

DEVICE ELECTRONICS FOR INTEGRATED CIRCUITS

RICHARD S. MULLER
THEODORE I. KAMINS

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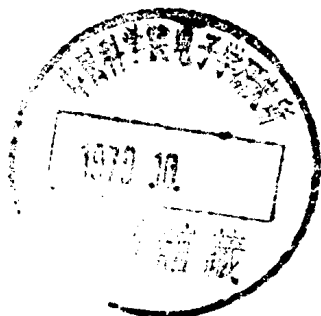
DEVICE ELECTRONICS FOR INTEGRATED CIRCUITS

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JOHN WILEY & SONS

New York • Santa Barbara • London • Sydney • Toronto

5505348

To our TEACHERS . . .
 who opened new vistas,
and to our STUDENTS . . .
 who will see further than we.

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Library of Congress Cataloging in Publication Data

Muller, Richard S

Device electronics for integrated circuits.

Includes index.

1. Semiconductors. 2. Integrated circuits.

I. Kamins, Theodore L., 1941- joint author.

II. Title.

TK7871.85.M86 621.381'71 77-1332

ISBN 0-471-62364-4

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

If, as we believe, the past is guide to the future, then major advances in integrated circuits will be made by individuals who have a firm grasp of device electronics. For this reason, a course emphasizing the concepts that underlie the operation and use of integrated-circuit devices has been offered to students at the University of California, Berkeley for several years. This book is the result of teaching students whose diverse interests range from basic device electronics to circuit design. About ten years' experience in teaching the course has given us the opportunity to experiment with a number of approaches to making manageable and, more important, making *digestible* the many facts of solid-state physics that are fundamental to device electronics.

After trying several alternatives, the course (and this book) evolved in the following way. First, we narrowed our focus to the most important devices used in silicon-integrated circuits. This selection left us with material that could be adequately introduced in roughly two quarters of the university calendar. Furthermore, this topical coverage has strong student appeal, a fact consistent with a precept that was well-phrased by Anatole France: "The whole art of teaching is only the art of awakening the natural curiosity of young minds."

Our next step in organizing the material was to prepare a list of the devices of interest and to order the list in a sequence determined by the number of physical concepts necessary for the reader to understand the underlying basic ideas. The book was then put together by alternating a discussion of physical concepts with a description of the application of these concepts to specific devices. Device applications are dealt with in the concluding sections of each chapter. This approach has the following major advantages:

1. It exposes the reader to material that has more permanence than does detailed description of a particular device.
2. It emphasizes the association between the facts of science and the techniques of engineering.
3. It presents the material in a *fact-application* sequence that stimulates understanding and retention.

However, in using this technique, it is difficult to maintain an even level of discussion. The problem arises because the operation of some devices can be explained in terms of only a few concepts, even though from many aspects the device is still quite complex. For example, only a few physical principles are needed to explain the Schottky-barrier diodes covered in Chapter 2, but the

complexity of analysis required for rigor in the discussion can make the topic seem unduly confusing. To avoid this problem and to provide a more uniform level of discussion, we have marked certain sections of the text with daggers ([†]). These sections contain material of generally higher sophistication than that in the unmarked sections and can be omitted by the reader who is interested mainly in first principles. Whenever results from the more sophisticated topics are necessary for the continuity of the presentation, we have included a more easily grasped explanation outside of the sections marked with a dagger.

A summary is given at the end of each chapter. It is intended to help the student review the material and to focus his attention on the essential concepts developed in the chapter.

Most of the problems at the end of each chapter give the reader an opportunity to apply the concepts that have been developed. There are only a few straightforward “plug-in” problems that follow directly from formulas developed in the text. These have usually been included to give the student a feeling for the magnitudes of various quantities. Some problems ask him to carry out portions of derivations only partially developed in the text. These can be used gainfully to emphasize certain treatments that may hold special interest. Challenging problems are occasionally included to extend the reader’s comprehension and to deduce new principles. We have marked with a dagger both these problems and those that emphasize material from a section marked with a dagger. An instructor’s manual containing solutions to all of the problems is available from the publisher. The manual contains suggestions on how to use some of the problems for lecture discussions.

