

U P D A T E D E D I T I O N

M

Mathematics

FOR ELEMENTARY TEACHERS

FIFTH EDITION

A CONTEMPORARY APPROACH



Gary L. Musser • William F. Burger • Blake E. Peterson

W / 1 CD

Fifth Edition

013

MATHEMATICS FOR ELEMENTARY TEACHERS

A Contemporary Approach

UPDATED EDITION

Gary L. Musser

Oregon State University

William F. Burger

Blake E. Peterson

Brigham Young University, Utah



JOHN WILEY & SONS, INC.

NEW YORK • CHICHESTER • WEINHEIM • BRISBANE • SINGAPORE • TORONTO

ACQUISITIONS EDITOR
MARKETING MANAGER
FULL SERVICE MANAGER
COVER PHOTO

Debbie Berridge
Katherine Hepburn
Jeanine Furino
Andy Washnik

This book was set by Meridian Creative Group and printed and bound by R.R. Donnelley & Sons, Inc.
The cover was printed by Phoenix Color Corporation.

This book is printed on acid-free paper.

Copyright 2001© John Wiley & Sons, Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158-0012, (212) 850-6011, fax (212) 850-6088, E-Mail: PERMREQ@WILEY.COM. To order books please call 1(800)-225-5945.

ISBN 0-471-38898-X

Printed in the United States of America

10 9 8 7 6 5 4 3

To

Irene, my wonderful wife of 38 years; Greg, my great son, and his wonderful daughter, Maranda, better known as BB (Bright and Beautiful); Marge, my mother, the best one could hope for; G.L., my father, who passed away before I could thank him enough; and Mary, the daughter of my original coauthor, Bill Burger—she's the greatest.

G. L. M.

Shauna, my beautiful wife, for her unwavering love and support; Quinn, Joelle, Taren, and Riley, my four children, for the joy and happiness they bring to my life; Mom and Dad, for their never-ending support, and for teaching me to shoot for the stars and how to work to reach them.

B. E. P.

A Special Introduction—Blake Peterson

I am very pleased to introduce my new coauthor. Blake was born and raised in Logan, Utah. He attended Logan High School and Utah State University, where he completed a BA degree in Mathematics Education. After graduation, he married Shauna, the woman of his dreams, and they moved to Southern California where he taught high school mathematics (and coached football and basketball) for two years. He returned to graduate school at Washington State University, where he completed a Masters degree in 1990 and a PhD in Mathematics in 1993. His areas of graduate study were number theory and geometry, with particular interest in Integer Polygons and Polyhedra. Since graduating, he has devoted his teaching and research to mathematics teacher education. From 1993 to 1996, Blake was on the mathematics faculty at Oregon State University, where he had been hired to replace my first coauthor, Bill Burger, who had passed away. I worked closely with Blake for a couple of years. He was a superb teacher and was revered and respected by his students who also appreciated his dedication to and enthusiasm for teaching. In 1996, he joined the mathematics department at Brigham Young University, where he is an assistant professor. Blake shares his life with Shauna and their four children, Quinn, Joelle, Taren, and Riley. He continues to enjoy playing basketball, as well as working in the yard and spending time with his family. Blake will continue our tradition of offering a book that has a high level of mathematical integrity as well as relevance for the classroom.

