PREPARATION FOR CALCULUS

3RD EDITION

S.L. Salas
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PREFACE

Years ago we wrote a little book that we called *Precalculus*. The purpose of that book was to enable students who did not have full command of high school mathematics to move on to calculus with as little delay as possible. Most of this algebra and trig, we figured, they've seen before. What they need is a reminder. For some students this approach worked well; for many it did not.

It is now our view that most students graduating from high school need more than a quick review of algebra and trigonometry to begin their study of calculus. They need thorough, detailed explanations supported by many examples and many, many exercises, some easy, some not so easy, some hard. *Preparation For Calculus* was written with these students in mind.

Much in this book was inspired by our reviewers. We are indebted to:

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A NOTE TO THE INSTRUCTOR

As you'll see, we have resisted the current fashion in precalculus of trying to discuss in depth the functions e^x and $\log_e x$. We have never encountered a calculus text that assumed that the students were familiar with these functions and have never heard a calculus teacher lament that the students did not understand these functions before they enrolled in the calculus course. In this text we give considerable weight to exponentials and logarithms but focus mostly on base 2 and base 10, leaving base e to the calculus where, in our opinion, it properly belongs.

S.L. Salas and C.G. Salas
Haddam, Connecticut

THE GREEK ALPHABET

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ϵ	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	o	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	ϕ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

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CHAPTER 1

NUMBERS AND ALGEBRAIC EXPRESSIONS

In the first two chapters we focus on those parts of high school algebra that are necessary for calculus.

1.1 CLASSIFICATION OF NUMBERS

We begin with the set of *natural numbers*: $\{1, 2, 3, \cdots\}$. With these numbers we can count:

$$\underset{1}{\bullet}\quad\underset{2}{\bullet}\quad\underset{3}{\bullet}\quad\underset{4}{\bullet}\quad\underset{5}{\bullet}\quad\underset{6}{\bullet}\quad\underset{7}{\bullet}\quad\cdots$$

add and multiply:

$$2 + 3 = 5, \qquad 2 \cdot 3 = 6$$

but we cannot always subtract:

$$5 - 8 \quad \text{is not a natural number}$$

and we cannot always divide:

$$5 \div 8 \quad \text{is not a natural number.}$$

To be able to subtract arbitrarily, we enlarge the number system to include the set of *integers*: $\{0, \pm1, \pm2, \pm3, \cdots\}$. Now, for instance,

$$5 - 8 = -3.$$

If we wish to divide, we must allow for fractions and use the set of *rational numbers;* a rational number, you will recall, is a number that can be written in the form

$$p/q \quad \text{with } p \text{ and } q \text{ integers, } q \neq 0.$$

The rational numbers include the integers ($n = n/1$) and the fractions, both *positive* and *negative*. They also include the *mixed numbers,* since these can be written as fractions:

$$2\tfrac{1}{3} = \tfrac{7}{3}, \qquad 4\tfrac{2}{5} = \tfrac{22}{5}, \quad \text{etc.}$$

With the rational numbers we can count, we can add and subtract:

$$\tfrac{5}{8} + \tfrac{1}{4} = \tfrac{5}{8} + \tfrac{2}{8} = \tfrac{7}{8}, \qquad \tfrac{5}{8} - \tfrac{1}{4} = \tfrac{5}{8} - \tfrac{2}{8} = \tfrac{3}{8}$$

multiply and divide:

$$\tfrac{5}{8} \cdot \tfrac{1}{4} = \tfrac{5}{32}, \qquad \tfrac{5}{8} \div \tfrac{1}{4} = \tfrac{5}{8} \cdot \tfrac{4}{1} = \tfrac{20}{8}$$

but we cannot always take roots and we cannot measure all distances. Consider, for example, a unit square (Figure 1.1.1). By the Pythagorean theorem,† the distance between the opposite vertices of this square must be $\sqrt{2}$, the square root of 2. As we show in Section 2.1, this is not a rational number. With only rational numbers at our disposal we

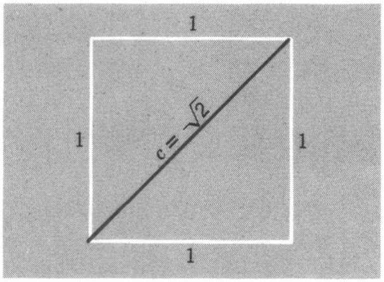

Figure 1.1.1

†The Pythagorean theorem states that in a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse (Figure 1.1.2). In the case of the unit square, $1^2 + 1^2 = c^2$, so that $c^2 = 2$ and $c = \sqrt{2}$.

