

# Holographic Interferometry

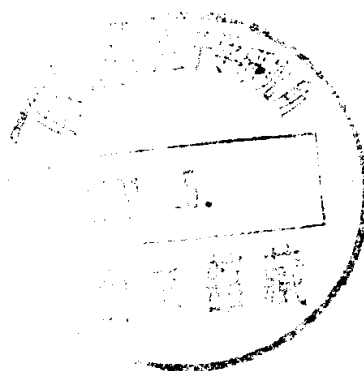
CHARLES M. VEST

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# Holographic Interferometry

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# Preface

In writing this book I have attempted to provide a unified, self-contained treatment of the theory, practice, and application of holographic interferometry and related coherent optical measurement techniques. Emphasis is placed on quantitative evaluation of holographic interferograms of both opaque and transparent objects. The technical level and the scope of the book are such that it should be of interest to workers and students in coherent optics and to engineers and scientists from other disciplines who wish to evaluate or use holographic interferometry as a measurement or diagnostic technique. Clear, simple, physical reasoning is applied in a manner that circumvents the use of complicated mathematics wherever possible. Practical information and data regarding light sources, recording media, the design and construction of holographic systems, and physical properties such as refractive index are given throughout the book and are made readily accessible by careful indexing. The book serves as a guide to the literature of the subject and contains almost 700 cited references. It could be used as a textbook or supplementary reference in senior-graduate level courses in holographic interferometry, optical measurement techniques, engineering measurements, or holography.

The principles of holographic interferometry were discovered very soon after the introduction of off-axis holography by Leith and Upatnieks in 1961. Much of the basic theory, practice, and future potential of the method were presented clearly in the early papers of Powell and Stetson, Burch, and Heflinger, Wuerker, and Brooks. This stage was followed by a surge of activity, some of which suggested that holographic interferometry was viewed by some as a panacea for problems encountered in engineering and scientific metrology. Activity diminished as the practical limitations of the method were recognized, but work continued; the basic theory was refined, and more emphasis was placed on quantitative interpretation of interferograms. During the last 5 years a steady diffusion of the literature of holographic interferometry from optics journals into the journals of other disciplines reflects a new, more mature period of application and

growth. It is my belief that this growth in the importance of holographic interferometry as a scientific and engineering tool will continue as specific applications for which it is uniquely suited are recognized. I hope that this book will contribute to the understanding required for future accomplishments. The current range of applications is remarkable. Readers of this book will learn that holographic interferometry is used in plasma experiments associated with the laser-fusion program, and has served to increase understanding of the mechanics of hearing, to measure relaxation rates of structural materials, to observe the vibration of turbine blades in operating jet engines, to design musical instruments, and to detect subsurface damage in fifteenth-century panel paintings.

Work in holographic interferometry has also stimulated the development, extension, or adaptation of several closely related measurement techniques based on the use of laser light. These include holographic contour generation, speckle photography and interferometry, projected fringe techniques, techniques incorporating television systems, and holographic photoelasticity. Because of their close, and often complementary, relationship to holographic interferometry, the theory, practice, and application of these techniques are also discussed in the book.

In organizing and presenting the material I have assumed that many readers will approach the topic without the benefit of a formal background in coherent optics or Fourier transform theory. For this reason I have written an introductory chapter in which the elements of coherent optics, holography, and holographic interferometry are presented. Although readers already conversant with coherent optics may wish to bypass all but the last section of this chapter, others will find that it contains virtually all the basic concepts required for the developments in the rest of the book. Also, simple physical and geometrical arguments are emphasized throughout the book. This is particularly evident in the analysis of fringe localization. Although all but the most elementary formalisms of Fourier optics and the mathematics associated with diffraction theory are bypassed, quantitative aspects of the subject are emphasized.

I have provided a substantial number of references to the published literature for those who wish to pursue particular topics in more detail. References are restricted almost exclusively to journal articles because I have found that most important results of research published in reports and symposia proceedings usually reappear in article form. There are, of course, exceptions and time lags, but that is a risk encountered in writing about a field in which applications are still developing.

