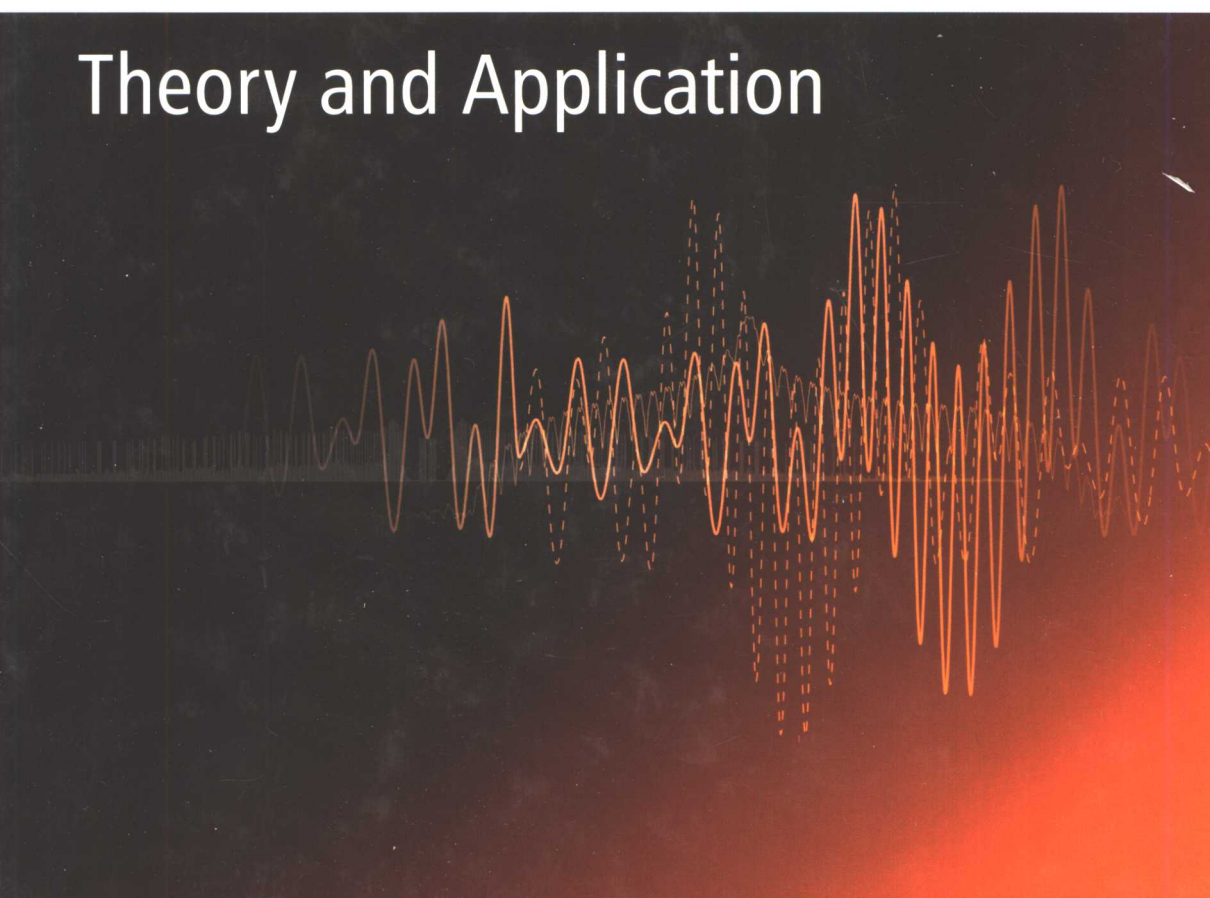


HANS-JÜRGEN ZEPERNICK | ADOLF FINGER

# Pseudo Random Signal Processing

Theory and Application



 WILEY



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**Hans-Jürgen Zepernick**

*Blekinge Institute of Technology, Sweden*

**Adolf Finger**

*Dresden University of Technology, Germany*



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# Preface

Pseudo random signal processing has emerged from space and military applications with a history of research and development in these areas spanning a period of more than 40 years. The main focus in these applications was on signal formats and processing techniques that can ensure signal integrity, especially immunity against jamming attacks. The evolution of digital mobile radio systems and the increasing demand for positioning systems along with the advances in integrated circuit complexity have resulted in frequent use of pseudo random signal processing as a viable technique for many civilian and commercial applications. Especially with the introduction of cellular mobile radio systems, pseudo random signal processing has received increased attention during the course of the late 1980s. The numerous features of these processing techniques that are important for cellular radio include the ability to eliminate or alleviate multipath propagation, the resistance to interference, and the potential of sharing allocated bandwidth with other users or even sharing it as an overlay with other communication systems. More recently, successful applications in commercial satellite navigation systems and third-generation mobile communication systems have proven the concepts of pseudo random signal processing as being an important and critical enabler of modern communication and information systems. In addition, the methodology of pseudo random signal processing has evolved into fields such as acoustics, biomedicine, and sensor systems, to mention just a few of the specialized application areas. It can be expected that an in-depth knowledge of pseudo random signal processing will provide the basis for the development of many new applications in communication, information, and computer technologies, in both the short and long term.

The objective of this book is to provide an important transition from covering the mathematical foundations to conveying the powerful engineering concepts of pseudo random signal processing. In particular, the far-reaching signal processing principles will serve as the connecting link between theory and practice. The book is intended to provide comprehensive coverage of the theoretical foundations of pseudo random signal processing, which makes it timeless and independent of the actual state of the art in circuit and system technology. The reader will also gain insights into the increasingly sophisticated applications of the described techniques in modern communication and information technologies such as mobile radio systems, navigation systems, scrambling, circuit testing, cryptology, and a number of selected specialized applications. The combination of theory and practice makes the book attractive as a practically oriented introduction to researchers and it provides essential reading for practicing engineers.

