

FUNDAMENTALS OF GASEOUS IONIZATION & PLASMA ELECTRONICS

ESSAM NASSER

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FUNDAMENTALS OF GASEOUS IONIZATION AND PLASMA ELECTRONICS

ESSAM NASSER

PROFESSOR OF ELECTRICAL ENGINEERING

IOWA STATE UNIVERSITY

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PREFACE

*Science is good, and,
its worth is everlasting, and,
it has not been disturbed
since the day of its creation*

Ptahhotep, Egyptian Scholar
ca. 2700 B.C.

One of the most fascinating and intriguing areas of science is that of ionized gases. Most of the matter in the universe is in the ionized state. In this state, *nature* often exhibits what may almost be termed *wonders*. It is with both admiration and awe that one should view these *wonders* of ionized gases whether in the dramatic lightning stroke, in the beautiful aurora borealis (northern lights), or in the magnificent solar corona that surrounds the sun and can only be seen during a total solar eclipse.

Ionized gases and plasmas are used in an increasing number of engineering applications ranging from such important devices as spark plugs and arc furnaces to thermonuclear fusion reactors and magneto-dynamic generators. In the realm of scientific research, plasma studies are conducted to understand the phenomena in the ionosphere, in the development of atmospheric electricity and the effects of the ionized gas, or coronas, surrounding antennas and transmission lines. Controlled ionization is used in radiation detectors and counters and numerous electronic tubes. Corona discharges are the essential organ of electronic precipitators that remove the solid pollutants from the exhaust gases of industrial plants.

In order to cover the fundamentals underlying this important and vast subject with so many applications in science and technology, a 3-quarter course was developed and taught for several years. Whereas excellent reference and

research books on the various specialized areas such as on collision phenomena, gaseous breakdown, coronas, microwave breakdown and plasma physics are available, there was no book suitable as a classroom textbook that covered all of those areas at a lower level in an introductory fashion.

Thus the main reason for writing this book lies in the need to introduce the senior student or first year graduate student, in electrical engineering as well as in physics, to this large and significant area of science. Such introductory material should also provide the new researcher in this area with the fundamentals he needs for the study of modern reference and research books to be found on almost each of the chapters covered.

With general knowledge of electromagnetic fields, this book can be used to teach engineering seniors without any difficulty. It incorporates much of the work published recently in German and Russian some of which is not to be found in the English literature.

The material is presented by proceeding gradually from the neutral gas to the fully ionized one in a consistent and unified treatment with emphasis on the transition, that is on the electrical breakdown of gases. The treatment starts with the neutral gas by reviewing the kinetic gas theory in Chapter 1 and the atomic structure in Chapter 2. The processes of ionization and deionization are covered in Chapters 3 and 4, respectively. After a brief review of the types of electron emission in Chapter 5, Chapter 6 offers a discussion of the motion of particles in a gas with low values of field to pressure ratios, that is where particles cannot gain much energy from the field between successive collisions. As this ratio is increased, inelastic collisions start to take place leading to an increase in ionized particle density. This new condition is treated in Chapter 7. The conditions making these processes stable or self-maintaining, also known as breakdown processes, are dealt with in Chapters 8 and 9. The transient development of ionization is outlined in Chapter 10 as based on recent research published mainly in the German literature. Chapter 11 covers the vast area of electrical coronas and had to be confined to the basic processes in atmospheric air. Chapter 12 presents a discussion of microwave and alternating-field breakdown. Chapters 13 and 14 are devoted to an analysis of plasmas. References are given to areas still subject to investigations.

Some chapters present novel material that has never been included in a book form. In particular Chapters 6, 10, 11, 12, 13 and 14 are based partially on newer research results. Chapter 13 was enriched by recent research work published in Russian, but available in part in English or German translations.

With the exception of the first two, each chapter is written in a way that takes the student, such as any physical science senior, from the fundamentals and directs him toward the advanced state of the art either in detailed coverage, or depending on the emphasis, just by giving sources for further study. Thus adequate, but not comprehensive, and pertinent bibliography is given at

the end of each chapter. The reason for including a well-balanced bibliography was to pave the way for the student or researcher to continue in any area he chooses with sound and broad knowledge of the elementary theory. Problems are given after each chapter and constitute an important part of the learning process.

It is with great pleasure and gratitude that I wish to express my thanks to all who have contributed directly or indirectly to this book. The inspiration to the study of physical science came from my teachers at Saidieh Secondary School and University of Cairo and the Technische Universitat of West Berlin. My interest in this area has been aroused by Professor Heinrich Winkelkemper who, besides being a great teacher and inspirer, is responsible for the Lichtenberg figure technique discussed in Chapter 11. My association with Professor Leonard B. Loeb at the University of California at Berkeley was a most fruitful and gratifying period.