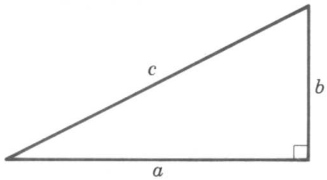

Figure 1.1.2

could not measure this distance. Nor could we find $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{13}$ (the cube root of 13), or $\sqrt[5]{5}$ (the fifth root of 5). The circle would also give us trouble. You are familiar with the formula

$$\text{circumference} = 2\pi r.$$

According to this formula, the distance around a unit circle ($r = 1$) would be 2π, but π is not rational and neither is 2π.† With only rational numbers, we could not even measure the distance around a wheel of radius 1.

To be able to take roots of all positive numbers and to be able to measure all distances, we must enlarge the number system once again. We must accept not only the *rational* numbers but also the *irrational* numbers. Together these comprise the set of *real numbers*.‡

Real Numbers and the Number Line

You can visualize the real numbers on a horizontal line, as in Figure 1.1.3. Choose a point for 0 and a unit of length. Then place the number r that number of units to the right of 0 if r is positive, and $-r$ units to the left of 0 if r is negative. The resulting pattern is the familiar *number line*, also called the *coordinate line*. Each point on it corresponds to a unique real number, and each real number corresponds to a unique point.

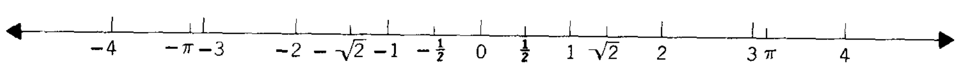

Figure 1.1.3

Decimal Expansions

Every decimal represents a real number:

$$n.a_1a_2a_3 \cdots = n + \frac{a_1}{10} + \frac{a_2}{100} + \frac{a_3}{1000} + \cdots$$

and every real number can be written as a decimal. Terminating decimals such as

$$0.5, \qquad 1.62, \qquad 5.0025$$

and repeating decimals such as

$$0.333 \cdots = 0.\overline{3}, \qquad 4.2121 \cdots = 4.\overline{21}, \qquad 0.1735735 \cdots = 0.1\overline{735}$$

represent rational numbers.¶ The nonrepeating infinite decimals represent the irrational numbers.

---

†You are asked to show this in Exercise 11.

‡To take even roots of negative numbers we must expand the number system once more, this time to include the *complex numbers*. You can find an introduction to the complex numbers in Section 8.6.

¶We use a bar to indicate the repeating block.

To find a decimal expansion for a rational number p/q we divide the *denominator* q into the *numerator* p.

- **Problem.** Express $\frac{13}{5}$ as a decimal.

Solution. We divide 5 into 13:

$$\begin{array}{r} 2.6 \\ 5\,\overline{)\,13.0} \\ \underline{10} \\ 30 \\ \underline{30} \\ 0 \end{array} \qquad \frac{13}{5} = 2.6. \ \square$$

- **Problem.** Express $\frac{5}{11}$ as a decimal.

Solution. We divide 11 into 5:

$$\begin{array}{r} 0.4545 \\ 11\,\overline{)\,5.0000} \\ \underline{4\,4} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 60 \\ \underline{55} \\ 50. \end{array}$$

The cycle goes on repeating itself. Once again we find ourselves dividing 11 into 50. Thus

$$\frac{5}{11} = 0.454545\cdots = 0.\overline{45}. \ \square$$

- **Problem.** Express $0.555\cdots = 0.\overline{5}$ as the quotient of two integers.