I wish to acknowledge my debt to Emmett Leith, who, together with my colleagues in the Department of Mechanical Engineering at The University of Michigan, has encouraged my activities in engineering applications of coherent optics over the past 10 years. Special thanks go also to Joseph

Goodman, Albert Macovski, and Daniel Bershader of Stanford University, who provided the opportunity and professional atmosphere in which the initial research and writing were done. Karl Stetson contributed immeasurably to this book through his excellent published work and his generous personal cooperation and assistance. Research carried out with Don Sweeney, Predrag Radulovic, Ron Boyd, and Soyoung Cha have contributed enormously to my understanding of this topic. Vicki Rothhaar produced many of the interferograms used to illustrate this book, and Karl Stetson, Soyoung Cha, and Bruce Hansche carefully read major portions of the manuscript and provided valuable criticisms. Madelyn Hudkins greatly eased the effort of writing by producing hundreds of pages of error-free typing with remarkable efficiency and cheerfulness. Finally, I want to express my sincere thanks to my family for their patience, interest, and encouragement.

CHARLES M. VEST

*Ann Arbor, Michigan  
December 1978*

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# 1

## Coherent Optics, Holography, and Holographic Interferometry

### 1.1 INTRODUCTION

In this introductory chapter a variety of concepts and procedures which are required for development of the theory, practice, and application of holographic interferometry are presented. The subject matter has been chosen so that the treatment of holographic interferometry in this book will be selfcontained and accessible to persons having little or no prior knowledge of coherent optics. To further this goal, descriptive physical approaches are used in preference to detailed analysis wherever possible. In particular, the topics of coherence, diffraction, and holography are discussed without the use of the Fourier transform.

After a general discussion of light, coherence, and interferometry, the topic of diffraction is introduced in order to develop the concept and nomenclature of spatial frequencies and the basic idea of spatial filtering. The phenomenon of laser speckle is discussed, and the manner in which it is analyzed quantitatively is described. This material is useful for understanding the holographic interferometry of diffusely scattering objects and is required for the discussion of speckle photography and interferometry in Chapter 7.

The theory and practice of holography are discussed briefly. Some emphasis is given to experimental considerations such as properties of photographic emulsions and selection of reference-to-object-beam ratios. Finally, the basic concepts of holographic interferometry are presented in order to establish nomenclature and prepare the reader for the detailed presentations in Chapters 2 to 6.

### 1.2 LIGHT, INTERFERENCE, AND COHERENCE

Light is a form of electromagnetic radiation. It is characterized by its amplitude, wavelength (or frequency), phase, polarization, speed of propagation, and direction of propagation. When light is scattered or reflected

by the surface of an opaque object, or when it is transmitted through a transparent medium, any or all of these characteristics may be altered. By measuring the changes in these characteristics, we obtain information about the state of the object, for example, its size, shape, temperature, velocity, density, or state of stress. Holographic interferometry is one important method for carrying out measurements of this type. Some basic concepts regarding light, interference, and coherence are required for development of the theory and practice of holographic interferometry. A brief introduction to these topics is given in this section.

An electromagnetic wave such as light can be described by specifying the temporal and spatial dependence of its electric intensity vector  $\mathbf{E}$ . A more complete description requires specification of the magnetic intensity  $\mathbf{H}$ , the electric displacement  $\mathbf{D}$ , and the magnetic induction  $\mathbf{B}$ , which are interrelated by the Maxwell equations. We restrict our attention to  $\mathbf{E}$  because we are interested in the form of the wave rather than its basic physics, and because the photographic recording materials used in holographic interferometry respond primarily to the  $\mathbf{E}$  field.

The simplest type of electromagnetic wave is the linearly polarized plane wave. If such a wave is polarized in the  $y$  direction and propagates in the  $z$  direction, the three components of  $\mathbf{E}$  are

$$\begin{aligned} E_x &= 0, \\ E_y &= A \cos(\omega t - kz), \\ E_z &= 0. \end{aligned} \quad (1.1)$$

Here  $A$  is the amplitude of the wave, and its *circular frequency*  $\omega$  and *wave number*  $k$  are given by

$$\omega = 2\pi\nu \quad (1.2)$$

and

$$k = \frac{2\pi}{\lambda}, \quad (1.3)$$

where  $\nu$  is the temporal frequency and  $\lambda$  is the wavelength. The frequency of light is on the order of  $10^{15}$  Hz, and visible light has wavelengths in the range  $0.38 < \lambda < 0.76 \mu\text{m}$ . The light wave travels at its phase speed  $v = \omega/k$ . This speed depends on the medium in which the light propagates. Its maximum value,  $3 \times 10^8$  m/s, occurs in a vacuum and is denoted by  $c$ .

