

QUANTUM ELECTRONICS

Third Edition

Amnon Yariv

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Amnon Yariv

California Institute of Technology



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Preface

The thirteen years that have intervened since the appearance of the second edition of this book have witnessed some important developments in the field of lasers and quantum electronics. Foremost among them are: phase-conjugate optics and its myriad applications, the long wavelength quaternary semiconductor laser, and the deepened understanding of the physics of semiconductor lasers—especially that applying to their current modulation and limiting linewidth, laser arrays and the related concept of supermodes, quantum well semiconductor lasers, the role of phase amplitude coupling in laser noise, and free-electron lasers. The present edition retains nearly all the material of the second edition. There are four new chapters on semiconductor lasers, quantum well lasers, free-electron lasers, and phase-conjugate optics. In addition, the chapters on laser noise and third-order nonlinear effects have been extensively revised.

I benefited from teaching the material and from feedback at Caltech by colleagues and students who aided me in tightening and improving numerous points throughout the book. To these individuals special thanks are due. I am grateful to my administrative assistant, Jana Mercado, for her assistance in preparing this edition. I also take pleasure in thanking the talented group of students and ex-students, especially K. Lau, K. Vahala, C. Harder, T. Koch, M. Cronin-Golomb, S. Kwong, and the late C. Lindsey, whose research efforts are responsible for much of the new material added to this edition. The material on quantum well lasers benefited greatly from my association with Y. Arakawa while David Crouch contributed a major part of the treatment of squeezed states.

Pasadena, California
September 1987

Amnon Yariv

Preface to the Second Edition

This textbook introduces the main principles involved in the study and practice of quantum electronics, which include the theory of laser oscillators, a wide range of optical phenomena, and devices that owe their existence to the intense and coherent optical fields made possible by the laser.

The emphasis is almost exclusively on fundamental principles. An attempt is made, however, to bridge the gap between theory and practice through the use of numerical examples based on real situations.

Approximately one-half of this edition is new. In addition, a number of topics related to microwave phenomena and magnetic resonance were omitted. The major changes are as follows.

1. The addition of treatments on Gaussian beam propagation in lenslike media, optical resonators, density matrix formulation of the interaction of light and matter, theory of laser oscillation, Van der Pol noise analysis of lasers, dye lasers, amplification in vibrational-rotational transitions, double heterojunction lasers, mode locking in homogeneously broadened lasers, *Q*-switching, saturated amplifiers and amplification of spontaneous emission, acoustooptic interactions, self-induced transparency, photon echoes, spontaneous parametric fluorescence, distributed feedback lasers, and mode coupling in dielectric waveguides.

2. The deletion of chapters dealing with microwave masers, magnetism, magnetic resonance, and microwave parametric oscillators.

3. An exclusive use of the meter-kilogram-second (MKS) unit system.

This text is primarily for the graduate student in physics and applied physics. The latter category often includes students in departments of electrical engineering and material science.

The typical Caltech student taking the course from which this book was developed has a background of a one-year rigorous course in quantum mechanics and one course in electromagnetic theory. These are courses taken by the more advanced students in their senior year but often in the first year of graduate school. A good familiarity with these two topics is assumed, although most of the prerequisite background material is included here.

The book can be used as a basis for a one-year course in quantum electronics or, alternatively, for these one-semester courses:

X PREFACE TO THE SECOND EDITION

1. **Lasers:** Chapters 5–13.
2. **Nonlinear Optical Effects and Stimulated Scattering Phenomena:** Chapters 14 (part dealing with acoustooptics), 15–18.
3. **Optical Modes and Propagation Phenomena:** Chapters 5–7, 14, 19.

Course 1 makes heavy use of quantum mechanics. In course 2 quantum mechanics is needed only in Chapter 15, while in course 3 it is not used at all. An electromagnetic background is needed in all three courses.

I apologize to any of my colleagues whose work has not been acknowledged or adequately represented in this book. Since this is primarily a textbook, the material was chosen mainly because of pedagogic considerations rather than chronological precedence.

I thank Ruth Stratton and Dian Rapchak for typing and proofreading the original manuscript and Paula Samazan for assisting with the references.

