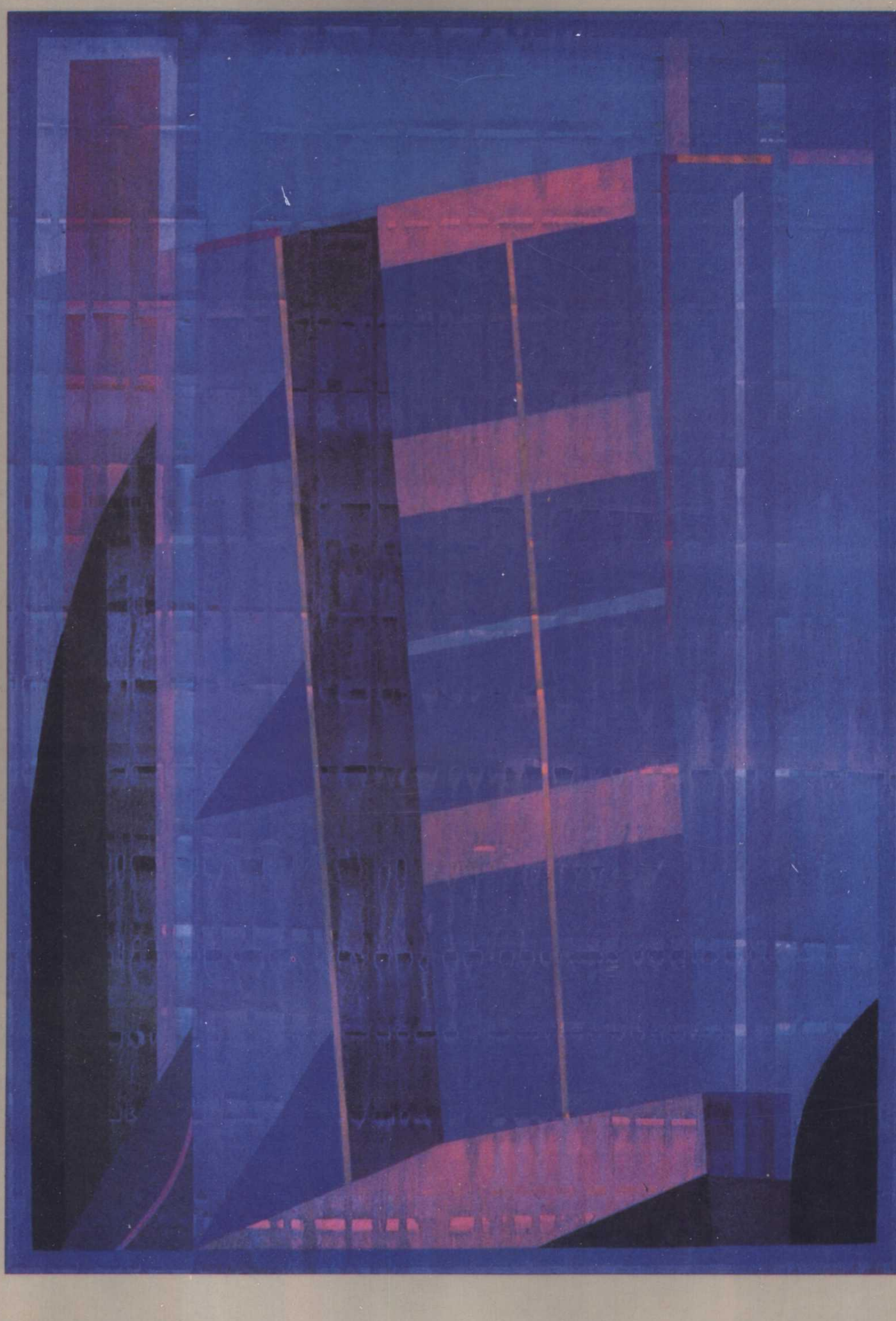
Linda Pulsinelli Patricia Hooper

Second Edition

INTERMEDIATE ALGEBRA
AN INTERACTIVE APPROACH



Intermediate Algebra

An Interactive Approach

SECOND EDITION

Linda Pulsinelli
Patricia Hooper
Department of Mathematics
Western Kentucky University

MACMILLAN PUBLISHING COMPANY
NEW YORK
Collier Macmillan Publishers
London

For two of our teachers who sparked our early interest in mathematics: Ann Hancock Collins and Norman E. Cromack

Printed in the United States of America

Macmillan Publishing Company
866 Third Avenue, New York, New York 10022

Collier Macmillan Canada, Inc.