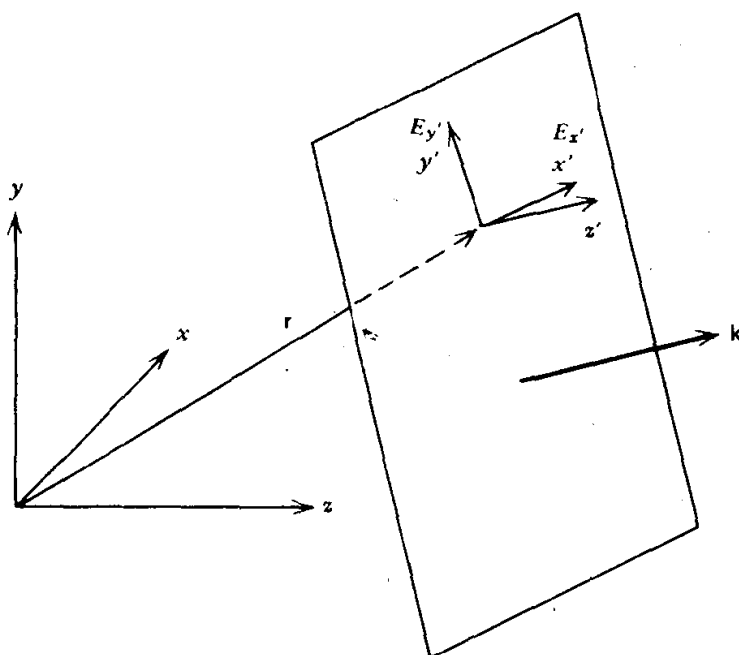
The wave described by equations 1.1 was termed a plane wave because at any instant of time  $\mathbf{E}$  has the same value at all points lying in the same plane,  $z = \text{constant}$ , normal to the direction of propagation. It was termed

linearly polarized because  $\mathbf{E}$  at any point is always directed along the same line parallel to the  $y$  axis. More generally, we describe the direction in which a light wave travels by its *propagation vector*  $\mathbf{k}$ , which has magnitude  $k = 2\pi/\lambda$  and points in the direction of propagation. A *plane wave* is a wave whose phase at any instant of time is constant at all points on any plane normal to  $\mathbf{k}$ . If  $\mathbf{r} = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z$  is the position vector of any point in space, as shown in Figure 1.1, the equation of a linearly polarized plane wave is

$$\begin{aligned} E_x &= 0, \\ E_y &= A \cos(\omega t - \mathbf{k} \cdot \mathbf{r}), \\ E_z &= 0. \end{aligned} \tag{1.4}$$

A surface over which phase is constant, in this case the planes  $\mathbf{k} \cdot \mathbf{r} = \text{constant}$ , is called a *wavefront*. Another wave of simple form which is important in optics is the *spherical wave* arising from a point source of light. In this case the wavefronts are concentric spheres,  $r = \text{constant}$ , about the point source. The amplitude of  $\mathbf{E}$  decreases in inverse proportion to the distance from the source.

