5, such as linear transformations, coupling, the eigenvalue problem, natural modes and orthogonality of modes, are presented in a more compact manner by means of matrix algebra. Then, in Sec. 7.8, the question of rigid-body motions is addressed. In Secs. 7.9 and 7.10, modal analysis is first developed in a rigorous manner and then used to obtain the response to initial excitations. Certain issues associated with the eigenvalue problem are discussed in Secs. 7.11 and 7.12. Section 7.13 is devoted to Rayleigh's quotient, a concept of great importance in vibrations. The response to external excitations is obtained in continuous time in Secs. 7.14 and 7.15 and in discrete time in Sec. 7.17. MATLAB programs are provided as follows: the solution of the eigenvalue problem for conservative systems and for nonconservative systems, both in Sec. 7.18, the response to initial excitations in Sec. 7.19 and the response to external excitations by the discrete-time transition matrix in Sec. 7.20.

This chapter, in full or in part, is suitable for a senior course on vibrations, and should be considered as an alternative to Ch. 5. The material rightfully belongs in a first-year graduate course.

Chapter 8. Distributed-Parameter Systems: Exact Solutions—In Sec. 8.1, the equations of motion for a set of lumped masses on a string are first derived by the Newtonian approach and then transformed in the limit into a boundary-value problem for a distributed string. The same boundary-value problem is derived in Sec. 8.2 by the extended Hamilton principle. In Sec. 8.3, the boundary-value problem for a beam in bending is derived by both the Newtonian approach and the extended Hamilton principle. Sections 8.4–8.8 are devoted to the differential eigenvalue problem and its solution. Rayleigh's quotient is used in Sec. 8.8 to develop the variational approach to the differential eigenvalue problem. The response to initial excitations and external excitations by modal analysis is considered in Secs. 8.9 and 8.10, respectively. A modal solution to the problem of a rod subjected to a boundary force is obtained in Sec. 8.11. The wave equation and its solution in terms of traveling waves and standing waves are introduced in Sec. 8.12, and in Sec. 8.13 it is shown that a traveling wave solution matches the standing waves solution obtained in Sec. 8.11.

Sections 8.1–8.5, 8.9 and 8.10 are suitable for a senior course or a first-year graduate course on vibrations. The balance of the chapter belongs in a second-year graduate course.

Chapter 9. Distributed-Parameter Systems: Approximate Methods—Sections 9.1–9.4 discuss four lumped-parameter methods, including Holzer's method and Myklestad's method. The balance of the chapter is concerned with series discretization techniques. Section 9.5 presents Rayleigh's principle, which is the basis for the variational approach to the differential eigenvalue problem identified with the Rayleigh-Ritz method, as expounded in Secs. 9.6–9.8. Sections 9.9 and 9.10 consider two weighted residuals methods, Galerkin's method and the collocation method, respectively. A MATLAB program for the solution of the eigenvalue problem for a nonuniform rod by the Rayleigh-Ritz method is provided in Sec. 9.11.

The material is suitable for a senior or a first-year graduate course on vibrations, with the exception of the second half of Sec. 9.6 and the entire Sec. 9.7, which are more suitable for a second-year graduate course.

Chapter 10. The Finite Element Method—Section 10.1 presents the formalism of the finite element method. Sections 10.2 and 10.3 consider strings, rods and shafts in terms of linear, quadratic and cubic interpolation functions. Then, Sec. 10.4 discusses beams in bending. Estimates of errors incurred in using the finite element method are provided in Sec. 10.5. In Secs. 10.6 and 10.7, trusses and frames are treated as assemblages of rods and beams, respectively. Then, system response by the finite element method is discussed in Sec. 10.8. A MATLAB program for the solution of the eigenvalue problem for a nonuniform pinned-pinned beam is provided in Sec. 10.9.

This chapter is suitable for a senior or a first-year graduate course on vibrations, with the exception of Sec. 10.3, which is optional, and Secs. 10.6 and 10.7, which are more suitable for a second-year graduate course.

Chapter 11. Nonlinear Oscillations—Sections 11.1–11.3 are concerned with qualitative aspects of nonlinear systems, such as equilibrium points, stability of motion about equilibrium, trajectories in the neighborhood of equilibrium and motions in the large. Section 11.4 discusses the van der Pol oscillator and the concept of limit cycle. Sections 11.5–11.7 introduce the perturbation approach and how to obtain periodic perturbation solutions by Lindstedt's method. Using the perturbation approach, the jump phenomenon is discussed in Sec. 11.8, subharmonic solutions in Sec. 11.9 and linear systems with time-dependent coefficients in Sec. 11.10. Section 11.11 is devoted to numerical integration of differential equations of motion by the Runge-Kutta methods. A MATLAB program for plotting trajectories for the van der Pol oscillator is provided in Sec. 11.12.