Library of Congress Cataloging in Publication Data

Pulsinelli, Linda Ritter.
Intermediate algebra.
Includes index.
1. Algebra. I. Hooper, Patricia (Patricia I.)
II. Title.
QA154.2.P85 1987 512.9 86-8497
ISBN 0-02-396990-3

Printing: 6 7 8 Year: 9 0 1 2 3 4

ISBN 0-02-396990-3

Preface

Intermediate Algebra: An Interactive Approach is designed to be useful to students in bridging the gap between high school and college algebra. It is generally assumed that a student using this textbook has been exposed to an introductory algebra course but is not ready to tackle traditional college algebra.

In order to make this mathematics textbook readable, we have worded explanations in clear, concise language understandable to students at this level. Our general approach is to start from a fundamental idea with which students are familiar and proceed to a related concept in the most straightforward, intuitive way possible.

Our experience with intermediate algebra students leads us to believe that they must practice each new skill as soon as it has been presented. For this reason the book includes several unique features that are designed to provide maximum reinforcement. Each chapter in the book follows the same basic structure.

Motivational Applied Problem

At the beginning of each chapter we have presented an applied problem that can be solved after the student has mastered the skills in that chapter. Its solution appears within the chapter.

Explanations

We have tried to avoid the "cookbook" approach to algebra by including a straightforward and readable explanation of each new concept. Realizing that students at the intermediate level become easily bogged down in reading lengthy explanations, we have attempted to make our explanations as brief as possible without sacrificing rigor.

Highlighting

Definitions, properties, theorems, and formulas are highlighted in boxes throughout the book for easy student reference. In most cases a rephrasing of a generalization in words accompanies the symbolic statement, and it is also highlighted in a box.

Examples

Immediately following the presentation of a new idea, several completely worked-out examples appear along with several partially worked-out examples with blanks to be filled in by the student. These examples are completed correctly at the end of each section and the student is advised to check his or her work immediately.

Trial Runs

Sprinkled throughout each section are several short Trial Runs, a list of six or eight problems to check on the student's grasp of a new skill. The answers appear at the end of the section.

Exercise Sets

Each section concludes with an extensive Exercise Set in which each odd-numbered problem corresponds closely to the following even-numbered problem.

Stretching the Topics

At the end of each Exercise Set there are several problems designed to challenge the better students by extending the skills learned in the chapter to the next level of difficulty.

Checkups

Following each Exercise Set, a list of about 10 problems checks on the student's mastery of the most important concepts in the section. Each Checkup problem is keyed to comparable examples in the section for restudy if necessary.

Problem Solving

One section of almost every chapter involves switching from words to algebra. By including such a section in each chapter, we are attempting to treat problem solving as a natural outgrowth of acquiring algebraic skills.

Chapter Summaries

Each chapter concludes with a summary in which the important ideas are again highlighted, in tables when possible. New concepts are presented in symbolic form and verbal form, accompanied by a typical example.

Speaking the Language of Algebra

Following the summary, we have included a group of sentences to be completed *with words* by the student. Algebra students (especially those in self-paced programs) often lack the opportunity to "speak mathematics." We hope that these short sections will help them develop a better mathematics vocabulary.

Review Exercises

A list of exercises reviewing all the chapter's important concepts serves to give the student an overview of the content. Each problem is keyed to the appropriate section and examples.

Practice Test

A Practice Test is included to help the student prepare for a test over the material in the chapter. Once again, each problem is keyed to the appropriate chapter sections and examples.

Sharpening Your Skills

Finally, we have included a short list of exercises that will provide a cumulative review of concepts and skills from earlier chapters. Retention seems to be a very real problem with students at this level, and we hope that these exercises will serve to minimize that problem. Each cumulative review exercise is keyed to the appropriate chapter and section.

Throughout the book we have adhered to a rather standard order of topics, making an attempt to connect new concepts to old ones whenever appropriate. This modified spiraling technique is designed to help students maintain and overview of the content. Success in future courses seems to us to hinge on students' seeing that algebra is a logical progression of ideas rather than a set of unrelated skills to be memorized and forgotten.

The answers to the odd-numbered exercises in the Exercise Sets appear in the back of the book together with answers for *all* items in Stretching the Topics, Checkups, Speaking the Language of Algebra, Review Exercises, Practice Tests, and Sharpening Your Skills.

More assistance for students and instructors can be found among the supplementary materials that accompany this book.