To discuss the concept of polarization further, we consider the time dependence of  $\mathbf{E}$  at a point in space, such as that located at  $\mathbf{r}$  in Figure 1.1. If  $x'$  and  $y'$  are any mutually orthogonal axes lying in the plane tangent to

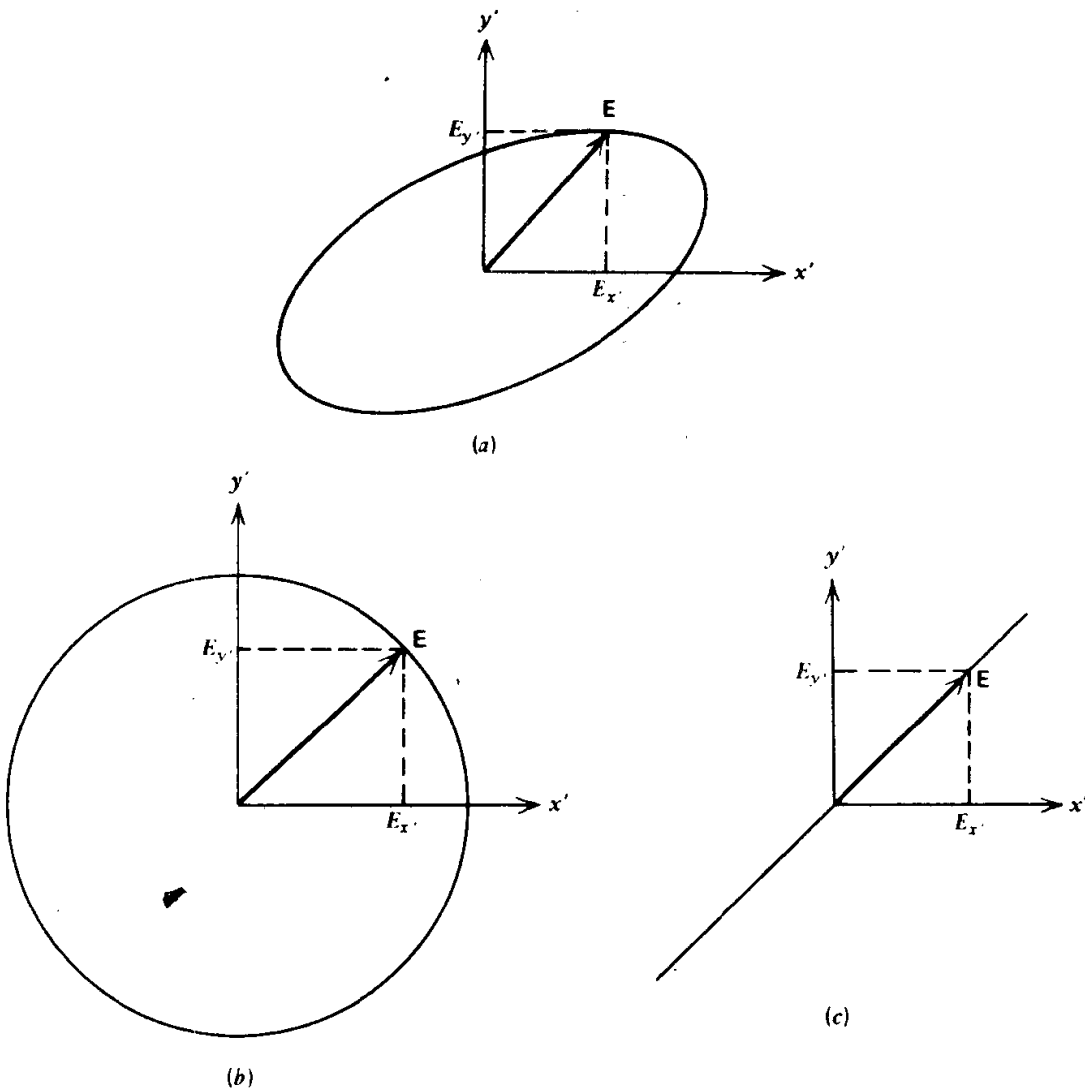


**Figure 1.1** Plane wave of light propagating in the direction specified by the propagation vector  $\mathbf{k}$ .

the wavefront at  $\mathbf{r}$ , then

$$\begin{aligned} E_{x'} &= A_{x'} \cos(\omega t - \mathbf{k} \cdot \mathbf{r}), \\ E_{y'} &= A_{y'} \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi), \\ E_{z'} &= 0, \end{aligned} \quad (1.5)$$

where  $\phi$  denotes the phase difference between the  $x'$  and  $y'$  components of  $\mathbf{E}$ . Since  $E_{x'}$  and  $E_{y'}$  both vary harmonically with time, the tip of the vector  $\mathbf{E}$ , which is their resultant, traces out a closed curve in the  $x'$ - $y'$  plane. Equations 1.5 are a parametric representation of this curve. A bit of algebra discloses that in general this equation describes an ellipse, as shown in Figure 1.2a. Such light is said to be *elliptically polarized*. Two