The book is based on our experience in the area of pseudo random signal processing obtained from numerous research projects, teaching senior graduate courses at universities, and delivering short courses to industry. The book is based on a course entitled "Digital Signal Structures," which has been taught by Adolf Finger. It also draws upon material from the senior graduate courses "Advanced Communications" and "Error Control Coding," which have been taught by Hans-Jürgen Zepernick.

These courses are aimed at students in their final year of studies to provide insights into the areas of advanced communications theory and how it can be utilized in practice.

The book is aimed at academics and students in the areas of electrical, electronic, and computer engineering as well as scientists and practicing engineers in research and development. It is suitable for a wide audience working in the fields of telecommunications, information technology, and computer science. The book is accessible to readers with at least an undergraduate electrical engineering or computer science background in signals and systems, communications, and electronics. The book is written at an advanced level and will enable the reader to access the more specialized technical articles and textbooks.

We are grateful to the many researchers whose original contributions form the foundations of pseudo random signal processing and who basically have made this book possible. We would also like to thank the anonymous reviewers who provided constructive suggestions and valuable comments that guided us in the early stages to shape the content of the book into its present form.

We would like to extend warm thanks to our students from Europe, the United States, Asia, and Australia, whose questions have helped us to refine the presentation. Special thanks go to our doctoral students and post-doctoral research fellows for the intellectual stimulation they have provided over the years. We would like to thank our friends and colleagues for many helpful discussions and support during the course of writing this book. We also wish to express our deep gratitude to Dr. Manora Caldera and Dr. Helmut Wiehl for proof-reading various parts of the manuscript and their valuable comments and suggestions which have helped us to improve the book.

Finally, we wish to thank the editorial and publishing team of John Wiley & Sons for their enormous assistance in the preparation of this book. In particular, we are very grateful to Birgit Gruber, Sarah Corney, Kathryn Sharples, Claire Twine, Simone Taylor, Emily Bone, and Wendy Hunter for guiding us safely through all the phases of the book project and for their professional work.