Most of the material of this book evolved from class notes written at Iowa State University during the past years. I would like to thank Dr. Mohamed S. Abou-Seada and Mr. Manfred Heiszler for making valuable suggestions and discussions; and Mrs. Linda Ghais for typing most of the manuscript. I wish to express my deepest gratitude to Dr. Warren B. Boast, Head of the Electrical Engineering Department, for providing the opportunity to write this book, for his continuous encouragement and support of the whole project. Finally, special thanks go to my wife for her consideration and invaluable assistance.

ESSAM NASSER

Ames, Iowa
May 1970

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CHAPTER 1

Theory of Gases

1.1 Introduction

Before a study is made of the behavior of gases under the influence of electric and magnetic fields it is appropriate to review the basic principles of the kinetic theory of gases as pertinent to the later studies of gaseous ionization and breakdown. Since the conduction of a current through a gas can only take place if charged particles are available, a review of the atomic structure and the elementary processes of atomic physics will be given in the next chapter.

The gas theory to be discussed in this chapter, however, assumes that all particles are electrically neutral. In all the actual cases of gaseous conduction conditions will differ appreciably because of the presence of charged particles together with neutral ones.

Although the problems under study can range from the low-pressure low-temperature discharge of a Geiger counter to the high-pressure high-temperature pinch arc used to trigger a thermonuclear fusion reaction, the quantitative treatment is reduced in most cases to the study of the particle behavior and the conditions of motion. In all these problems an exact knowledge of the mode of motion of the neutral molecules is a prerequisite. From there the various concepts of ionic and electronic behavior can be inferred. This chapter is devoted therefore to the review of the kinetic theory of gases. Further studies can be made using the literature cited at the end of this chapter.

1.2 Boyle's Law

Robert Boyle (1627–1691) found by experiment that the product of the pressure and volume, for a given amount of enclosed gas, is a constant if the temperature remains unchanged. This law can be written as

$$pV = C = \text{constant} \quad (1.1)$$

2 Theory of Gases

where p is the pressure, V is the volume, and C depends on the temperature and mass of the gas.

This law as given by (1.1) holds very accurately for “*ideal*” or “*perfect*” gases only. Many gases, such as oxygen, nitrogen, hydrogen and helium, can be considered “*ideal*,” especially under low pressure. Equation 1.1 will hold approximately for gases that are not too close to condensation. The easily liquified vapors, such as water vapor and carbon dioxide, depart appreciably from this law, especially at high pressure. Even small impurities of these gases can cause a permanent gas to show appreciable departure.

Later, experiments were undertaken by Charles, Mariotte, and Gay-Lussac to study the behavior of a gas under constant pressure and constant volume as the temperature was changed. At constant pressure it was found that the volume follows the equation

$$V = V_0 [1 + \alpha(\theta - \theta_0)] \quad (1.2)$$

where θ is the new temperature and V_0 is the volume at a certain reference temperature θ_0 ; α is called the coefficient of volume expansion and has the same value for all permanent gases, provided the initial temperature θ_0 is the same.

1.3 Gay-Lussac's Law

Investigations of the thermal expansion of gases showed that

$$\alpha = \frac{1}{273 + \theta_0} \quad (1.3)^*$$

where θ_0 is the initial temperature in degrees Celsius ($^{\circ}\text{C}$).

Substituting (1.3) into (1.2), we get

$$\frac{V}{V_0} = \frac{273 + \theta}{273 + \theta_0} \quad (1.4)$$

where θ and θ_0 are temperatures in $^{\circ}\text{C}$. Equation 1.4 is known as the Gay-Lussac's law, and can be used to determine the gas volume at different temperatures and constant pressure.

This law suggests that if a gas is cooled to -273°C , its volume will shrink to zero. Such behavior is not true because all gases will be liquified at temperatures appreciably above -273°C . Furthermore, this temperature cannot be reached experimentally and it is therefore called the *absolute zero of temperature*.