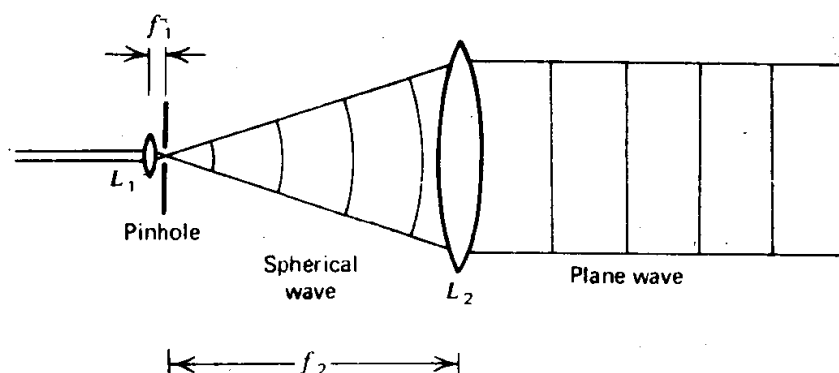


**Figure 1.2** Polarization defined by the curve traced by the tip of the  $\mathbf{E}$  vector: (a) elliptical polarization; (b) circular polarization; (c) linear polarization.

cases of great practical importance arise when the relative phase  $\phi$  and the amplitudes  $A_x$  and  $A_y$  have special values. The first is *circularly polarized* light, for which  $A_x = A_y$  and  $\phi = \pm(2N+1)\pi/2$ , where  $N$  is any positive integer or zero. The second is *linearly polarized* light, for which  $\phi = \pm N\pi$ . The term “plane polarization” is used synonymously with “linear polarization.” The corresponding curves traced out by  $\mathbf{E}$  in these cases are shown in Figures 1.2*b* and 1.2*c*, respectively.

In the vast majority of applications of holographic interferometry a laser is used as the light source. Lasers emit light waves of unusually simple form, having parameters which are quite constant with time and can be measured with high precision. Lasers emit narrow beams of nearly monochromatic light with almost perfectly plane wavefronts. Most lasers emit linearly polarized light. Light from a typical He-Ne continuous wave (cw) laser used for holographic interferometry has a wavelength  $\lambda = 632.8$  nm, which is constant to within about  $5 \times 10^{-4}$  nm. It is emitted in a beam of about 2 mm diameter, which diverges at an angle less than 0.7 mrad and is linearly polarized to better than 1 part in  $10^3$ . The most important characteristic of laser light for the applications discussed in this book is its high coherence. Coherence is discussed later in this section.

Spherical and plane waves used in holographic interferometry can be produced from a narrow beam of laser light, as shown in Figure 1.3. The beam is passed through a small positive lens such as a microscope objective of short focal length  $f_1$ . After passing through the focal point, the rays diverge to form a spherical wave. If desired, this wave can be *collimated* by using a second lens of larger focal length  $f_2$ . If this lens is placed a distance  $f_2$  from the origin of the spherical wave, a plane wave is formed, as shown in the figure. Typically, laser light is linearly polarized in the vertical direction. If desired, this can be converted to circularly polarized light by passing it through a *quarter-wave plate*. (See Section 7.6.)



**Figure 1.3** Thin collimated wave is expanded by lens  $L_1$  of focal length  $f_1$  to form a spherical wave. A plane wave is then formed by a second lens  $L_2$  of focal length  $f_2$ .