**Hans-Jürgen Zepernick**  
**Adolf Finger**

# List of abbreviations

3GPP	Third-generation partnership project
AACF	Aperiodic autocorrelation function
ACCF	Aperiodic crosscorrelation function
ACF	Autocorrelation function
ACL	Asynchronous connectionless
ADSL	Asymmetric digital subscriber line
AMPS	Advanced mobile phone service
AO	Auto-optimal
ASIC	Application-specific integrated circuit
ATM	Asynchronous transfer mode
AU	Astronomical unit
AWGN	Additive white Gaussian noise
BCD	Binary-coded decimal
BCH	Bose–Chaudhuri–Hocquenghem
BER	Bit error rate
B-ISDN	Broadband integrated services digital network
BIST	Built-in self-test
BOC	Binary offset carrier
BOT	Broadcast online television
BPF	Bandpass filter
BPSK	Binary phase shift keying
BS	Base station
CA	Conditional access
CCD	Charge-coupled device
CCD-PNMF	CCD pseudo noise matched filter
CCF	Crosscorrelation function
CD	Collision detection
CDMA	Code-division multiple-access
CIW	Container identification word
CMOS	Complementary metal oxide semiconductor
CO	Cross-optimal
CQI	Channel-quality indication
CS	Commercial service
CSMA	Carrier sense multiple-access
CSMA-CD	CSMA with collision detection

CSS	Content scrambling system
CUT	Circuit-under-test
CVBS	Composite video blanking and synchronization
CW	Control word
DAB	Digital audio broadcast
DAC	Digital-to-analog converter
DC	Direct current
DECT	Digital enhanced cordless telecommunications
DEMUX	Demultiplexer
DES	Data encryption standard
DGPS	Differential GPS
DH	Data high
DK	Distribution key
DLL	Delay-lock loop
DMT	Discrete multi-tone
DPCCH	Dedicated physical control channel
DPDCH	Dedicated physical data channel
DS	Direct-sequence
DS-CDMA	Direct-sequence code-division multiple-access
DSL	Digital subscriber line
DSP	Digital signal processing
DSR	Digital satellite radio
DSS	Distributed sample scrambling
DSSS	Direct-sequence spread spectrum
DVB	Digital video broadcast
DVD	Digital versatile disc
ECL	Emitter coupled logic
EEG	Electroencephalograph
EGNOS	European geostationary navigation overlay service
EK	Entitlement key
EOE	Equivalent odd and even
EPROM	Erasable programmable read-only memory
ESA	European Space Agency
ETSI	European Telecommunications Standards Institute
EU	European Union
EUVE	Extreme Ultraviolet Explorer
FBG	Fiber Bragg grating
FBI	Feedback information
FCC	Federal Communications Commission
FCSR	Feedback with carry shift register
FDD	Frequency-division duplex
FDMA	Frequency-division multiple-access
FFH	Fast frequency hopping
FH	Frequency hopping
FH-CDMA	Frequency hopping CDMA
FHS	Frequency hop synchronization
FHSS	Frequency hopping spread spectrum
FLL	Frequency-lock loop

FM	Frequency modulation
FPGA	Field programmable logic gate array
FSK	Frequency shift keying
FSS	Frame synchronous scrambling
FZC	Frank–Zadoff–Chu
Gbps	Gigabits per second
GLONASS	Global navigation satellite system
GMW	Gordon–Mills–Welch
GPS	Global positioning system
GRO	Gamma Ray Observatory
GSM	Global system for mobile communication
GSRx	Ground station reference receiver
HARQ	Hybrid automatic repeat request
HARQ-ACK	HARQ acknowledgment
HBI	Horizontal blanking interval
HDSL	High-bit-rate digital subscriber line
HEC	Header error control
HPA	High-power amplifier
HPSK	Hybrid PSK
HS-DPCCH	High-speed dedicated physical control channel
HS-PDSCH	High-speed dedicated physical downlink shared channel
IEEE	Institute of Electrical and Electronics Engineers
IF	Intermediate frequency
IFW	Interference-free window
IR	Infrared
IS-95	Interim Standard 95
ISDN	Integrated services digital network
ISI	Intersymbol interference
ISM	Industrial, scientific, and medical
ITU	International Telecommunications Union
LA	Large-area
LAB	Logic array block
LAP	Lower address part
LAS	Large-area synchronous
LC	Linear complexity
LED	Light-emitting diode
LEO	Low earth orbit
LFSR	Linear feedback shift register
LORAN	Long-range navigation
LPF	Lowpass filter
LS	Loosely synchronous codes
LSB	Least significant bit
LSE	Least sidelobe energy
LTI	Linear time-invariant
LUT	Look-up table
MAC	Medium access control
MAI	Multiple access interference
MASER	Microwave amplification by stimulated emission of radiation