Instructor's Manual with Test Bank

The Instructor's Manual contains the answers for all exercises in the Exercise Sets and Stretching the Topics. In addition there are six Chapter Tests (four open-ended and two multiple choice) for each chapter and three Final Examinations (two open-ended and one multiple choice). Answers to these tests and examinations also appear in the Instructor's Manual.

Student's Solutions Manual

The Student's Solutions Manual, written by Rebecca Stamper, contains step-by-step solutions for the even-numbered exercises in the Exercise Sets and for *all* items in the Review Exercises, Practice Tests, Sharpening Your Skills, and exercises involving word problems. Using the same style as appears in the text, these solutions emphasize the procedure as well as the answer.

Video Tapes

A series of 10 video tapes (each 20 to 30 minutes in length) provides explanations for some of the more difficult topics in the course.

Audio Tapes

A series of 10 audio cassettes (each 20 to 30 minutes in length) also offer explanations for the more difficult topics. Keyed to examples in the text, these cassettes encourage students to work along.

Computerized Test Generator

A set of computer-generated tests is available for producing either a 10-item test for each chapter *or* tests of any length from objective-referenced items. Cumulative tests and final exams may also be constructed using the objective-referenced items.

Acknowledgments

The writing of this book would not have been possible without the assistance of many people. We express our appreciation to our indefatigable typist Maxine Worthington, to Becky Stamper for carefully working all our problems, to our families for tolerating our obsessive work schedules, to our Mathematics Editors Gary Ostedt and Bob Clark for their enthusiastic support, and to our Production Supervisor Elaine Wetterau for her efficiency and expertise.

We also thank our reviewers: Mary Jean Brod, University of Montana; Donald R. Johnson, Scottsdale Community College; Adele Le Gere, Oakton Community College; Gerald J. LePage, Bristol Community College; Lois Miller, El Camino College; Harold M. Nerr, University of Wisconsin; Dan Streeter, Portland State University; and Jack W. Rotman, Lansing Community College; for their careful scrutiny and helpful comments.

L.R.P.
P.I.H.

Contents

Working with Numbers and Variables 1

Charlie is a part-time college student. He must pay \$50 per course credit hour and he has saved \$800. Write an algebraic expression for the amount Charlie will have left after enrolling for *h* credit hours.

We review operations with real numbers here at the start, so that we can use them with confidence throughout the remainder of the text. In this chapter we recall how to:

1. Recognize different kinds of numbers.
2. Perform operations with real numbers.
3. Use the properties of the real numbers.
4. Work with variables and constants in algebraic expressions.
5. Find the absolute value of a quantity.
6. Switch from word expressions to algebraic expressions.

1.1 Working with Sets of Numbers

In the same way that your knowledge of different kinds of numbers has changed throughout your life, mathematicians have introduced new sets of numbers as they were needed. We shall use standard set notation to represent our number sets, naming each with a capital letter and listing the members of each set with commas between them and enclosed in braces.

Recognizing Sets of Numbers

The first set of numbers ever used for measuring and counting was the set of **natural numbers**, represented by N.

Natural Numbers

$N = \{1, 2, 3, 4, \ldots\}$

The dots following the number 4 indicate that this set continues forever; it is an **infinite set** containing no last element. However, N is a **well-defined** set, which means that we can determine exactly what numbers belong to the set without listing all its members.

The need for a symbol to represent *none* of some measure led to the introduction of **zero**, and the set of **whole numbers** was invented. We shall use W to represent this set.

Whole Numbers

$W = \{0, 1, 2, 3, 4, \ldots\}$

Notice that the set of whole numbers is also a well-defined, infinite set. Notice, too, that every natural number is a whole number.

In order to measure losses, mathematicians expanded their set of numbers to include the opposites (or **negatives**) of the natural numbers, and the set of **integers** came into existence. We represent the set of integers with the letter J.

Integers

$J = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

This set is also well defined and infinite, and it contains all the whole numbers.

Let's take a moment here to introduce some mathematical shorthand. To say that a number belongs to a set (or is an **element** of a set) we use the symbol "ϵ."

Symbol	Meaning	Examples
$\in$	Is an element of	$5 \in N$ $-6 \in J$
$\notin$	Is not an element of	$-2 \notin W$ $\frac{1}{2} \notin J$
$=$	Is equal to	$2 + 3 = 5$ $7 \cdot 4 = 28$
$\neq$	Is not equal to	$-6 \neq 0$ $\frac{2}{3} \neq \frac{3}{2}$

A need to represent *parts* of integers led mathematicians to define the set of **rational numbers** containing every number that can be expressed as a fraction with an integer as the numerator and a nonzero integer as the denominator. Using a to represent the numerator, b to represent the denominator, and Q to represent the set of rational numbers, we define this new set.