A Special Dedication to Don Miller

I met Don at an NCTM Annual Conference in Orlando when this book was in its first edition. Don had been working on some problem-solving materials and he sought my advice about publishing them. He had already published an excellent soft-cover book on calculators, so I encouraged him to seek out a similar publisher. After several months, Don called to say that he had not found the right publisher. I asked him if he would like to write a problem-solving supplement for Musser/Burger. He said that he would, so the two of us planned the format and I sent Don a twenty-pound box full of problems that I had collected. As it turned out, Don did not use one problem I had sent him. He proceeded to produce some of the best problems I had seen, all organized around the problem-solving strategy flow in our book. I field tested his book in a problem-solving course for elementary teachers in Oregon State. The students loved Don's book, so we rushed it out as a supplement. Bill Burger and I continued to use the book in our problem solving class for years with great success. During the preparation of this edition, I needed to contact Don to have him sign a renewal contract. I called his department at St. Cloud University and was saddened to find that he had passed away on October 10, 1998. Don was one of the most creative mathematics educators I have known. He was a member of the mathematics faculty at SCSU from 1966 through 1997 when he retired. He was a very active mathematics educator and was a major contributor to the strong mathematics program for elementary teachers at his university. We are pleased and proud that we can continue to have Don's book, *A Guide to Problem Solving*, as a supplement to our book. I regard his book to be a classic in problem solving. Proceeds from the sale of his supplement will go to the Department of Mathematics at SCSU to be used to enhance the education of teachers of mathematics in Don's memory. Personally and professionally, I will miss Don greatly.

—Gary L. Musser

Preface

Welcome to a world of mathematical understanding that we hope you will find stimulating, rewarding, enlightening, and fun. We salute you for choosing teaching as a profession and hope that your experiences with this book will help prepare you to be the best possible teacher of mathematics that you can be. We have presented this elementary mathematics material from a variety of perspectives so that you will be more able to address the broad range of learning styles that you will encounter in your future students. This book also encourages prospective teachers to gain an understanding of the underlying concepts of elementary mathematics while maintaining an appropriate level of mathematical precision.

We have also sought to present this material in a manner consistent with the recommendations in (1) *A Call for Change: Recommendations on the Mathematics Preparation of Teachers*; prepared by the Mathematical Association of America's Committee on the Mathematical Education of Teachers; and (2) the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics*. In addition, we have received valuable advice from many of our colleagues around the United States through questionnaires, reviews, focus groups, and personal communications. We have taken great care to respect this advice and to ensure that the content of the book has mathematical integrity and is accessible and helpful to the variety of students who will use it. As always, we look forward to hearing from you about your experiences with the text.

Gary L. Musser
Blake E. Peterson

Content Features

Number Systems: Insofar as possible, number topics are covered sequentially to parallel their development in the school curriculum. Fractions and integers are each treated as extensions of whole numbers. Rational numbers are developed briskly as extensions of both the fractions (by adjoining their opposites) and the integers (by adjoining their reciprocals). The mathematical structure of an ordered field continues to serve to unify this presentation. The important applications of statistics and probability serve as a capstone to the study of number systems.

Approach to Geometry: Geometry is organized from the point of view of the five-level van Hiele model of a child's development in geometry. After studying shapes and measurement, geometry is approached more formally through Euclidean congruence and similarity, coordinates, and transformations. The Epilogue provides an eclectic approach by solving geometry problems using a variety of techniques.

Underlying Themes

Problem Solving: An extensive collection of problem-solving strategies is progressively developed; these strategies can be applied to a generous supply of problems in the exercise/problem sets. The depth of problem-solving coverage can be varied by the number of strategies selected throughout the book and by the problems assigned.

Deductive Reasoning: The use of deduction is promoted throughout the book. The approach is gradual, with later chapters having more multistep problems. In particular, the last sections of Chapters 14, 15, and 16 and the Epilogue offer a rich source of interesting theorems and problems in geometry.

Technology: The Math Explorer calculator, which is used in many schools, is used to show its capability to do long division with remainder and calculate using fractions as well as to perform the many functions of a scientific calculator. The Technology Section includes a study of Logo, computer exploration software for geometry, and graphics calculators.

Additional Topics

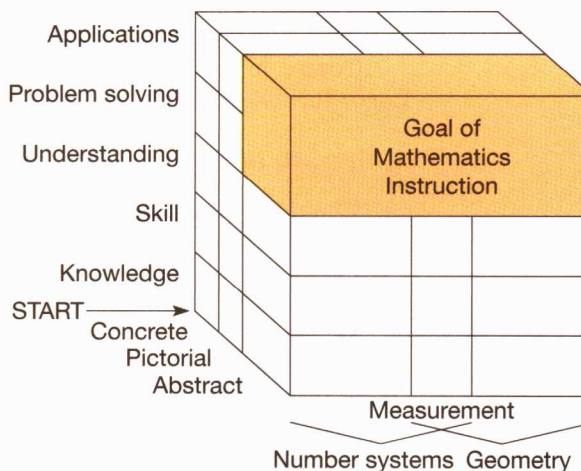
Since reviewers were split concerning where the following topics should appear in text, they are placed near the end of the book to allow for maximum flexibility.