MC	Multi-carrier
MCC	Master control center
MC-CDMA	Multi-carrier CDMA
MEO	Medium earth orbit
MF	Merit factor
MFSK	<i>M</i> -ary FSK
MLSSA	Maximal-length sequence system analyzer
MPDU	MAC sublayer protocol data unit
MPEG	Moving Pictures Experts Group
MS	Mobile station
MSAC	Mean-square out-of-phase aperiodic autocorrelation
MSAS	MT sat-based augmentation system
MSB	Most significant bit
MSCC	Mean-square aperiodic crosscorrelation
MSPD	Maximum peak-to-side-peak distance
MSPR	Maximum peak-to-side-peak ratio
MT-CDMA	Multi-tone CDMA
MUI	Multi-user interference
MUX	Multiplexer
NASA	National Aeronautics and Space Administration
NAVSTAR	Navigation system time and ranging
NCO	Number-controlled oscillator
NLES	Navigation land earth station
OCQPSK	Orthogonal complex quadrature phase shift keying
OFDM	Orthogonal frequency-division multiplexing
OOK	On-off keying
OQPSK	Offset quadrature phase shift keying
OS	Open services
OSI	Open system interconnection
OVSF	Orthogonal variable spreading factor
PACF	Periodic autocorrelation function
PAL	Phase alternate line
PAR	Peak-to-average ratio
PCCF	Periodic crosscorrelation function
PCM	Pulse code modulation
PCPCH	Physical common packet channel
PCS	Personal communications system
PD	Photo diode
PDF	Probability density function
PHY	Physical layer
PLA	Programmable logic array
PLCP	Physical layer convergence protocol
PLD	Programmable logic device
PLL	Phase-lock loop
PN	Pseudo noise
PPDU	PLCP protocol data unit
PRACH	Physical random access channel
PRBS	Pseudo random bit sequence

PRC	Pseudo random code
PRN	Pseudo random noise
PRPG	Pseudo random pattern generator
PROM	Programmable read-only memory
PRS	Public regulated services
PSC	Primary synchronization code
P-SCH	Primary SCH
PSD	Power spectral density
PSDU	PLCP service data unit
PSK	Phase shift keying
QAM	Quadrature amplitude modulation
QoS	Quality-of-service
QPSK	Quadrature phase shift keying
RAM	Random access memory
RARASE	Recursion-aided RASE
RASE	Rapid acquisition by sequential estimation
RF	Radio frequency
RIMS	Ranging and monitoring station
RNSS	Radio navigation satellite system
ROM	Read-only memory
RSA	Rivest–Shamir–Adleman
SA	Selective availability
SAR	Search and rescue
SAW	Surface acoustic wave
SCH	Synchronization channel
SDH	Synchronous digital hierarchy
SDU	Service data unit
SFD	Start frame delimiter
SFH	Slow frequency hopping
SIG	Special interest group
SK	Service key
SNR	Signal-to-noise ratio
SOH	Section overhead
SOL	Safety-of-life
S/P	Serial-to-parallel
SS	Spread spectrum
S-SCH	Secondary SCH
SSS	Self-synchronous scrambling
STM	Synchronous transport module
TD-CDMA	Time-division CDMA
TDD	Time-division duplex
TDL	Tau-dither loop
TDMA	Time-division multiple-access
TDRSS	Tracking and data relay satellite system
TFC	Transport format combination
TFCI	Transport format combination indicator
THSS	Time hopping spread spectrum
TIA	Telecommunications Industry Association

TPC	Transmit power control
TPG	Test pattern generator
TTL	Transistor-transistor logic
TV	Television
UAP	Upper address part
UHF	Ultra high frequency
UMTS	Universal mobile telecommunications system
U-NII	Unlicensed national information infrastructure
UTRA	UMTS terrestrial radio access
VBI	Vertical blanking interval
VCO	Voltage controlled oscillator
VDSL	Very high-bit-rate digital subscriber line
VEP	Visual evoked potential
VHDL	Very high-speed integrated circuit hardware description language
WAAS	Wide area augmentation system
WCDMA	Wideband CDMA
WLAN	Wireless local area network
WPAN	Wireless personal area network
WSS	Wide-sense stationary
XNR	Exclusive-NOR
ZC	Zadoff-Chu
ZCZ	Zero correlation zone