The material is suitable for a senior or a graduate course on nonlinear vibrations.

Chapter 12. Random Vibrations—Sections 12.1–12.3 introduce such concepts as random process, stationarity, ergodicity, mean value, autocorrelation function, mean square value and standard deviation. Sections 12.4 and 12.5 are concerned with probability density functions. Properties of the autocorrelation function are discussed in Sec. 12.6. Sections 12.7–12.11 are devoted to the response to random excitations using frequency domain techniques. Sections 12.12–12.15 are concerned with joint properties of two random processes. The response of multi-degree-of-freedom systems and distributed systems to random excitations is discussed in Secs. 12.16 and 12.17, respectively.

The material is suitable for a graduate course on random vibrations.

Appendix A. Fourier Series—The material is concerned with the representation of periodic functions by Fourier series. Both the real form and the complex form of Fourier series are discussed.

Appendix B. Laplace Transformation—The appendix contains an introduction to the Laplace transformation and its use to solve ordinary differential equations with constant coefficients, such as those encountered in vibrations.

Appendix C. Linear Algebra—The appendix represents an introduction to matrices, vector spaces and linear transformations. The material is indispensable to an efficient and rigorous treatment of multi-degree-of-freedom systems.

In recent years, computational algorithms of interest in vibrations have matured to the extent that they are now standard. Examples of these are the QR method for solving algebraic eigenvalue problems and the method based on the discrete-time transition matrix for computing the response of linear systems. At the same time, computers capable of handling such algorithms have become ubiquitous. Moreover, the software for the implementation of these algorithms has become easier to use. In this regard, MATLAB must be considered the software of choice. It is quite intuitive, it can be used interactively and it possesses an inventory of routines, referred to as functions, which simplify the task of programming even more. This book contains 14 MATLAB programs solving typical vibrations problems; they have been written using Version 5.3 of MATLAB. The programs can be used as they are, or they can be modified as needed, particularly the data. In addition, a number of MATLAB problems are included. Further information concerning MATLAB can be obtained from:

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760

It should be stressed that the book is independent of the MATLAB material and can be used with or without it. Of course, the MATLAB material is designed to enhance the study of vibrations, and its use is highly recommended.

The author wishes to express his appreciation to William J. Atherton, Cleveland State University; Amr M. Baz, University of Maryland; Itzhak Green, Georgia Institute of Technology; Robert H. Lipp, University of New Orleans; Hayrani Ali Öz, Ohio State University; and Alan B. Palazzolo, Texas A&M University, for their extensive review of the manuscript and their many useful suggestions. He also wishes to thank Timothy J. Stemple, Virginia Polytechnic Institute and State University, for producing the computer-generated figures and for reviewing an early version of the manuscript. Special thanks are due to İlhan Tuzcu, Virginia Polytechnic Institute and State University, for his major role in developing the MATLAB programs, as well as for his thorough review of the manuscript. Last but not least, the author would like to thank Norma B. Guynn for typing the book essentially as it appears in its final form; the book places in evidence the excellent quality of her work.

Leonard Meirovitch

INTRODUCTION

Dynamics is the branch of physics concerned with the motion of bodies under the action of forces. For the problems of interest in this text, relativistic effects are extremely small, so that the motions are governed by the laws of Newtonian mechanics. *Vibrations*, or *oscillations*, can be regarded as a subset of dynamics in which a system subjected to restoring forces swings back and forth about an equilibrium position, where a system is defined as an assemblage of parts acting together as a whole. The restoring forces are due to elasticity, or due to gravity.

For the most part, engineering systems are so complex that their response to stimuli is difficult to predict. Yet, the ability to predict system behavior is essential. In such cases, it is necessary to construct a simplified model acting as a surrogate for the actual system. The process consists of identifying constituent components, determining the dynamic characteristics of the individual components, perhaps experimentally, and assembling the components into a model representative of the whole system. Models are not unique, and for a given system it is possible to construct a number of models. The choice of a model depends on its use and on the system mass and stiffness properties, referred to as parameters. For example, in preliminary design, a simple model predicting the system behavior reasonably well may suffice. On the other hand, in advanced stages of design, a very refined model capable of predicting accurately the behavior of the actual system may be necessary. Many systems can be simulated by models whose behavior is described by a single ordinary differential equation of motion, i.e., by *single-degree-of-freedom* models. This is the case when the model consists of a single mass undergoing translation in one direction, or rotation about one axis. Many other systems must be modeled by an array of masses connected elastically. The behavior of such models is described by a set of ordinary differential equations, and are known as *multi-degree-of-freedom* models. They are commonly referred to as *discrete systems*, or *lumped-parameter systems*. Then, there are systems with distributed mass and stiffness properties. They can be represented by lumped-parameter models, or by *distributed-parameter models*, where the behavior of the latter is described by partial differential equations. Occasionally, we encounter systems with both lumped and distributed properties. Modeling is an important part of engineering vibrations.