The phenomenon of *interference* is central to the subject matter of this book. In the rest of this section we discuss interference and the related property of light referred to as coherence. It was noted above that the frequency of light is approximately  $10^{15}$  Hz. Practical detectors such as photographic film, photodiodes, or the retina of an eye are not capable of responding to such extremely rapid variations. Rather, they respond to *irradiance*, which is the time-average energy flux of the light wave. We denote irradiance by  $I$ . Using electromagnetic theory, it can be shown that

$$I = \epsilon v \langle \mathbf{E}^2 \rangle, \quad (1.6)$$

where  $\epsilon$  is the electrical permittivity of the medium in which the light travels, and  $v$  is the speed of propagation. The key point is that  $I$  is proportional to the time average of  $\mathbf{E}^2$ , so the proportionality constant  $\epsilon v$  in equation 1.6 will be dropped in the rest of our discussion. To initiate consideration of interference, we suppose that two different light waves  $\mathbf{E}_1$  and  $\mathbf{E}_2$  of the same frequency are superimposed. Since  $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ , the irradiance will be

$$I = \langle \mathbf{E}^2 \rangle = \langle \mathbf{E}_1^2 \rangle + \langle \mathbf{E}_2^2 \rangle + 2\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle. \quad (1.7)$$

For simplicity we assume that both waves are linearly polarized in the same direction. We then have a simple scalar computation involving

$$E_1 = A_1 \cos(\omega t - \mathbf{k}_1 \cdot \mathbf{r}) \quad (1.8)$$

and

$$E_2 = A_2 \cos(\omega t - \mathbf{k}_2 \cdot \mathbf{r} + \phi), \quad (1.9)$$

where  $\phi$  is a constant relative phase between the two waves. Combining equations 1.7 to 1.9 and carrying out the averaging, we find that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta \quad (1.10)$$

where  $I_1 = A_1^2$ ,  $I_2 = A_2^2$ , and

$$\delta = \mathbf{k}_2 \cdot \mathbf{r} - \mathbf{k}_1 \cdot \mathbf{r} - \phi \quad (1.11)$$

is the phase difference between the two waves at any location. The irradiance varies from a minimum value  $I_{\min} = I_1 + I_2 - 2(I_1 I_2)^{1/2}$  at points where  $\delta = 2N + 1$  to a maximum value  $I_{\max} = I_1 + I_2 + 2(I_1 I_2)^{1/2}$  at points where  $\delta = 2N\pi$ ,  $N$  being an integer. The irradiance pattern in any plane can be recorded simply by exposing a sheet of photographic film to the

light. It can also be viewed on a diffusing screen such as a plate of ground glass. In either case a pattern consisting of alternate light and dark fringes will be observed. This fringe pattern enables one to measure the spatial distribution of phase difference between the two waves.

A specific, important example of interference is that of two spherical waves emanating from two point sources of light,  $S_1$  and  $S_2$ , as shown in Figure 1.4. Assume that  $S_1$  and  $S_2$  radiate in phase, that is,  $\phi=0$ . The irradiance at any point  $P$  in space is given by equation 1.10 with the phase difference

$$\delta = k(r_1 - r_2).$$

The locus of points forming a surface of maximum irradiance is determined by setting  $\delta = 2\pi N$ :

$$r_1 - r_2 = \frac{2\pi N}{k} = N\lambda, \quad N = 0, 1, 2, \dots \quad (1.12)$$

This is the equation of a family of hyperboloids of revolution about the axis  $\overline{S_1 S_2}$  connecting the two point sources. The interference fringes which would be observed by placing a detector surface such as a sheet of film into the field is the intersection of these hyperboloids with that surface. For example, the intersections with a plane containing  $\overline{S_1 S_2}$  are shown in Figure 1.4. The fringes formed on a plane will always be straight lines, or arcs of circles or hyperbolas, depending on the location and orientation of the plane.