# List of common symbols

$a_i$	Element $i$ of sequence $\{a_i\}$
$a_i^{(k)}$	Element $i$ of $k^{th}$ sequence $\{a_i^{(k)}\}$
$a_i^{(l)}$	Element $i$ of $l^{th}$ sequence $\{a_i^{(l)}\}$
$\{a_i\}$	Unipolar binary, multilevel, or $q$ -ary sequence
$\{a_i^{(k)}\}$	$k^{th}$ unipolar binary, multilevel, or $q$ -ary sequence
$\{a_i^{(l)}\}$	$l^{th}$ unipolar binary, multilevel, or $q$ -ary sequence
<b>a</b>	Unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
<b>a</b> <sup>(k)</sup>	$k^{th}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
<b>a</b> <sup>(l)</sup>	$l^{th}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
$a(x)$	Polynomial
$a^*(x)$	Reciprocal polynomial of $a(x)$
$a(x, x^{-1})$	Laurent polynomial
$a(D)$	Power series representation of sequence $\{a_i\}$
$\arg(c)$	Argument of a complex number $c = c_1 + jc_2$
$A_s$	Agreements for discrete-time shift $s$
$b_i$	Element $i$ of sequence $\{b_i\}$
$b_i^{(k)}$	Element $i$ of $k^{th}$ sequence $\{b_i^{(k)}\}$
$b_i^{(l)}$	Element $i$ of $l^{th}$ sequence $\{b_i^{(l)}\}$
$\{b_i\}$	Unipolar binary, multilevel, or $q$ -ary sequence
$\{b_i^{(k)}\}$	$k^{th}$ unipolar binary, multilevel, or $q$ -ary sequence
$\{b_i^{(l)}\}$	$l^{th}$ unipolar binary, multilevel, or $q$ -ary sequence
<b>b</b>	Unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
<b>b</b> <sup>(k)</sup>	$k^{th}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
<b>b</b> <sup>(l)</sup>	$l^{th}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
$b(x)$	Polynomial
$b^*(x)$	Reciprocal polynomial of $b(x)$
$b(x, x^{-1})$	Laurent polynomial
$b(D)$	Power series representation of sequence $\{b_i\}$
$\mathcal{B}_i$	Conjugacy class of $\beta^i$ with respect to a given Galois field
$c_i$	Element $i$ of sequence $\{c_i\}$
$c_i^{(k)}$	Element $i$ of $k^{th}$ sequence $\{c_i^{(k)}\}$
$c_i^{(l)}$	Element $i$ of $l^{th}$ sequence $\{c_i^{(l)}\}$