The students using this book at the University of California, Berkeley, are mostly seniors, but some juniors and a few graduate students are also enrolled each quarter. All students are required to have a standard sophomore background in the sciences that encompasses basic chemistry and physics including introductory quantum mechanics, courses in differential and integral calculus, a first course in electric circuit principles, and a course in the properties of materials that contains a discussion of basic crystal structure. We have found that the most important background needed for the course is an understanding of electromagnetic theory and of the elements of modern physics, particularly the notion of allowed energy values in systems of electrons.

Below is an overview of the topics covered in the book presented according to chapter.

Chapter 1 ELECTRONICS AND TECHNOLOGY. This chapter deals with the physical electronics of solids that comprise the necessary background material for the book. Included are sections on material and processing technology that are optional to further discussion. The device described in this chapter is an integrated-circuit resistor.

Chapter 2 METAL-SEMICONDUCTOR CONTACTS. In this chapter the student is introduced to the concept of thermal equilibrium between dissimilar solids. The electronics of Schottky-barrier contacts are described in detail. Optional sections present a rigorous derivation of the current-voltage relationship for the junction, the nature of Schottky ohmic contacts, and surface effects at metal-semiconductor contacts. Practical Schottky diodes are considered in the device section.

Chapter 3 *pn* JUNCTIONS. The topic of this chapter is the electronics associated with inhomogeneously doped semiconductors. The behavior of *pn* junctions under reverse bias is described in detail. A section on junction breakdown is included, but may be omitted without loss of continuity. In the concluding section the junction field-effect transistor is discussed.

Chapter 4 CURRENTS IN *pn* JUNCTIONS. In this chapter, the continuity equations for free carriers are derived. The concepts of generation and recombination are introduced and then applied to describe the electronics of *pn* junctions under forward (injecting) bias. A discussion of minority-carrier storage and circuit modeling of diodes provides a foundation for the later development of transistor equivalent circuits. Optional sections include a full description of Shockley-Hall-Read recombination theory and space-charge-region generation and recombination. The device discussion focuses on the integrated-circuit uses of junction diodes, particularly for the junction isolation of devices.

Chapter 5 BIPOLAR TRANSISTORS I: BASIC OPERATION. The framework for the discussion of junction transistors in this chapter is the integrated-charge model. The familiar homogeneously doped transistor is treated as a special case of the general theory. This approach to bipolar transistors creates a smooth transition into topics such as bias ranges, current gain, and the Ebers-Moll model. The device discussion focuses on properties of planar integrated-circuit transistors for amplifying and switching purposes. Only one section of this chapter, that dealing with reciprocity in the transistor, is properly optional to further discussion.

Chapter 6 BIPOLAR TRANSISTORS II: LIMITATIONS AND MODELS. Most of this chapter could be omitted from a first course. The material discussed is, however, very much concerned with the practical aspects of bipolar transistors for integrated circuits. The topical coverage is oriented toward computer modeling of the transistor. For example, the Early effect is introduced in terms of the Early voltage, and effects at low and high emitter-base bias are considered with the use of the integrated-charge model. The same approach is taken to describe the electronics of the base transit time. This topic leads naturally to the charge-control model for the transistor and to examples of its use. The small-signal transistor model (hybrid- π) is deduced from the charge-control model, and equivalences between the various representations for the transistor are discussed. The final

model discussed is for computer simulation. The device discussion focuses on *pnp* transistors for integrated circuits, and introduces circuit applications such as integrated-injection logic.

Chapter 7 PROPERTIES OF THE OXIDE-SILICON SYSTEM. This chapter provides the necessary additional background in physical electronics that is needed for a discussion of MOS devices. The control exercised by the gate to accumulate, deplete or invert the semiconductor surface is described, and the concepts underlying the flat-band and threshold voltages in an MOS system are presented. An optional section describes surface effects on *pn* junctions. The chapter concludes with a discussion of MOS capacitors for integrated circuits and charge-coupled devices (CCDs).

Chapter 8 THE INSULATED-GATE FIELD-EFFECT TRANSISTOR. The properties of IGFETs are deduced initially from an approximate charge-control model, and then more rigorously from a distributed model. The parameters of the device and a model for circuit design are described. A discussion of technologies and an optional section concerned with circuit applications are included. Also optional is a discussion of second-order effects in the device. A final section describes the design and properties of ion-implanted, *n*-channel IGFETs.