The response of a system to given excitations depends on the system characteristics, as reflected in the differential equations of motion. If the response increases proportionally to the excitation, then the system is said to be *linear;* otherwise it is *nonlinear*. Linearity is of paramount importance to a system, as it dictates the approach to the solution of the equations of motion. Indeed, in the case of linear systems the *principle of superposition* applies, which can simplify the solution greatly. The superposition principle does not apply to nonlinear systems.

Different types of excitations call for different methods of solution, particularly the external excitations. By virtue of the superposition principle, the response of linear systems to initial excitations and to external excitations can be obtained separately and then combined linearly. Because for all practical purposes the response to initial excitations decays with time, it is referred to as *transient*. In the case of sinusoidal excitations, it is more advantageous to treat the response in the frequency domain, through frequency response plots, rather than in the time domain. Periodic excitations can be represented as a combination of sinusoidal functions by means of Fourier series, and the response can be obtained as a corresponding combination of sinusoidal responses. Because in both cases time plays no particular role, the response to sinusoidal excitations and the response to periodic excitations are said to be *steady state*. Arbitrary excitations can be regarded as superpositions of impulses of varying magnitude, so that the response can be obtained as corresponding superpositions of impulse responses. The Laplace transformation method yields the same results, perhaps in a less intuitive manner. In linear system theory, the most common approach to the response is to cast the equations of motion in state form and then solve them by a technique based on the state transition matrix. For the most part, the response to arbitrary excitations must be obtained numerically on a computer, which implies discrete-time processing. Random excitations require entirely different approaches, and the response can be obtained in terms of statistical quantities.

Although the preceding discussion applies to all types of models, multi-degree-of-freedom systems and distributed-parameter systems require further elaboration. The equations of motion for multi-degree-of-freedom systems are more efficiently derived by means of Lagrange's equations than by direct application of Newton's second law. Linear, or linearized equations of motion are best expressed in matrix form. Because these are simultaneous equations, the coefficient matrices, albeit symmetric, are fully populated. Their solution can only be carried out by rendering the equations independent by means of modal analysis. This involves the solution of an algebraic eigenvalue problem and an orthogonal transformation using the modal matrix, all made possible by developments in linear algebra. The independent modal equations resemble those for a single-degree-of-freedom and can be solved accordingly. Although different in appearance, partial differential equations describing distributed-parameter systems can be solved in an analogous manner, the primary difference being that they require the solution of a differential eigenvalue problem instead of an algebraic one.

For the most part, differential eigenvalue problems do not admit analytical solutions, so that they must be solved approximately, which amounts to reducing them to algebraic eigenvalue problems. This implies the construction of a discrete model approximating the distributed-parameter system, which can be done through parameter lumping or series discretization. Among series discretization methods, we include the Rayleigh-Ritz method, the Galerkin method and the finite element method, the latter being perhaps the most important development in structural dynamics in the last half a century.

The fact that the superposition principle does not hold for nonlinear systems causes difficulties in producing solutions. If the interest lies only in qualitative stability characteristics, rather than in the system response, then such information can be obtained by linearizing the equations of motion about a given equilibrium point, solving the corresponding eigenvalue problem and reaching stability conclusions from the nature of the eigenvalues. For systems with small nonlinearities, more quantitative results can be obtained by means of perturbation techniques, which permit solutions using once again methods of linear analysis. For nonlinearities of arbitrary magnitude, solutions can only be obtained numerically on a computer. To this end, the Runge-Kutta methods are quite effective.

Difficulties of a different kind arise in the case of random excitations. The response to random excitations is also random and can only be defined in terms of statistical quantities. This situation is much better for Gaussian random processes, for which the probability that the response will remain below a certain value can be defined by means of two statistics alone: the mean value and the standard deviation. The latter is the more important one and can be computed working in the frequency domain using Fourier transforms, rather than in the time domain.

Finally, it should be noted that the numerical work involved in this vibrations study can be programmed for computer evaluation using MATLAB software. In fact, this book contains MATLAB programs for a variety of vibrations problems, which can be regarded as the foundation for a vibrations toolbox.

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