$\{c_i\}$	Unipolar binary, multilevel, or $q$ -ary sequence
$\{c_i^{(k)}\}$	$k^{\text{th}}$ unipolar binary, multilevel, or $q$ -ary sequence
$\{c_i^{(l)}\}$	$l^{\text{th}}$ unipolar binary, multilevel, or $q$ -ary sequence
$\mathbf{c}$	Unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
$\mathbf{c}^{(k)}$	$k^{\text{th}}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
$\mathbf{c}^{(l)}$	$l^{\text{th}}$ unipolar binary, multilevel, or $q$ -ary sequence of length or period $N$
$c(x)$	Characteristic polynomial
$C_{am}$	Maximum out-of-phase aperiodic autocorrelation magnitude
$C_{cm}$	Maximum aperiodic crosscorrelation magnitude
$C_{max}$	Maximum nontrivial aperiodic correlation value
$C_{a,a}(s)$	Aperiodic autocorrelation function of $\mathbf{a}$
$C_{a,b}(s)$	Aperiodic crosscorrelation function of $\mathbf{a}$ and $\mathbf{b}$
$C_{x,x}(s)$	Aperiodic autocorrelation function of $\mathbf{x}$
$C_{x,y}(s)$	Aperiodic crosscorrelation function of $\mathbf{x}$ and $\mathbf{y}$
$C_{u,u}(s)$	Aperiodic autocorrelation function of $\mathbf{u}$
$C_{u,v}(s)$	Aperiodic crosscorrelation function of $\mathbf{u}$ and $\mathbf{v}$
$C_{k,k}(s)$	Aperiodic autocorrelation function of $k^{\text{th}}$ sequence $(\cdot)^{(k)}$
$C_{k,l}(s)$	Aperiodic crosscorrelation function of $k^{\text{th}}$ sequence $(\cdot)^{(k)}$ and $l^{\text{th}}$ sequence $(\cdot)^{(l)}$
$\mathcal{C}_i$	Cyclotomic coset
$D$	Shift operator
$D_s$	Disagreements for discrete-time shift $s$
$\det(\cdot)$	Determinant of a matrix
$\deg[a(x)]$	Degree of polynomial $a(x)$
$\exp(\cdot)$	Exponential function
$E_x$	Energy of $x(t)$
$E\{\cdot\}$	Expectation
$f$	Frequency
$f_c$	Chip rate
$f(x)$	General polynomial
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathcal{F}^{-1}\{\cdot\}$	Inverse Fourier transform
$g(x)$	Generator polynomial
$\gcd(a, b)$	Greatest common divisor of $a$ and $b$
$GF$	Galois field
$GF(p)$	Galois field or prime field of order $p$
$GF(q)$	Galois field or extension field of order $q = p^m$
$GF(p)[x]$	Set of polynomials $f(x)$ of arbitrary degree over $GF(p)$
$H(\cdot)$	Histogram
$i$	Discrete-time variable
$\lim$	Limit
$\text{lcm}(a, b)$	Least common multiple of $a$ and $b$
$m_x$	Mean of a random process $X(t)$
$m(x)$	Minimal polynomial
$\max\{\mathcal{A}\}$	Largest element of a set $\mathcal{A} = \{a_1, a_2, \dots, a_A\}$
$\min\{\mathcal{A}\}$	Smallest element of a set $\mathcal{A} = \{a_1, a_2, \dots, a_A\}$
$\text{mod}$	Modulo operation
$n$	Polynomial degree

$N$	Length or period of a sequence
$N_p(n)$	Number of monic irreducible polynomials $p(x)$ in $GF(p)[x]$ of degree $n$
$ord(\beta)$	Order of an element $\beta$ of a finite field
$p(x)$	Irreducible or primitive polynomial
$P_e$	Bit error rate
$P_x$	Power of $x(t)$
$PG$	Processing gain
$r_{x,x}(\tau)$	Normalized autocorrelation coefficient of $x(t)$
$r_{x,y}(\tau)$	Normalized crosscorrelation coefficient of $x(t)$ and $y(t)$
$rect(\cdot)$	Rectangular pulse
$R_{ac}$	Average mean-square out-of-phase aperiodic autocorrelation value
$R_{am}$	Maximum out-of-phase periodic autocorrelation magnitude
$\hat{R}_{am}$	Maximum out-of-phase periodic odd autocorrelation magnitude
$R_{cc}$	Average mean-square aperiodic crosscorrelation value
$R_{cm}$	Maximum periodic crosscorrelation magnitude
$\hat{R}_{cm}$	Maximum periodic odd crosscorrelation magnitude
$R_{max}$	Maximum nontrivial periodic correlation value
$R_{a,a}(s)$	Periodic autocorrelation function of <b>a</b>
$R_{a,b}(s)$	Periodic crosscorrelation function of <b>a</b> and <b>b</b>
$R_{x,x}(s)$	Periodic autocorrelation function of <b>x</b>
$R_{x,y}(s)$	Periodic crosscorrelation function of <b>x</b> and <b>y</b>
$R_{u,u}(s)$	Periodic autocorrelation function of <b>u</b>
$\hat{R}_{u,u}(s)$	Odd periodic autocorrelation function of <b>u</b>
$R_{u,v}(s)$	Periodic crosscorrelation function of <b>u</b> and <b>v</b>
$\hat{R}_{u,v}(s)$	Odd periodic crosscorrelation function of <b>u</b> and <b>v</b>
$R_{x,x}(\tau)$	Autocorrelation function of $x(t)$
$R_{x,y}(\tau)$	Crosscorrelation function of $x(t)$ and $y(t)$
$R_{u,u}(\tau)$	Autocorrelation function of $u(t)$
$R_{u,v}(\tau)$	Crosscorrelation function of $u(t)$ and $v(t)$
$R_{X,X}(\tau)$	Autocorrelation function of a wide-sense stationary random process $X(t)$
$R_{X,X}(t_i, t_j)$	Autocorrelation function of a random process $X(t)$
$R_{X,Y}(t_i, t_j)$	Crosscorrelation function of two random processes $X(t)$ and $Y(t)$
$R_d(n)$	Residue or remainder
$\mathcal{R}_r$	Residue class
$\Re\{c\}$	Real part of a complex number $c = c_1 + jc_2$
$s$	Discrete-time shift variable
$sgn(\cdot)$	Signum function
$sinc(\cdot)$	Sinc function
$step(\cdot)$	Step function
$S_{x,x}(f)$	Power spectral density of $x(t)$
$S_{u,u}(f)$	Power spectral density of $u(t)$
$S_{x,x}(k)$	Autocorrelation spectrum of <b>x</b>
$S_{x,y}(k)$	Crosscorrelation spectrum of <b>x</b> and <b>y</b>
$t$	Continuous-time variable
$T$	Period of continuous-time signal
$T_c$	Chip duration
$u_i$	Element $i$ of sequence $\{u_i\}$