The development of this book was greatly aided by the supportive comments of many colleagues at the University of California. Earlier versions of the manuscript were used in teaching by Professors T. Van Duzer, R. W. Broderick, T. E. Everhart, and C. Hu. Each made useful suggestions for improvement. Professors A. R. Neureuther, R. G. Meyer, and D. A. Hodges provided helpful criticism of the later chapters in the book. The manuscript was also used at Stanford University by one of the authors (T. I. Kamins) and by Professors J. B. Angell, R. W. Dutton, and J. G. Linvill. Several helpful suggestions came from colleagues in industrial laboratories. These include Dr. J. Kerr at Signetics Corporation, Dr. J. Gaul at Siemens Research Laboratory, Munich, Germany, Mr. E. H. Nicollian at the Bell Telephone Laboratories, Murray Hill, New Jersey, Mr. H. Jakobsen at the Central Institute for Industrial Research, Oslo, Norway, and Mr. T. Masuhara at the Hitachi Research Laboratory, Tokyo, Japan. Many comments from students were influential in improving the manuscript, and special appreciation is expressed to several graduate students at the University of California, Berkeley, who provided useful, detailed criticism. These include R. Amantea, S. H. Kwan, W. Loesch, K. W. Yeh, S. P. Fan, and especially R. Coen who very thoroughly checked several of the chapters and who diligently read and solved many of the problems. The problems were checked further and the solutions manual was written with meticulous care by K. C. Hsieh. F. Kashkooli of Signetics Corporation and J. Solomon of National Semiconductor responded generously to requests for illustrations.

As any author knows, textbooks are only harvested after months of nurture. It seems appropriate to acknowledge here the contribution of the University sabbatical system which afforded the opportunity to carry a rather large effort to completion. The staff at Wiley, our editor G. Davenport and his assistant Mrs. E. King, our copy editor E. Patti, and our production manager P. Klein have all been extremely helpful in guiding this project through its publication phase. Our appreciation is also extended to Bettye Fuller who ably typed several sections. Mrs. Joyce R. Muller gave devoted effort to shaping the manuscript and typed most of it several times. Her careful reading, copy editing, typing, and suggestions for revision and rewording have been responsible for improvements throughout the entire book.

Richard S. Muller
Theodore I. Kamins

PHYSICAL CONSTANTS

(in units frequently used in semiconductor electronics)

Electronic charge	q	$1.602 \times 10^{-19} \text{ C}$
Speed of light in vacuum	c	$2.998 \times 10^{10} \text{ cm s}^{-1}$
Permittivity of vacuum	ϵ_0	$8.854 \times 10^{-14} \text{ F cm}^{-1}$
Free electron mass	m_0	$9.11 \times 10^{-31} \text{ kg}$
Planck constant	h	$6.625 \times 10^{-34} \text{ J s}$
		$4.135 \times 10^{-15} \text{ eV s}$
Boltzmann constant	k	$1.38 \times 10^{-23} \text{ J } ^\circ\text{K}^{-1}$
		$8.62 \times 10^{-5} \text{ eV } ^\circ\text{K}^{-1}$
Avogadro's number	A_0	$6.022 \times 10^{23} \text{ molecules (g mole)}^{-1}$
Thermal voltage	$V_t = kT/q$	
at 80.6°F (300°K)		0.025860 V
at 68°F (293°K)		0.025256 V