Thanks are due to Dr. Jack Comly who made important contributions to Chapters 15 and 18 and to Mr. H. W. Yen who has gone over the whole manuscript rederiving the results and checking the internal consistency of the text.

Amnon Yariv

About the Author

Amnon Yariv, a native of Israel, received his higher education at the University of California at Berkeley. After four years at the Bell Telephone Laboratories, he joined the California Institute of Technology (Caltech) where he is the Thomas G. Myers Professor of Electrical Engineering and Applied Physics. At Caltech he studies with a team of doctoral students and postdoctoral fellows, a number of research topics in laser physics, nonlinear optics, and optoelectronics. Some of his major contributions include the invention and co-invention of the fields of integrated optoelectronic circuits, phase conjugate optics, and the authoring of the first papers on the theory of mode locking and nonlinear quantum optics. Dr. Yariv is also the founder and chairman of the board of ORTEL Corporation, a semiconductor laser company in Alhambra, California.

When not working, which is rare, he spends time with his family (wife—Fran and three daughters) as well as with a piano, a tennis racket, and a windsurfer.

Contents

CHAPTER 1	
Basic Theorems and Postulates of Quantum Mechanics	1
1.0 Introduction	1
1.1 The Schrödinger Wave Equation	1
1.2 The Time-Independent Schrödinger Wave Equation	7
CHAPTER 2	
Some Solutions of the Time-Independent Schrödinger Equation	18
2.0 Introduction	18
2.1 Parity	18
2.2 The Harmonic Oscillator	19
2.3 The Schrödinger Equation in Spherically Symmetric Potential Fields	27
2.4 The Angular Momentum Operators and Their Eigenfunctions	30
CHAPTER 3	
Matrix Formulation of Quantum Mechanics	34
3.0 Introduction	34
3.1 Some Basic Matrix Properties	34
3.2 Transformation of a Square Matrix	35
3.3 Matrix Diagonalization	36
3.4 Representations of Operators as Matrices	36
3.5 Transformation of Operator Representations	38
3.6 Deriving the Eigenfunctions and Eigenvalues of an Operator by the Matrix Method	39
3.7 The Heisenberg Equations of Motion	41
3.8 Matrix Elements of the Angular Momentum Operators	42
3.9 Spin Angular Momenta	45
3.10 Addition of Angular Momentum	45
3.11 Time-Independent Perturbation Theory	47

xiv CONTENTS

3.12 Time-Dependent Perturbation Theory—Relation to Line Broadening	50
3.13 Density Matrices—Introduction	56
3.14 The Density Matrix	56
3.15 The Ensemble Average	57
3.16 Time Evolution of the Density Matrix	58
3.17 The Time Evolution Operator-Feynman Diagrams	58

CHAPTER 4

Lattice Vibrations and Their Quantization	68
4.0 Introduction	68
4.1 Motion of Homogeneous Line	68
4.2 Wave Motion of a Line of Similar Atoms	69
4.3 A Line with Two Different Atoms	71
4.4 Lattice Sums	74
4.5 Quantization of the Acoustic Branch of Lattice Vibrations	76
4.6 Average Thermal Excitation of Lattice Modes	80

CHAPTER 5

Electromagnetic Fields and Their Quantization	83
5.0 Introduction	83
5.1 Power Transport, Storage, and Dissipation in Electromagnetic Fields	83
5.2 Propagation of Electromagnetic Waves in Anisotropic Crystals	87
5.3 The Index Ellipsoid	90
5.4 Propagation in Uniaxial Crystals	92
5.5 Normal Mode Expansion of the Electromagnetic Field in a Resonator	94
5.6 The Quantization of the Radiation Field	96
5.7 Mode Density and Blackbody Radiation	99
5.8 The Coherent State	100

CHAPTER 6

The Propagation of Optical Beams in Homogeneous and Lenslike Media	106
6.0 Introduction	106
6.1 The Lens Waveguide	106
6.2 The Identical-Lens Waveguide	111
6.3 The Propagation of Rays Between Mirrors	111
6.4 Rays in Lenslike Media	112
6.5 The Wave Equation in Quadratic Index Media	115
6.6 The Gaussian Beam in a Homogeneous Medium	116