$u_i^{(k)}$	Element $i$ of $k^{\text{th}}$ sequence $\{u_i^{(k)}\}$
$u_i^{(l)}$	Element $i$ of $l^{\text{th}}$ sequence $\{u_i^{(l)}\}$
$\{u_i\}$	Complex-valued or polyphase sequence
$\{u_i^{(k)}\}$	$k^{\text{th}}$ complex-valued or polyphase sequence
$\{u_i^{(l)}\}$	$l^{\text{th}}$ complex-valued or polyphase sequence
$\mathbf{u}$	Complex-valued or polyphase sequence of length or period $N$
$\mathbf{u}^{(k)}$	$k^{\text{th}}$ complex-valued or polyphase sequence of length or period $N$
$\mathbf{u}^{(l)}$	$l^{\text{th}}$ complex-valued or polyphase sequence of length or period $N$
$u(t)$	Complex-valued signal
$v_i$	Element $i$ of sequence $\{v_i\}$
$v_i^{(k)}$	Element $i$ of $k^{\text{th}}$ sequence $\{v_i^{(k)}\}$
$v_i^{(l)}$	Element $i$ of $l^{\text{th}}$ sequence $\{v_i^{(l)}\}$
$\{v_i\}$	Complex-valued or polyphase sequence
$\{v_i^{(k)}\}$	$k^{\text{th}}$ complex-valued or polyphase sequence
$\{v_i^{(l)}\}$	$l^{\text{th}}$ complex-valued or polyphase sequence
$\mathbf{v}$	Complex-valued or polyphase sequence of length or period $N$
$\mathbf{v}^{(k)}$	$k^{\text{th}}$ complex-valued or polyphase sequence of length or period $N$
$\mathbf{v}^{(l)}$	$l^{\text{th}}$ complex-valued or polyphase sequence of length or period $N$
$v(t)$	Complex-valued signal
$wt(\mathbf{a})$	Weight of $\mathbf{a}$
$x_i$	Element $i$ of sequence $\{x_i\}$
$x_i^{(k)}$	Element $i$ of $k^{\text{th}}$ sequence $\{x_i^{(k)}\}$
$x_i^{(l)}$	Element $i$ of $l^{\text{th}}$ sequence $\{x_i^{(l)}\}$
$\{x_i\}$	Bipolar binary or real-valued sequence
$\{x_i^{(k)}\}$	$k^{\text{th}}$ bipolar binary or real-valued sequence
$\{x_i^{(l)}\}$	$l^{\text{th}}$ bipolar binary or real-valued sequence
$\mathbf{x}$	Bipolar binary or real-valued sequence of length or period $N$
$\mathbf{x}^{(k)}$	$k^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$\mathbf{x}^{(l)}$	$l^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$x(t)$	Real-valued signal
$X$	Random variable
$X(t)$	Random process
$X(f)$	Fourier transform of $x(t)$
$y_i$	Element $i$ of sequence $\{y_i\}$
$y_i^{(k)}$	Element $i$ of $k^{\text{th}}$ sequence $\{y_i^{(k)}\}$
$y_i^{(l)}$	Element $i$ of $l^{\text{th}}$ sequence $\{y_i^{(l)}\}$
$\{y_i\}$	Bipolar binary or real-valued sequence
$\{y_i^{(k)}\}$	$k^{\text{th}}$ bipolar binary or real-valued sequence
$\{y_i^{(l)}\}$	$l^{\text{th}}$ bipolar binary or real-valued sequence
$\mathbf{y}$	Bipolar binary or real-valued sequence of length or period $N$
$\mathbf{y}^{(k)}$	$k^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$\mathbf{y}^{(l)}$	$l^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$y(t)$	Real-valued signal
$Y$	Random variable
$Y(t)$	Random process
$Y(f)$	Fourier transform of $y(t)$