Solution

$$\begin{array}{rl} 10(0.\overline{5}) &= 5.\overline{5} \\ -\quad 0.\overline{5} &= 0.\overline{5} \\ \hline 9(0.\overline{5}) &= 5 \end{array}$$

Therefore $0.\overline{5} = \frac{5}{9}$. $\square$

● **Problem.** Express $0.141414\cdots = 0.\overline{14}$ as the quotient of two integers.

Solution. In the previous problem the repeating block had length 1 and we multiplied by 10. Here the repeating block has length 2 and we multiply by 100. If the repeating block had length 3, we would multiply by 1000; and so on.

$$\begin{array}{rrcr} & 100(0.\overline{14}) & = & 14.\overline{14} \\ - & 0.\overline{14} & = & 0.\overline{14} \\ \hline & 99(0.\overline{14}) & = & 14 \end{array}$$

Therefore, $0.\overline{14} = \frac{14}{99}$. □

EXERCISES

(The starred exercises have answers at the back of the book.)

A

1. Draw a number line and mark on it the points $0, \frac{1}{2}, -1, 1, 2, \frac{7}{3}, 3.3, -4$.

2. Draw a number line and mark on it the points 0, 10, 50, -65, and -100.

3. Show that the number is rational by expressing it as the quotient of two integers:
*(a) $6\frac{4}{5}$. (b) $-2\frac{1}{4}$. *(c) $2 - \frac{1}{5}$. (d) $-2 - \frac{1}{5}$.
*(e) 3.1. (f) 1.35. *(g) 3.176. (h) -2.115.

4. Write the fraction in *lowest terms* by canceling all positive integer factors common to the numerator and denominator:
*(a) $\frac{16}{28}$. (b) $\frac{120}{144}$. *(c) $\frac{350}{490}$. (d) $\frac{625}{1000}$.
*(e) $\frac{81}{240}$. (f) $\frac{210}{225}$. *(g) $\frac{16}{1296}$. (h) $\frac{480}{1350}$.

5. Fractions are added, subtracted, multiplied, and divided according to the following rules:

$$\frac{p}{q} + \frac{r}{s} = \frac{ps + qr}{qs}, \qquad \frac{p}{q} - \frac{r}{s} = \frac{ps - qr}{qs}$$

$$\frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs}, \qquad \frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \cdot \frac{s}{r} = \frac{ps}{qr} \text{ provided } r \neq 0.$$

Carry out the indicated operations expressing your answer as a fraction in lowest terms:
*(a) $\frac{1}{2} + \frac{1}{3}$. (b) $\frac{1}{2} - \frac{1}{3}$. *(c) $\frac{5}{8} + \frac{3}{10}$. (d) $\frac{7}{9} - \frac{4}{15}$.
*(e) $\frac{1}{2} \cdot \frac{4}{9}$. (f) $\frac{3}{25} \cdot \frac{5}{21}$. *(g) $\frac{1}{2} \div \frac{3}{2}$. (h) $\frac{6}{25} \div \frac{12}{5}$.
*(i) $\frac{7}{12} - \frac{4}{9}$. (j) $\frac{7}{12} \div \frac{4}{9}$. *(k) $\frac{7}{12} \cdot \frac{4}{9}$. (l) $\frac{7}{12} \div \frac{5}{120}$.

6. Express the fraction as a decimal:
*(a) $\frac{14}{25}$. (b) $\frac{1}{3}$. *(c) $\frac{4}{11}$. (d) $\frac{11}{4}$.
*(e) $\frac{5}{9}$. (f) $\frac{3}{8}$. *(g) $\frac{2}{7}$. (h) $\frac{1}{111}$.

7. Write the decimal as the quotient of two integers:
*(a) $0.444\cdots = 0.\overline{4}$. (b) $0.212121\cdots = 0.\overline{21}$.
*(c) $0.373737\cdots = 0.\overline{37}$. (d) $4.5123123\cdots = 4.5\overline{123}$.
*(e) $0.a_1a_1a_1\cdots = 0.\overline{a_1}$. (f) $0.a_1a_2a_1a_2\cdots = 0.\overline{a_1a_2}$.