CONVERSION FACTORS

$$1 \text{ mil} = 10^{-3} \text{ inch} = 25.4 \mu\text{m}$$

$$1 \text{ eV} \approx 1.602 \times 10^{-19} \text{ J}$$

$$1 \text{ J} = 10^7 \text{ erg}$$

MATERIAL PROPERTIES

Properties of Semiconductors and Insulators—pp 32, 33

Properties of Silicon—p 34

Free-Carrier Mobilities in Silicon—p 26

Resistivity versus Doping in Silicon—p 24

SELECTED LIST OF SYMBOLS

Symbol	Definition	Page
a	gradient of dopant density	121
BV	breakdown voltage	133
C_s	surface concentration of dopant	46
$C_{je}, (C_{je})$	small-signal junction capacitance at the collector (emitter)	244, 281
C_{ox}	oxide capacitance	314
C_D	diffusion capacitance	276
$D_n, (D_p)$	diffusion constant for electrons (holes)	30, 31
D'_s	density of surface states	94
$E_a, (E_d)$	acceptor (donor) energy	13, 12
$E_c, (E_v)$	energy at the conduction-band (valence-band) edge	12, 13
E_f	Fermi energy	17
E_i	intrinsic Fermi energy	20
E_0	vacuum reference energy	69
\mathcal{E}_l	electric field at which drift velocity reaches a limiting value	251
$\mathcal{E}_s, (\mathcal{E}_{ox})$	electric field in a semiconductor (an oxide)	322, 327
$f_D(E)$	Fermi-Dirac distribution function	17
f_T	frequency at which β_F approaches unity	282
$g(E)$	density of allowed states	17
$g_m, (g_{m\text{ sat}})$	transconductance (in the current saturation region)	143, 144
$G, (R)$	generation (recombination) rate	15
$i_B, (i_C), (i_E)$	total base (collector) (emitter) current	265
I_n	electron current traversing base	204
I_{pE}	emitter hole current ($n p n$ transistor)	213
I_{rB}	base recombination current	212
$I_D, (I_{D\text{ sat}})$	drain current (in saturation)	139, 142
$I_{CS}, (I_{ES})$	collector (emitter) saturation current	223
$J_n, (J_p)$	electron (hole) current density	23
k'	constant for simplified MOS equations	359
l	carrier mean-free path	29
$L, (L')$	channel length (channel length to pinch-off point)	352, 377
$L_n, (L_p)$	diffusion length for minority-carrier electrons (holes)	168
L_D	Debye length	85
$m_n^*, (m_p^*)$	effective mass of an electron (a hole)	12, 19
m_0	free electron mass	3
$n, (p)$	electron (hole) density	11
$n', (p')$	excess electron (hole) density	161
$n_i, (p_i)$	intrinsic electron (hole) density	11
$n_o, (p_o)$	electron (hole) density at thermal equilibrium	161
$n_s, (p_s)$	surface electron (hole) density	162
$N_a, (N_d)$	acceptor (donor) density	13, 12
$N_c, (N_v)$	effective density of states in the conduction (valence) band	16
N_t	density of trapping states	156
q	electronic charge	3

xii Selected List of Symbols

$q_F, (q_R)$	forward (reverse) charge component in charge-control theory	263, 269
$q_{VE}, (q_{VC})$	charge at the emitter-base (collector-base) junction	263
Q_d	space charge in a depletion region	319
Q_s	space charge in a semiconductor	74
Q_B	integrated base majority charge	203
R_{\square}	sheet resistance	55
s	surface recombination velocity	164
T_{tr}	transit time of carriers in a surface channel	351
U	recombination rate	159
v_d	drift velocity	23
v_l	limiting drift velocity	27
v_{th}	thermal velocity	21
V_a	applied voltage	74
V_o	characteristic "turn-on voltage" in a diode under forward bias	98
V_t	thermal voltage	275
V_A	Early voltage	242
V_T	turn-off voltage (JFET), threshold voltage (IGFET)	143, 323
V_{FB}	flat-band voltage	309
x_d	depletion-layer width	73
x_j	junction depth	54
$x_n, (-x_p)$	boundary of space-charge region in n -type (p -type) semiconductor	114
x_{ox}	oxide thickness	39
$x_B, (x_E)$	width of quasi-neutral base (emitter) region	203, 213
$\alpha_n, (\alpha_p)$	ionization coefficient of electrons (holes)	130, 131
$\alpha_F, (\alpha_R)$	forward (reverse) current gain I_C/I_E	217, 223
α_T	base transport factor	213
$\beta_F, (\beta_R)$	forward (reverse) current gain I_C/I_B	217, 221
γ	emitter efficiency	216
δ	defect factor	274
$\epsilon_s, (\epsilon_{ox})$	permittivity of a semiconductor (oxide)	73
$\mu_n, (\mu_p)$	electron (hole) mobility	23
ρ	resistivity, space-charge density	1, 110
σ	conductivity	23
$\sigma_n, (\sigma_p)$	capture cross section for electrons (holes)	157
$\tau_n, (\tau_p)$	lifetime of electrons (holes)	161
τ_r	dielectric relaxation time	183
$\tau_{cn}, (\tau_{cp})$	mean scattering time of electrons (holes)	21, 23
τ_B	base transit time	260
ϕ_i	built-in potential at a junction	73
$\phi_n, (\phi_p)$	potential associated with n -type (p -type) silicon	318
ϕ_s	surface potential	318
$\Phi_M, (\Phi_S)$	potential equivalent for the work function in a metal (semiconductor)	71
X	electron affinity	71