Rational Numbers

$$Q = \left\{\frac{a}{b} : a \in J, b \in J, b \neq 0\right\}$$

To be a member of Q a number must be expressible as a quotient of integers. You should agree that

$$\frac{2}{3}, \frac{1}{2}, \frac{7}{93}, \frac{-5}{8}, \frac{17}{11}, \frac{-1}{10}, \frac{-9}{5}$$

are all rational numbers.

Since every integer can be expressed as a quotient of integers, every integer is also a rational number. Some decimal numbers are also rational numbers; in particular, any decimal number that *terminates* or *repeats* in a fixed block of digits is a rational number.

$6 \in Q$	because	$6 = \frac{6}{1}$
$-3 \in Q$	because	$-3 = \frac{-3}{1}$
$0 \in Q$	because	$0 = \frac{0}{1}$
$0.37 \in Q$	because	$0.37 = \frac{37}{100}$
$0.333 \ldots \in Q$	because	$0.333 \ldots = \frac{1}{3}$
$-0.2727 \ldots \in Q$	because	$-0.2727 \ldots = \frac{-3}{11}$

To recognize members of the set of rational numbers, we must look for numbers that are integers or common fractions or repeating decimals or terminating decimals.

Although you may think that we have considered all the possible sets of numbers, there are many numbers, called **irrational numbers**, which are *not* expressible as quotients of integers. Some examples might jog your memory:

$$\sqrt{2}, \quad \sqrt{3}, \quad -\sqrt{5}, \quad \pi, \quad 3 + \sqrt{7}$$

are members of the set of irrational numbers, H.

If we consider all the rational numbers together with all the irrational numbers, we can form the very important set of **real numbers**, represented by R.

Real Numbers

R = {numbers that are rational *or* irrational}

Every number that we have discussed so far belongs to the set of real numbers. Until further notice, all our work in algebra will deal with real numbers.

Example 1. To which number sets does 3 belong?

Solution

$3 \in N$

$3 \in W$

$3 \in J$

$3 \in Q$

$3 \in R$

Now try Example 2.

Example 2. To which number sets does -0.5 belong?

Solution

$-0.5 \in$ ____

$-0.5 \in$ ____

Check your work on page 9. ▶

You complete Example 3.

Example 3. For the set $\{-5, 0, 0.1, \sqrt{5}, \pi, 4, \frac{24}{5}\}$, complete the following list.

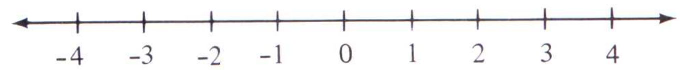
Natural numbers:	4
Whole numbers:	0, 4
Integers:	____, ____, ____
Rational Numbers:	____, ____, ____, ____, ____
Irrational numbers:	____, ____

Check your work on page 9. ▶

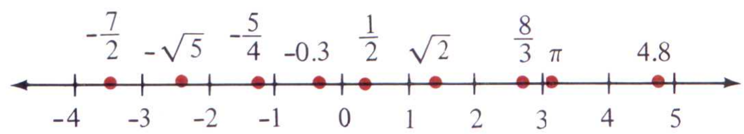
Using the Number Line

To better understand the different sets of numbers, we can use a **real number line**. To construct a number line, we draw a straight line and then choose a zero point and a length to represent

1 unit. All points spaced 1 unit apart are labeled to correspond to the integers in order, with positive integers to the right of zero and negative integers to the left of zero.



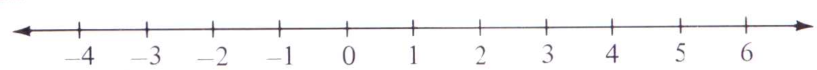
The arrows at either end of the number line show that the line extends indefinitely in both directions, so **we can locate a point corresponding to any rational or irrational number on this number line**. Although our location of irrational numbers will not be completely accurate, we shall be satisfied with a good approximation. On the number line, we locate points corresponding to real numbers by placing a solid dot in the appropriate position. This is called **graphing a number**.



Always be sure to label your points so that a reader will not be confused about your meaning. See if you can graph the numbers in Example 4.

Example 4. On a real number line, graph the numbers 3, 1.1, $-\sqrt{2}$, $\frac{9}{2}$, -2.5, and 0.