8. Given that

$$r_1 = 0.66, \qquad r_2 = 0.33, \qquad r_3 = 0.66\cdots, \qquad r_4 = 0.33\cdots,$$

carry out the indicated operations expressing your answer as a fraction in lowest terms.
*(a) $r_1 + r_2$. (b) $r_1 - r_2$. *(c) r_1r_2. (d) r_1/r_2.
*(e) $r_3 - r_4$. (f) $r_3 + r_4$. *(g) r_3/r_4. (h) r_3r_4.
*(i) $r_1 + r_4$. (j) $r_1 - r_4$. *(k) $r_2 + r_3$. (l) $r_3 - r_2$.
*(m) r_1r_4. (n) r_1/r_4. *(o) r_2r_3. (p) r_3/r_2.

B

*9. Given a unit length, use right triangles and the Pythagorean theorem to construct line segments of lengths $\sqrt{3}$ and $\sqrt{5}$. (HINT: First construct a line segment of length $\sqrt{2}$.)

10. The set of rational numbers is closed under addition and multiplication; that is,

if r_1 and r_2 are rational, then $r_1 + r_2$ and r_1r_2 are rational.

Is this true for the set of irrational numbers? Explain.

11. Show that, since π is irrational, 2π is irrational. (HINT: Show that, if 2π were rational, then π would be rational.)

12. Find r given that r is rational and, for some irrational number z, rz is rational.

1.2 THE ARITHMETIC OF REAL NUMBERS

Here is a list of the basic arithmetic properties of the real-number system with here and there a comment attached. We assume that you've seen all this before in one form or another. Nevertheless we urge you to give this material at least a quick run through.

1 Addition

Addition is *associative*:

$$(a + b) + c = a + (b + c)$$

and *commutative*:

$$a + b = b + a.$$

(Therefore, when we add numbers, we can group them as we please and add them in any order we choose.)

The number 0 is an *additive identity*:

$$a + 0 = 0 + a = a.$$

Every real number a has an *additive inverse* $-a$:

$$a + (-a) = (-a) + a = 0.$$

The additive inverse of $-a$ is a:

$$-(-a) = a.$$

II Multiplication

Multiplication is *associative*:

$$(ab)c = a(bc)$$

and *commutative*:

$$ab = ba.$$

(Therefore, when we multiply real numbers, we can group them as we please and multiply them in any order we choose.)

The number 1 is a *multiplicative identity*:

$$a \cdot 1 = 1 \cdot a = a.$$

Every nonzero real number a has a *multiplicative inverse* $1/a$:

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1.$$

The multiplicative inverse of $1/a$ is a:

$$\frac{1}{1/a} = a.$$

III Subtraction and Division

Subtracting b is the same as adding $-b$:

$$a - b = a + (-b)$$

and subtracting $-b$ is the same as adding b:

$$a - (-b) = a + b.$$

Dividing by a nonzero number b is the same as multiplying by $1/b$:

$$\frac{a}{b} = a \cdot \frac{1}{b}$$

and dividing by $1/b$ is the same as multiplying by b:

$$\frac{a}{1/b} = ab.$$

IV The Distributive Law (Two Forms)

Multiplication *distributes* over addition and subtraction:

$$a(b \pm c) = ab \pm ac \qquad \text{and} \qquad (a \pm b)c = ac \pm bc.$$

(This is the basis for factoring.)

V More on Products and Quotients

The number 0 times any number is 0:

$$0 \cdot a = a \cdot 0 = 0.$$

To multiply by -1 is to change the sign:

$$(-1) \cdot a = a \cdot (-1) = -a.$$

If the product of two factors is 0, then at least one of the factors is 0:

$$\text{if} \quad ab = 0, \quad \text{then} \quad a = 0 \quad \text{or} \quad b = 0.$$

(If, for example, we are given that $(x - 3)(x - 5) = 0$, then we can conclude immediately that $x = 3$ or $x = 5$.)

We can test whether two quotients are equal by *cross multiplication:* for $b \neq 0$ and $d \neq 0$

$$\frac{a}{b} = \frac{c}{d} \quad \text{if and only if} \quad ad = bc.$$

To add or subtract quotients we first give them a common denominator:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}.$$

To multiply quotients we multiply numerators and denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

To divide by a nonzero quotient is to multiply by the reciprocal of that quotient:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$