Topic 1, "Elementary Logic," may be used anywhere in a course.

Topic 2, "Clock Arithmetic: A Mathematical System," uses the concepts of opposite and reciprocal and hence may be most instructive after Chapter 6, "Fractions" and Chapter 8, "Integers" have been completed. This last section also contains an introduction to modular arithmetic.

Pedagogical Features

The general organization of the book was motivated by the following mathematics learning cube.:



The three dimensions of the cube—cognitive levels, representational levels, and mathematical content—are integrated throughout the textual material as well as in the problem sets and chapter tests. Problem sets are organized into exercises (to support knowledge, skill, and understanding) and problems (to support problem solving and applications).

Course Options

The material in this book has been organized to allow for a wide variety of courses. At Oregon State University, all preservice elementary teachers are required to take 12 quarter-hours of mathematics. This book is used for the first 9 quarter-hours, a one-year sequence. Each week, students attend 3 one-hour lectures and a 1-hour lab in which materials from the *Student Resource Handbook* are covered. This book, together with *A Guide to Problem-Solving*, is also used in a three quarter-hour junior level course in problem solving for elementary teachers. This course is devoted entirely to developing the students' facilities in applying Pólya's four-step process and the strategies introduced at the beginning of each chapter.

Since many schools have special mathematical requirements, the following are suggested chapters and topics to fit any particular course.

Basic course: Chapters 1–7.

Basic course with logic: Topic 1, Chapters 1–7.

Basic course with informal geometry: Chapters 1–7, 12.

Basic course with introduction to geometry and measurement: Chapters 1–7, 12, 13.

Course through real numbers: Chapters 1–9.

Course through real numbers with applications: Chapters 1–9, 10, and/or 11.

Course through real numbers with geometry: Chapters 1–9, 12, 13, 14 (Sections 1 and 2), 15 (Sections 1 and 2), 16 (Sections 1 and 2), Epilogue.

Course in geometry: Chapters 12–16 (with or without “Programming in Logo, Turtle Geometry” and exploration software)

Summary of Changes in the Fifth Edition

- Chapter 10 has been revised extensively. It now includes a description and application of percentiles as well as scatterplots and regression.
- Quotative and partitive division are discussed in Section 3.2.
- The use of base-10 blocks to model the division algorithm is discussed in Section 4.2.
- Relations has been moved from Topics and integrated into Section 2.4.
- Advanced Counting Techniques has been moved from Topics and integrated into Chapter 11.
- Number lines have been added to different bases in Section 2.3.
- A 100-grid visual approach to percents has been added to Section 7.4.
- A visual approach to dividing fractions has been added to Section 6.3.
- A visual approach to $a \mid b$ has been added to Chapter 5 and is followed throughout.
- A buildup model (as a subset of the prime factorization model) has been added to Section 5.2 to find the LCM.
- Chapter tests have been extensively revised and expanded.
- Many new Reflections from Research have been added and a bibliography for these is provided at the end of each chapter.

Summary of Popular Features Continuing from the Fourth Edition

- Each chapter is launched with a one-page historical vignette.
- Problem solving using strategies is integrated throughout.
- Each chapter has a relevant problem near the beginning to introduce a new strategy.
- Margins include connections to the NCTM Standards as well as Reflections from Research.
- A Student Page Snapshot for an elementary textbook series is included in each chapter to add relevance.
- Every section ends with a rich Mathematical Morsel, which integrates history in a natural way.
- Problem sets are separated into Part A (with answers in the back of the book) and Part B (with answers in the *Instructor's Resource Manual*). Also, exercises and problems are differentiated.
- Problems for Writing/Discussion are included at the end of every problem set.
- A rich pair of historical vignettes involving People in Mathematics is included at the end of each chapter.
- A comprehensive Chapter Review is found near the end of every chapter as well as an extensive Chapter Test.