$z_i$	Element $i$ of sequence $\{z_i\}$
$z_i^{(k)}$	Element $i$ of $k^{\text{th}}$ sequence $\{z_i^{(k)}\}$
$z_i^{(l)}$	Element $i$ of $l^{\text{th}}$ sequence $\{z_i^{(l)}\}$
$\{z_i\}$	Bipolar binary or real-valued sequence
$\{z_i^{(k)}\}$	$k^{\text{th}}$ bipolar binary or real-valued sequence
$\{z_i^{(l)}\}$	$l^{\text{th}}$ bipolar binary or real-valued sequence
$\mathbf{z}$	Bipolar binary or real-valued sequence of length or period $N$
$\mathbf{z}^{(k)}$	$k^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$\mathbf{z}^{(l)}$	$l^{\text{th}}$ bipolar binary or real-valued sequence of length or period $N$
$z(t)$	Real-valued signal
$\alpha$	Primitive element of a Galois field
$\beta$	Element of a Galois field
$\gamma$	Element of a Galois field
$\delta(\cdot)$	Dirac impulse
$\eta$	Energy efficiency
$\mu(\cdot)$	Möbius function
$\mu_{ij}$	Covariance coefficient
$\phi(\cdot)$	Euler's totient function
$\psi_j(x)$	Cyclotomic polynomial
$\rho_{ij}$	Correlation coefficient
$\tau$	Continuous-time shift variable
$\Delta t$	Chip duration
$\Lambda(\cdot)$	Triangular pulse
$\mathbb{C}$	Set of complex numbers
$\mathbb{F}$	Finite field
$\mathbb{N}$	Set of natural numbers
$\mathbb{N}^+$	Set of nonzero natural numbers
$\mathbb{R}$	Set of real numbers
$\mathbb{Z}$	Set of integer numbers
$ \cdot $	Absolute value or magnitude
$\ \cdot\ $	Cardinality or size of a set
$(\cdot)^T$	Transposition of a vector or matrix
$(\cdot)^{-1}$	Inversion of a matrix
$(\cdot)^*$	Complex conjugation
$\lfloor x \rfloor$	Greatest integer less than or equal to $x$
$\lceil x \rceil$	Smallest integer greater than or equal to $x$
$\oplus$	Modulo addition
$\otimes$	Kronecker product
$\wedge$	Conjunction
$\vee$	Disjunction
$\forall$	For all
$\in$	Is an element of
$\cup$	Union
$\subset$	Subset
$\neg$	Negation
$\equiv$	Equivalence
$\circ\!\!\!\rightarrow$	Fourier transform

$a!$	Factorial function
$a b$	$a$ divides $b$
$\binom{a}{b}$	Binomial coefficient
$\left(\frac{a}{b}\right)$	Legendre symbol
$a(t) * b(t)$	Convolution
$a(t) \star b(t)$	Correlation
$\mathcal{A} \setminus \mathcal{B}$	Difference of set $\mathcal{A}$ and set $\mathcal{B}$
$\infty$	Infinity