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CHAPTER 1

ELECTRONICS AND TECHNOLOGY

1.1 Physics of Semiconductor Materials

Band Model of Solids

Holes

Bond Model

Donors and Acceptors

Thermal Equilibrium Statistics

1.2 Free Carriers in Crystals

Drift Velocity

Mobility and Scattering

Diffusion Current

1.3 Semiconductor Technology

Crystal Growth[†]

Planar Technology[†]

1.4 Device: Integrated-Circuit Resistor

Conductance

Sheet Resistance

Summary

Problems

From everyday experience we know that the electrical properties of materials vary widely. If we measure the current I flowing through a bar of homogeneous material with uniform cross section when a voltage V is applied across it, we can find its resistance $R = V/I$. The resistivity ρ —a basic electrical property of the material comprising the bar—is related to the resistance of the bar by a geometric ratio

$$\rho = R \frac{A}{L} \quad (1.1)$$

where L and A are the length and cross-sectional area of the sample.

The resistivities of common materials used in solid-state devices cover a very wide range of values. For example, there is the range of resistivities encountered at room temperature for the materials used to fabricate typical silicon integrated

circuits. Deposited metal stripes, made from very low-resistivity materials, connect elements of the integrated circuit; aluminum, which is most frequently used, has a resistivity at room temperature of about $10^{-6} \Omega\text{-cm}$. On the other end of the resistivity scale are the insulating materials, such as silicon dioxide, which serve to isolate portions of the integrated circuit. Silicon dioxide has a resistivity 22 orders of magnitude higher than does aluminum—about $10^{16} \Omega\text{-cm}$. The resistivity of the plastics often used to encapsulate integrated circuits may be as high as $10^{18} \Omega\text{-cm}$. Thus, a typical integrated circuit may contain materials with resistivities varying over 24 orders of magnitude—an extraordinarily wide range for a common physical property.

For electrical applications, materials are generally classified according to their resistivities. Those with resistivity values less than about $10^{-2} \Omega\text{-cm}$ are called conductors, while those having resistivities greater than about $10^5 \Omega\text{-cm}$ are called insulators. In the intermediate region is a class of materials called semiconductors that have profound electrical importance because their resistivities can be varied by design under very precise control. Equally important, they can be made to conduct by one of two types of current carriers. The utilization of these unique properties of semiconductors is the primary topic of this book.

In the first portion of this chapter we consider what gives rise to the enormous range of resistivities in solids. We then briefly review some important concepts from semiconductor physics. To focus on devices, however, our discussion of materials is brief, and we assume that the reader has had some exposure to basic semiconductor and solid-state physics such as that contained in reference 1. The treatment here is merely a review of some of the more important concepts in a form useful for later application.

After this brief review of semiconductor physics we look in detail at silicon planar technology. Perfected by more than a decade of refinement, this technology, in which all operations are performed on one surface of a single crystal of silicon (hence the name *planar*), is still being improved. The capabilities of the planar process and the accumulated experience in employing these capabilities assure a continuing importance to an understanding of the electronics of devices made by this process.

1.1 PHYSICS OF SEMICONDUCTOR MATERIALS

An understanding of the physics of electrons in solids can be attained by first considering electrons in an isolated atom. By perturbing the electrons in this system in a logical manner we can obtain many of the important electronic properties of solids.

We begin therefore by considering the allowed energies of an electron influenced only by an isolated atomic core. Then we look at the effect of bringing other atoms

near the first atom so that the core of one atom influences the electrons associated with the other atoms, as is true in a solid material. In this manner we can study the effect of the atomic cores in a crystal on the behavior of the associated electrons. We then investigate the effect on the electrons of applying an electric field to the solid material.