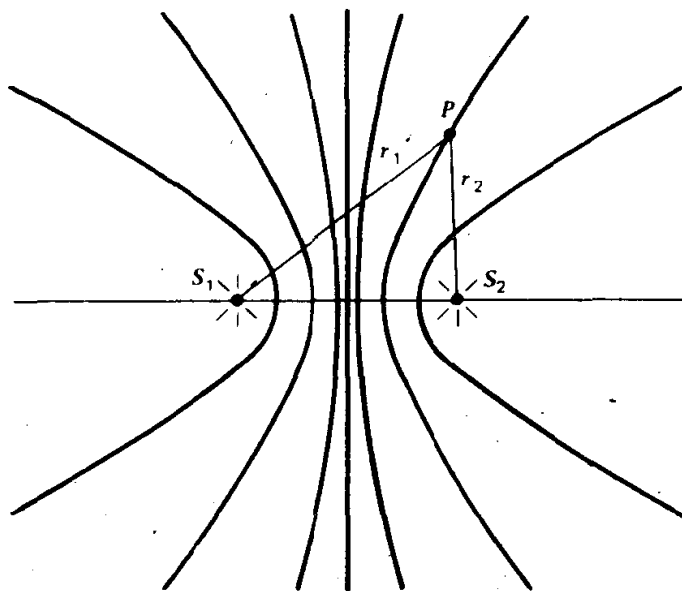


Figure 1.4 Interference of light emitted by two point sources,  $S_1$  and  $S_2$ .



Two light waves which are capable of interfering with each other are said to be *coherent* (a term which will be defined more precisely below). Because of coherence requirements most interference experiments are conducted using two images of the same physical source. These images are produced by an instrument called an *interferometer*. There are two basic types of interferometers: *division of wavefront* interferometers and *division of amplitude* interferometers. We will consider briefly one example of each.

A simple division of wavefront interferometer is that used to form *Young's fringes*. This is shown in Figure 1.5. It is simply an opaque screen in which two small holes (or parallel slits) separated by a distance  $b$  have been cut. This screen is illuminated by a small point source located a distance  $l_s$  behind the screen and a small distance  $y_s$  above the axis of symmetry. The light diffracted by the two holes (or slits) forms an interference pattern which can be observed on a screen placed some distance  $l_o$  away. In practice,  $y_s$ ,  $y$ , and  $b$  are much smaller than  $l_s$  and  $l_o$ . The irradiance of each of the light waves at  $y$  will therefore be nearly equal,  $I_1 = I_2 = I_0$ , so that equation 1.10 becomes

$$\begin{aligned} I &= 2I_0(1 + \cos \delta) \\ &= 4I_0 \cos^2\left(\frac{\delta}{2}\right). \end{aligned} \quad (1.13)$$

Here the phase difference  $\delta = (2\pi/\lambda)\Delta l$ , where  $\Delta l$  is the difference in distance the light travels from the source  $S$  to the observation point at  $y$ :

$$\begin{aligned} \Delta l &= \left\{ \left[ l_s^2 + \left( \frac{b}{2} - y_s \right)^2 \right]^{1/2} + \left[ l_o^2 + \left( \frac{b}{2} - y \right)^2 \right]^{1/2} \right\} \\ &\quad - \left\{ \left[ l_s^2 + \left( \frac{b}{2} + y_s \right)^2 \right]^{1/2} + \left[ l_o^2 + \left( \frac{b}{2} + y \right)^2 \right]^{1/2} \right\}, \end{aligned}$$

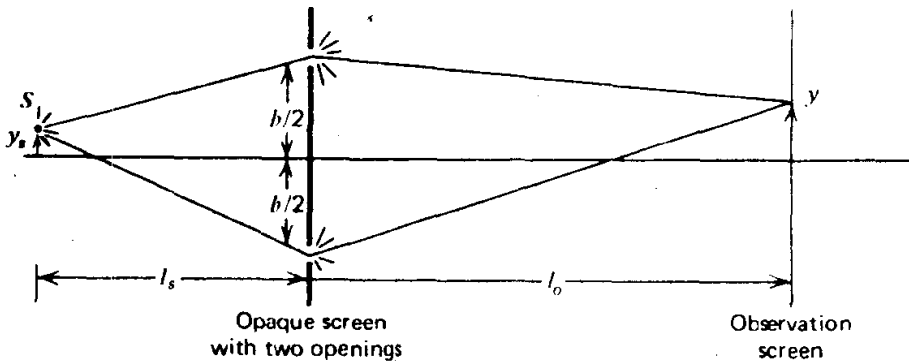


Figure 1.5 Interferometer to form Young's fringes.