- There is a rich Epilogue following Chapter 16 that provides an eclectic approach to geometry.
- Technology is integrated throughout. In addition, there is an instructional Technology Section near the end of the book.
- Some optional topics are contained in the Topics section near the end of the book.

Acknowledgments

We would like to acknowledge the following people for their assistance in the preparation of the first four editions of this book: Ron Bagwell, Julie Borden, Sue Borden, Tommy Bryan, Juli Dixon, Christie Gilliland, Dale Green, Kathleen Seagraves Higdon, Hester Lewellen, Roger Maurer, David Metz, Naomi Munton, Tilda Runner, Karen Swenson, Donna Templeton, Lynn Trimpe, Rosemary Troxel, Virginia Usnick, and Kris Warloe.

We also want to give special thanks and acknowledgment to Marcia Swanson, Karen Swenson, Don Miller, Lynn Trimpe, Roger Mauer, and Vikki Mauer for their authorship of our written supplements; and to Debra Pharo for her authorship of the Computerized Test Item File. During the preparation and production of our book, Pearson purchased Prentice-Hall. As a condition of purchase, the United States Department of Justice forced Pearson to sell over 50 titles and ours was one of them. Thus, although we began work on this edition with Prentice-Hall, we finished some of the production with our new owner, John Wiley. We thank Bayani DeLeon, Prentice-Hall, for the excellent job he did in taking our book through its production phase and Sally Simpson, our Prentice-Hall editor, for her contribution to this project. Very special thanks are also due to our new editor at John Wiley, Debbie Berridge, and Ann Berlin, John Wiley's Vice President, College Production and Manufacturing, for taking hold of this project and carrying it through its final phases with very short notice. Finally, comments from colleagues are encouraged and welcome. Please feel free to send suggestions to Gary Musser at musser@math.orst.edu or garymusser@aol.com, or Blake Peterson at peterston@math.byu.edu.

G. L. M.
B. E. P.

REVIEWERS OF OUR FIRST EDITION AND PORTIONS OF OUR SECOND EDITION

Peter Braunfeld, University of Illinois	Alan Hoffer, University of California, Irvine
Tom Briske, Georgia State University	Joe Kennedy, Miami University
Thomas Butts, University of Texas, Dallas	Robert S. Matulis, Millersville University
John Dossey, Illinois State University	Joe K. Smith, Northern Kentucky University
Ruhama Even, Michigan State University	J. Phillip Smith, Southern Connecticut State University
Iris B. Fetta, Clemson University	Judy Sowder, San Diego State University
Majorie Fitting, San Jose State University	Larry Sowder, San Diego State University
Susan Friel, Math/Science Education Network, University of North Carolina	Lynn Trimpe, Linn-Benton Community College
John G. Harvey, University of Wisconsin-Madison	Bruce Vogeli, Columbia University
	Kenneth C. Washinger, Shippensburg University

QUESTIONNAIRE RESPONDENTS FOR THE SECOND EDITION

Susan Baniak, Otterbein College	Charles R. Luttrell, Frederick Community College
James Bierden, Rhode Island College	Carl Maneri, Wright State University
Peter Braunfeld, University of Illinois	George F. Mead, McNeese State University
Randall Charles, San Jose State University	James A. Nickel, University of Texas, Permian Basin
Deann Christianson, University of the Pacific	Bill W. Oldham, Harding University
Henry A. Culbreth, Southern Arkansas University-El Dorado	Debra Pharo, Northwestern Michigan College
Gregory Davis, University of Wisconsin-Green Bay	Tom Richard, Bemidji State University
Roger Engle, Clarion University	Bill Rudolph, Iowa State University
Julie Guelich, Normandale Community College	Lee K. Sanders, Miami University, Hamilton
Brother Joseph Harris, C.S.C., St. Edward's University	Ann Savonen, Monroe County Community College
Patricia Henry, Weber State College	Karen Sharp, Mott Community College
Linda Hill, Idaho State University	Keith Shuert, Oakland Community College
Pat Jones, Methodist College	Ron Smit, University of Portland
Susan Key, Meridien Community College	Raymond E. Spaulding, Radford University
Mary Kilbridge, Augustana College	Sister Carol Spiegel, BVM, Clarke College
Judith Koenig, California State University, Dominguez Hills	