The Kelvin temperature scale (named after Lord Kelvin, or William Thompson, 1824–1907) is therefore widely used in gaseous physics. Each

* 273 will be used throughout although the exact value is 273.16°C

degree is the same as the Celsius degree, but the zero is shifted to -273°C . Therefore, If T is the temperature in degrees Kelvin ($^{\circ}\text{K}$), then

$$T = 273 + \theta \quad (1.5)$$

The Gay-Lussac's law becomes

$$\frac{V}{V_0} = \frac{T}{T_0} \quad (1.6)$$

This law gives the volume of a certain amount of a gas at different temperature if the pressure is kept constant. It is apparent that (1.1) and (1.6) define the state of a gas under any condition. These two equations can be combined as follows: From 1.6 we get

$$\frac{pV}{pV_0} = \frac{T}{T_0}$$

but $pV = C$ and $pV_0 = C_0$; thus

$$\frac{T}{T_0} = \frac{C}{C_0} \quad (1.7)$$

which means that the constant C is proportional to the temperature in $^{\circ}\text{K}$. If the ratio C_0/T_0 is called R , then (1.7) can be written

$$C = RT \quad (1.8)$$

Since this relation is valid for all ideal or perfect gases, R must be independent of the kind of gas. From (1.1) and (1.8) we find that

$$pV = RT$$

In (1.1) the amount of gas is not specified. If now we assume that the mass of the gas under consideration is equal to a kilogram molecule* or kilomole, (1.1) can be written in a more general form as

$$pV = nC$$

where n is the number of kilomoles of gas. Thus the general equation of the state of an ideal gas is

$$pV = nRT \quad (1.9)$$

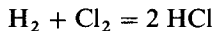
R , called the universal gas constant, is equal to $8314 \text{ J}/^{\circ}\text{K}$.

* A kilogram molecule (kilomole) is the amount of the gas whose mass in kilograms is numerically equal to the gas molecular weight. At 0°C and 760 torr (NTP) the volume of 1 kilomole is 22.4 m^3 .

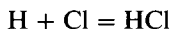
4 Theory of Gases

1.4 Avogadro's Hypothesis

Avogadro formulated his rule in 1811 as a result of Lavoisier's law of conservation of mass and Dalton's law of multiple proportions. These early chemical investigations led Avogadro to conclude that there is a difference between the atoms and the molecules of a gas. He could then explain a great stumbling block of that time: One volume of hydrogen and one volume of chlorine resulted in *two* volumes of hydrochloric acid and not one, as scientists then expected. This meant that the chemical reaction obeys the law.



and not



From this conclusion Avogadro stated his hypothesis: "Under the same conditions of temperature and pressure, equal volumes of different gases contain the same number of *molecules* and not of atoms." The word molecule must be construed to include the case of the monoatomic gas.

Various experimental results have confirmed this statement. Its greatest significance lies in the complete coordination of the structural formulas of many chemical compounds [1, 2].

The number of molecules in a gram molecule of any gas is therefore a constant, known as Avogadro's number, N_0 . It is equal to 6.02×10^{23} molecules/mole. A kilomole has, of course, a thousand times as many molecules.

Avogadro's number relates macroscopic quantities to atomic magnitudes and enables us to find atomic quantities by measuring macroscopic or laboratory-size magnitudes. Take for example an electrolysis experiment. By measuring the weight of a metallic deposit, the current and the time it took to obtain it, the total charge collected can be found. This must be equal to the number of atoms deposited multiplied by their electric charge. Once the charge and weight are known, the number of atoms can be computed. High-frequency electromagnetic waves can also be used to determine the spacing between two adjacent atoms of a solid, enabling the computation of the number of atoms per unit volume or weight.

It is possible to rewrite (1.9) in terms of the gas density N in a volume V containing a total N' molecules. Now $N' = nN_0$, so that (1.9) becomes

$$\frac{N'}{V} = N = \frac{N_0}{R} \frac{p}{T}$$

R/N_0 is a universal constant known as the Boltzmann constant κ . Hence

$$p = N\kappa T \quad (1.10)$$

which is a very useful simple form of the universal gas law.

1.5 Law of Partial Pressures

If two gases have volumes V_1 and V_2 at the same pressure and temperature, the mixture of the two gases at the *same* pressure and temperature will have a volume V where

$$V = V_1 + V_2$$

or, in general,

$$V = V_1 + V_2 + \cdots + V_n \quad (1.11)$$

This mixture obeys Boyle's law exactly as the two gases did separately.

The law of partial pressures states that in a mixture of gases each gas behaves as if the others were not present at all or the total pressure exerted by the mixture is equal to the sum of pressures that each gas would exert if it occupied the vessel alone. Using (1.9) in (1.11),

$$V = \frac{n_1 RT}{p} + \frac{n_2 RT}{p} + \cdots + \frac{n_n RT}{p}$$

since different quantities of the various gases are present in the mixture. Rearranging,

$$p = \frac{n_1 RT}{V} + \frac{n_2 RT}{V} + \cdots + \frac{n_n RT}{V}$$

or

$$p = p_1 + p_2 + \cdots + p_n \quad (1.12)$$

where p_1, p_2, \dots, p_n are designated the partial pressures of gases 1, 2, \dots , n . The partial pressure of a gas in a mixture can now be defined as the pressure of that gas quantity at the same temperature if it would occupy the volume V alone. Equation 1.12 is known as the law of partial pressures.