Two complementary models are used in the discussion of semiconductor physics: the *energy-band model* and the *crystal-bonding model*. Let us first consider the energy-band description and bring in the bonding model later to introduce other concepts.

Band Model of Solids

We know that an electron acted on by the Coulomb potential of an atomic core may have only certain allowed energies (Fig. 1.1). In particular, the electron can occupy one of a series of energy levels

$$E_n = \frac{-Z^2 m_0 q^4}{8\epsilon_0^2 h^2 n^2} \quad (1.1.1)$$

below a reference energy taken as zero. In Eq. (1.1.1) Z is the net charge on the core, m_0 the free electron mass, q the magnitude of its charge, ϵ_0 the permittivity of free space, h Planck's constant, and n a positive integer. For the hydrogen atom with $Z = 1$, the allowed energies are $-2.19 \times 10^{-18}/n^2$ joules or $-13.6/n^2$

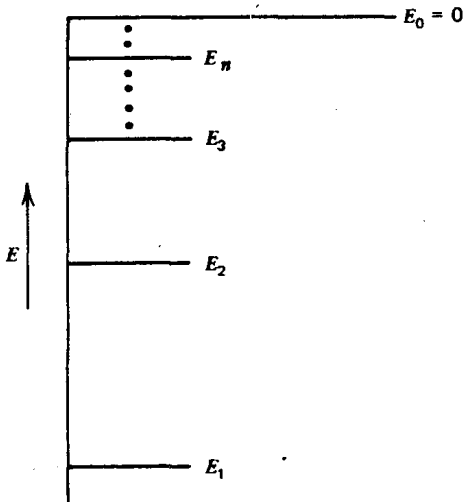


Figure 1.1 Allowed energy levels of an electron acted on by the Coulomb potential of an atomic core.

electron volts (eV) below the zero reference level.* At very low temperatures, when more than one electron are associated with the atom, the electrons fill the allowed levels starting with the lowest energies. By the Pauli exclusion principle, at most two electrons (of opposite spins) may occupy any energy level.

Let us now consider the electron in the highest occupied energy level of an atom and neglect the lower filled levels. When two isolated atoms are separated by a very large distance, the electron associated with each atom has the energy E_n given by Eq. (1.1.1). If two atoms approach one another, however, there will be a force on the first electron resulting from the second atomic core. The potential that determines the energy levels of the electron is therefore changed. All allowed energy levels for the electron are consequently modified because of this change in potential.

We must also consider the Pauli exclusion principle when describing a two-atom system. An energy level E_n , which can contain at most two electrons of opposite spin, is associated with each isolated atom so that the total system can contain at most four electrons. When the two atoms are brought together to form one system, however, only two electrons can be associated with the allowed energy level E_n . Therefore, the allowed energy level E_n of the isolated atoms must split into two levels with slightly different energies in order to retain space for a total of four electrons. Therefore, bringing two atoms close together not only slightly perturbs each energy level of the isolated atom but also splits each of the energy levels of the isolated atoms into two slightly separated energy levels. As the two atoms are brought closer together, stronger interactions are expected and the splitting becomes greater.

As more atoms are added to form a crystalline structure, the forces encountered by each electron are altered further and additional changes in the energy levels occur. Again, the Pauli exclusion principle demands that each allowed electron energy level have a slightly different energy so that many distinct, closely spaced energy levels characterize the crystal. Each of the original quantized levels of the isolated atom is split many times, and each contains one level for each atom in the system. When N atoms are included in the system, the original energy level E_n will split into N different allowed levels, forming an *energy band*, which may contain at most $2N$ electrons (because of spin degeneracy). Since the number of atoms in a crystal is generally large—of the order of 10^{22} cm^{-3} —and the total extent of the energy band is of the order of a few electron volts, the separation between the N different energy levels within the bands is very much smaller than the thermal energy possessed by an electron at room temperature, and the electron

* The use of the unit electron volt ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules}$; see the conversion factors on the inside front cover) is very convenient in semiconductor physics since it often avoids cumbersome exponents. Although not a pure rationalized unit, the electron volt is widely employed and is used extensively in our discussion.