Viji Sundar, California State
University, Stanislaus
Karen Swenson, George Fox College
Barbara Walters, Ashland
Community College

Joyce Wellington, Southeastern
Community College
Stanley J. Zehm, Heritage College

COMPUTER TEST BANK CONTRIBUTORS FOR THE SECOND EDITION

Darrel Austin, Anderson University
Susan Baniak, Otterbein College
Deann Christianson, University of
the Pacific
Gregory Davis, University of
Wisconsin–Green Bay
Roger Engle, Clarion University
Mary Kilbridge, Augustana College
Carl Maneri, Wright State University

James A. Nickel, University of Texas,
Permian Basin
Karen Sharp, Mott Community
College
Sister Carol Spiegel, BVM, Clarke
College
Barbara Walters, Ashland
Community College

REVIEWERS OF THE THIRD EDITION

Christine Browning, Western
Michigan University
Tommy Bryan, Baylor University
Lucille Bullock, University of Texas
Ann Dinkheller, Xavier University
John Dossey, Illinois State
University
Sheryl Ettlich, Southern Oregon
State College
Virginia Ellen Hanks, Western
Kentucky University
Richard Kinson, University of South
Alabama

John Koker, University of Wisconsin
Josephine Lane, Eastern Kentucky
University
Mercedes McGowen, Harper
College
Flora Alice Metz, Jackson State
Community College
Barbara Moses, Bowling Green State
University
James Riley, Western Michigan
University

QUESTIONNAIRE RESPONDENTS FOR THE THIRD EDITION

Mary Alter, University of Maryland
James Barnard, Western Oregon
State College
Judy Bergman, University of
Houston–Clearlake
Harold Brockman, Capital
University
Judith Brower, North Idaho College
Harmon Brown, Harding University
Christine Browning, Western
Michigan University

Joyce W. Bryant, St. Martin's College
Lynn Cleary, University of Maryland
Sister Marie Condon, Xavier
University
Lynda Cones, Rend Lake College
Greg Crow, John Carroll University
Doris Edwards, Northern State
College
Ann Farrell, Wright State University
Margaret Friar, Grand Valley State
College

Judy Gibbs, West Virginia University	Susan Novelli, Kellogg Community College
Anna Mae Greiner, Eisenhower Middle School	Jan Odell, Richland College
Virginia Hanks, Western Kentucky University	Debra Pharo, Northwestern Michigan College
Dave Hansmire, College of the Mainland	Robert Preller, Illinois Central College
Joseph Harris, St. Edward's University	Tom Richard, Bemidji State University
John Harvey, University of Wisconsin	Jan Rizzuti, Central Washington University
Ina Lee Herer, Tri-State University	Frances Rosamond, National University
Sandra Hsieh, Pasadena City College	Albert Roy, Bristol Community College
Jo Johnson, Southwestern College	Joseph Shields, St. Mary's College, MN
Patricia Johnson, Ohio State University	Lawrence Shirley, Townson State University
Mike Kilgallen, Lincoln Christian College	Rick Simon, Idaho State University
Josephine Lane, Eastern Kentucky University	James Smart, San Jose State University
Don Larsen, Buena Vista College	Larry Sowder, San Diego State University
Louise Lataille, Westfield State College	Ruthi Sturdevant, Lincoln University, MO
Vernon Leitch, St. Cloud State University	Ann Sweeney, College of St. Catherine, MN
Lawrence Levy, University of Wisconsin	Martha Van Cleave, Linfield College
Betty Long, Appalachian State University	Barbara Walters, Ashland Community College, KY
Nancy Maushak, William Penn College	Jerry Wilkerson, Missouri Western State College
Edith Maxwell, West Georgia College	Delbert Williams, University of Mary Hardin-Baylor
Kent Morris, Cameron University	Chris Wise, University of Southwestern Louisiana
Barbara Moses, Bowling Green State University	Makia Zimmer, Bethany College
Gale Nash, Western State College of Colorado	
Jerry Neft, University of Dayton	