1.6 The Kinetic Interpretation of Pressure

Simplifying assumptions must be made in order to comprehend the behavior of gases using classical mechanics. These assumptions, although not quite justified in view of the advances in modern atomic physics, can still be used in obtaining valuable and accurate results. Their limitations, however, should be always borne in mind.

The derivation of the kinetic theory of gases was made by J. C. Maxwell and more rigorously and accurately by Ludwig Boltzmann in the middle of the nineteenth century. In spite of Maxwell's simplifying assumptions, both methods deliver the same results.

The conditions in a gas chamber can be summarized as follows:

6 Theory of Gases

1. A gas consists of molecules* that have the mass m . For the sake of this derivation they are assumed to be solid elastic spheres.

2. The molecules are in continual random chaotic motion at various velocities and in all directions.

3. While moving the molecules approach each other to a degree which affects their respective motion. Such encounters are elastic collisions with each other and with the walls of the container when there is no loss of kinetic energy to another form. These collisions are assumed to obey the laws of classical mechanics.

4. In ideal or perfect gases, the mean distance between the molecules can be considered much greater than their diameter.

5. No appreciable forces exist between the molecules or between a molecule and the wall of the container except during collisions. Thus the path described by a molecule between two successive collisions can be assumed to be a straight line.

Unlike in solids and liquids, the molecules in gases interact only weakly and for relatively short times. These interactions or collisions are highly important because they are responsible for the observed behavior of the gas, such as its thermal equilibrium.

The derivation of an expression for the molecular speed in terms of the macroscopic, measurable quantities can now be attempted by using some simplifying approaches that fortunately do not affect the resulting expression. We shall assume that the molecules make collisions only with the walls of the container, that is, they do not undergo collisions with each other.

If the container is a rectangular box whose sides have the lengths a , b and c and if it contains N' identical molecules all of mass m , we can assume that all particles are traveling with the same speed v and that $N'/3$ molecules are

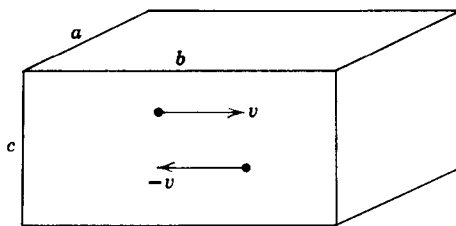


Fig. 1.1 A rectangular container with sides of lengths a , b and c . All gas molecules are assumed to be traveling at the average speed v .

* Molecules are used henceforth to denote molecules or atoms depending upon whether the gas is molecular or monoatomic. This eliminates repetition of "or atoms."

moving in the direction of each side or axis. This second assumption is based on the fact that the particles have three degrees of freedom of motion and holds for molecules having other velocities.

When a molecule, traveling with the velocity v in the x -direction, collides with the wall on the right-hand side, the change of its momentum is $2mv$. In one second a molecule moving parallel to the side b will traverse the container v/b times. The number of collisions on each side will be v/b per second.

Since $N'/3$ molecules are moving in this direction, the total number of collisions are $vN'/6b$.

The force F exerted on the side is equal to the total rate of change of momentum. Thus

$$F = (2mv) \left(\frac{vN'}{6b} \right) = N' \frac{mv^2}{3b}$$

Since the pressure is the force per unit area,

$$p = \frac{F}{ac} = \frac{N'mv^2}{3abc} = \frac{N'}{3} \frac{mv^2}{V}$$

where $V = abc$ = volume of the container. Therefore

$$pV = \frac{N'}{3} mv^2 \quad (1.13)$$

The importance of (1.13) lies in the fact that it relates the macroscopic properties p and V that can be easily measured with the microscopic, molecular or atomic quantities N' , m , and v .

Using Avogadro's number N_0 (see section 1.4), the number of molecules in a container can also be written in terms of N_0 where

$$N' = nN_0$$

where n is the number of kilomoles.

Equation 1.13 thus becomes

$$pV = n \frac{N_0}{3} mv^2 \quad (1.14)$$

Now, because we know that the kinetic energy of each molecule is $\frac{1}{2}mv^2$, (1.14) can be rewritten as

$$pV = n \left(\frac{2N_0}{3} \right) \left(\frac{1}{2}mv^2 \right) \quad (1.15)$$