Solution

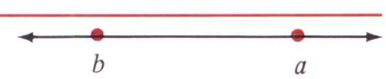
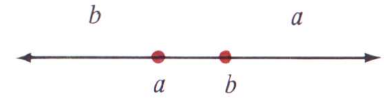
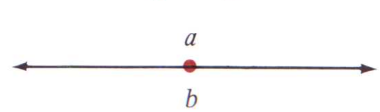


Check your work on page 9. ▶

The number line provides us with a handy means of visualizing the set of real numbers and allows us to make observations about *order* within that set. Suppose that we let a and b represent any real numbers.

> If a lies to the *right* of b on the number line, a must be **greater than** b.
> If a lies to the *left* of b on the number line, a must be **less than** b.
> If a and b occupy the *same* position on the number line, a must **equal** b.

Mathematicians have invented symbols to represent these three situations.

Symbols	Meaning	Number Line
$a > b$	a is greater than b	
$a < b$	a is less than b	
$a = b$	a is equal to b	

Because of the orderly nature of the real numbers, it is always possible to compare two real numbers using exactly one of these three statements.

Trichotomy Principle. Given any two real numbers, a and b, exactly *one* of the following statements must be true:

$$a > b \quad \text{or} \quad a < b \quad \text{or} \quad a = b$$

The statements $a > b$ and $a < b$ are called **inequalities**, and the number line helps us see that the following property for inequalities makes sense.

Transitive Property. Let a, b, and c be real numbers. If $a < b$ and $b < c$, then $a < c$.

For instance, if we know that $\sqrt{2} < 1.5$ and $1.5 < \sqrt{3}$, we may safely conclude that $\sqrt{2} < \sqrt{3}$.

Example 5. Compare the numbers using the symbols $<$, $>$, or $=$.

$$-2 < 4$$

$$0 > -1$$

$$3 < \pi$$

$$\sqrt{25} = 5$$

Now you try Example 6.

Example 6. Compare the numbers using the symbols $<$, $>$, or $=$.

-6 ____ -5.5

2 ____ $\sqrt{3}$

$\sqrt{9}$ ____ 3

Check your work on page 9. ▶

If a real number lies to the *right* of zero on the number line, we know that it is *positive*. If a real number lies to the *left* of zero on the number line, we know that it is *negative*. Therefore, we can state whether a number is positive or negative using our symbols of inequality.

$a > 0$	means	"a is a positive real number."
$a < 0$	means	"a is a negative real number."

Example 7. Use an inequality to state whether each number is positive or negative.

$$5, \quad 0.01, \quad -\pi$$

Solution

$$5 > 0$$

$$0.01 > 0$$

$$-\pi < 0$$

Now you try Example 8.

Example 8. Use an inequality to state whether each number is positive or negative.

$$\frac{-9}{2}, \quad \sqrt{3}, \quad -\sqrt{3}$$

Solution

$\frac{-9}{2}$ ____ 0

$\sqrt{3}$ ____ 0

$-\sqrt{3}$ ____ 0

Check your work on page 9. ▶

Suppose that we look at the real numbers 5 and -5. On the number line, each of these numbers corresponds to a point 5 units from zero. The point corresponding to 5 lies 5 units to the *right* of zero; the point corresponding to -5 lies 5 units to the *left* of zero. Such numbers are called **opposites** of each other and we say that

-5 is the opposite of 5

5 is the opposite of -5

Every real number has an opposite.
Let's complete Example 9.

Example 9. Complete each statement.

-2 is the opposite of ____ .   ____ is the opposite of π.

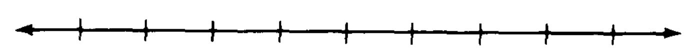
1.6 is the opposite of ____ .   ____ is the opposite of $-\sqrt{2}$.

____ is the opposite of $\frac{2}{3}$.   ____ is the opposite of 0.

Check your work on page 9. ▶

Trial Run

1. On the number line plot the points corresponding to the numbers $\frac{7}{2}$, $-\sqrt{3}$, 2.2, -3.1, 0 and 1.



Compare the numbers using $<$, $>$, or $=$.

2. 4 ____ $\sqrt{10}$ **3.** $\frac{5}{2}$ ____ 3   **4.** -2 ____ -2.5

Use an inequality to state whether each number is positive or negative.

________ **5.** 0.53   ________ **6.** $-\sqrt{5}$ ________ **7.** $\frac{2}{3}$

Complete each statement.

8. 3 is the opposite of ____ .

9. ____ is the opposite of $\frac{-4}{5}$.

10. -0.25 is the opposite of ____ .

Answers are on page 10.