QUESTIONNAIRE RESPONDENTS FOR THE FOURTH EDITION

Dr. J. Altinger, Youngstown State University	Harmon Brown, Harding University
Jamie Whitehead Ashby, Texarkana College	Judith Colburn, Lindenwood College
Dr. Donald Balka, Saint Mary's College	Sister Judith Costello, Regis College
Jim Ballard, Montana State University	H. Coulson, California State University
	Carl Cuneo, Essex Community College

- Cynthia Davis, Truckee Meadows
Community College
- Louise Deaton, Johnston
Community College
- Mary DeYoung, Hope College
- Randall L. Drum, Texas A&M
University
- P.R. Dwarka, Howard University
- Kathy Ernie, University of Wisconsin
- Ron Falkenstein, Mott Community
College
- Chris Ferris, University of Akron
- Cathey Funk, Valencia Community
College
- Dr. Amy Gaskins, Northwest
Missouri State University
- Daniel Green, Olivet Nazarene
University
- Dr. Noal Herbertson, California
State University
- Scott H. Hochwald, University of
North Florida
- Susan S. Hollar, Kalamazoo Valley
Community College
- Pat Jones, Methodist College
- Judy Kasabian, El Camino College
- Vincent Kayes, Mt. St. Mary College
- Joe Kennedy, Miami University
- Steven C. Leth, University of
Northern Colorado
- Robert Lewis, Linn-Benton
Community College
- Lois Linnan, Clarion University
- Melissa Shepard Loe, University of
St. Thomas
- C.A. Lubinski, Illinois State
University
- Pamela Lundin, Lakeland College
- Wilbur Mellema, San Jose City
College
- Clarence E. Miller, Jr., Johns
Hopkins University
- Diane Miller, Middle Tennessee
State University
- Bill Moody, University of Delaware
- Lisa Morrison, Western Michigan
University
- Katherine Muhs, St. Norbert College
- Fran Moss, Nicholls State University
- T. Neelor, California State
University
- Kathy Nickell, College of DuPage
- Jon O'Dell, Richland Community
College
- Jim Paige, Wayne State College
- Wing Park, College of Lake County
- Shahla Peterman, University of
Missouri
- Gary D. Peterson, Pacific Lutheran
University
- Dr. William Price, Niagara
University
- Kim Prichard, University of North
Carolina
- Janice Rech, University of Nebraska
- David Roland, University of Mary
Hardin-Baylor
- Richard Ross, Southeast Community
College
- Bernadette Russell, Plymouth State
College
- B. Signer, St. John's University
- Gayle Smith, Lane Community
College
- Debbie Stokes, East Carolina
University
- Carla Tayeh, Eastern Michigan
University
- Janet Thomas, Garrett Community
College
- S. Thomas, University of Oregon
- Mary Beth Ulrich, Pikeville College
- Dr. Howard Wachtel, Bowie State
University
- Dr. Mary Wagner-Krankel, St.
Mary's University
- Bill Weber, Eastern Arizona College
- Paula White, Marshall University
- Heide G. Wiegel, University of
Georgia
- Jane Wilburne, West Chester
University
- Jerry Wilkerson, Missouri Western
State College
- Jack D. Wilkinson, University of
Northern Iowa
- Carole Williams, Seminole
Community College
- Mary Wolfe, University of Rio
Grande
- Vernon E. Wolff, Moorhead State
University
- Maria Zack, Point Loma Nazarene
College

REVIEWERS FOR THE FOURTH EDITION

Chuck Beals, Hartnell College
Donna Erwin, Salt Lake Community
College
Louise Lataille, Springfield College
Kathy Nickell, College of DuPage

Dennis Parker, The University of the
Pacific
Lawrence Small, L.A. Pierce College
John Wilkins, California State
University-Dominguez Hills

REVIEWERS FOR THE FIFTH EDITION

Anne Brown, Indiana
University-South Bend
Carol Dyas, University of Texas-San
Antonio
Gerald Gannon, California State
University-Fullerton
Patricia L. Hayes, Utah State
University-Uintah Basin Branch
Campus

Barnabas Hughes, California State
University-Northridge
Maura Murray, University of
Massachusetts
Eric Rowley, Utah State University
Karen Spike, University of Northern
Carolina-Wilmington

Text Preview

The following pages highlight the special features of the text and its supplements

Focus On ►

Each chapter opens with an interesting historical feature which introduces ideas and concepts central to the chapter.

► Chapter

1

Introduction to Problem Solving

FOCUS ON

George Pólya: The Father of Modern Problem Solving

George Pólya was born in Hungary in 1887. He received his Ph.D. at the University of Budapest. In 1940 he came to Brown University and then joined the faculty at Stanford University in 1942.

In his studies, he became interested in the process of discovery, which led to his famous four-step process for solving problems:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back.

He died in 1985, leaving mathematics with the important legacy of teaching problem solving. His “Ten Commandments for Teachers” are as follows:

1. Be interested in your subject.
2. Know your subject.
3. Try to read the faces of your students; try to see their expectations and difficulties; put yourself in their place.
4. Realize that the best way to learn anything is to discover it by yourself.
5. Give your students not only information, but also know-how, mental attitudes, the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come—try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once—let the students guess before you tell it—let them find out by themselves as much as is feasible.
10. Suggest; do not force information down their throats.



George Pólya

Pólya wrote over 250 mathematical papers and three books that promote problem solving. His most famous book, *How to Solve It*, which has been translated into 15 languages, introduced his four-step approach together with heuristics, or strategies, which are helpful in solving problems. Other important works of Pólya are *Mathematical Discovery*, Volumes 1 and 2, and *Mathematics and Plausible Reasoning*, Volumes 1 and 2.

Enrichment Features

Problem Solving Strategies ►

Each chapter introduces a new Problem Solving Strategy; a comprehensive list is included at the beginning of each chapter

Problem-Solving Strategies

1. Guess and Test
2. Use a Variable
3. Draw a Picture
4. Look for a Pattern
5. Make a List
6. Solve a Simpler Problem
7. Draw a Diagram
8. Use Direct Reasoning
9. Use Indirect Reasoning
10. Use Properties of Numbers

Strategy

USE PROPERTIES OF NUMBERS

Understanding the intrinsic nature of numbers is often helpful in solving problems. For example, knowing that the sum of two even numbers is even and that an odd number squared is odd may simplify checking some computations. The solution of the initial problem will seem to be impossible to a naive problem solver who attempts to solve it using, say, the Guess and Test strategy. On the other hand, the solution is immediate for one who understands the concept of divisibility of numbers.

Initial problem

A major fast-food chain held a contest to promote sales. With each purchase a customer was given a card with a whole number less than 100 on it. A \$100 prize was given to any person who presented cards whose numbers totaled 100. The following are several typical cards. Can you find a winning combination?

3 9 12 15 18 27 51 72 84

▲ Initial Problem

At the beginning of each chapter, an initial problem is posed that can be solved by using the strategy introduced in that chapter.

NCTM Standards 2000 Number and Operations Grades 6–8

All students should use factors, multiples, prime factorization, and relatively prime numbers to solve problems.

▲ NCTM Standards 2000

NCTM Principles and Standards for School Mathematics, referred to as Standards 2000, are called out in the margins where the standard listed is being used in the text.

Reflections from Research ►

Encouraging the recognition of the research that is being drawn on throughout the text, these marginal notes provide a great portfolio of relevant research.

Reflection from Research

When students are allowed to use calculators to generate data and are encouraged to examine the data for patterns, they often discover divisibility rules on their own (Bezuszka, 1985).