6.7	The Fundamental Gaussian Beam in a Lenslike Medium—The <i>ABCD</i> Law	120
6.8	A Gaussian Beam in a Lens Waveguide	123
6.9	High-Order Gaussian Beam Modes in a Homogeneous Medium	124
6.10	High-Order Gaussian Beam Modes in Quadratic Index Media	125
6.11	Propagation in Media with a Quadratic Gain Profile	127
6.12	Elliptic Gaussian Beams	129

CHAPTER 7**Optical Resonators** 136

7.0	Introduction	136
7.1	Spherical Mirror Resonators	136
7.2	Mode Stability (Confinement) Criteria and the Self-Consistent Resonator Solutions	141
7.3	The Resonance Frequencies	145
7.4	Losses in Optical Resonators	147
7.5	Unstable Optical Resonators	149

CHAPTER 8**Interaction of Radiation and Atomic Systems** 155

8.0	Introduction	155
8.1	Density Matrix Derivation of the Atomic Susceptibility	155
8.2	The Significance of $\chi(\nu)$	162
8.3	Spontaneous and Induced Transitions	164
8.4	The Gain Coefficient	169
8.5	The Einstein Treatment of Induced and Spontaneous Transitions	171
8.6	Homogeneous and Inhomogeneous Broadening	173
8.7	Gain Saturation in Systems with Homogeneous and Inhomogeneous Broadening	176

CHAPTER 9**Laser Oscillation** 183

9.0	Introduction	183
9.1	The Laser Oscillation Condition	183
9.2	Laser Oscillation—General Treatment	189
9.3	Power Output from Lasers	191

CHAPTER 10**Some Specific Laser Systems** 202

10.0	Introduction	202
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xvi CONTENTS

10.1 Pumping and Laser Efficiency	202
10.2 The Ruby Laser	202
10.3 The Nd^{3+} :YAG Laser	208
10.4 The Neodymium-Glass Laser	211
10.5 The He-Ne Laser	214
10.6 The Carbon Dioxide Laser	216
10.7 Organic-Dye Lasers	224

CHAPTER 11

Semiconductor Diode Lasers 232

11.0 Introduction	232
11.1 Some Semiconductor Background	232
11.2 Optically Induced Band-to-Band Transitions in Semiconductors	236
11.3 Diode Lasers	243
11.4 GaInAsP Lasers	251
11.5 Some Real Lasers	251
11.6 Direct-Current Modulation of Semiconductor Lasers	255

CHAPTER 12

Quantum Well Lasers 264

12.0 Introduction	264
12.1 The Quantum Mechanics	264
12.2 Gain in Quantum Well Lasers	269
12.3 Some Numerical Considerations	271

CHAPTER 13

The Free-Electron Laser 277

13.0 Introduction	277
13.1 The Kinematics of Free-Electron-Photon Interaction	277
13.2 Theory of Optical Gain in Free-Electron Lasers	283
13.3 The Pondermotive Potential	289

CHAPTER 14

The Modulation of Optical Radiation 298

14.0 Introduction	298
14.1 The Electrooptic Effect	298
14.2 Electrooptic Retardation	307
14.3 Electrooptic Amplitude Modulation	310
14.4 Phase Modulation of Light	313
14.5 Transverse Electrooptic Modulators	315

14.6	High-Frequency Modulation Considerations	318
14.7	Electrooptic Beam Deflection	323
14.8	The Photoelastic Effect	325
14.9	Bragg Diffraction of Light by Acoustic Waves	327
14.10	Deflection of Light by Sound	335
14.11	Bragg Scattering in Naturally Birefringent Crystals	337

CHAPTER 15**Coherent Interactions of a Radiation Field and An Atomic System**

		342
15.0	Introduction	342
15.1	Vector Representation of the Interaction of a Radiation Field with a Two-Level Atomic System	342
15.2	Superradiance	352
15.3	Photon Echoes	355
15.4	Self-Induced Transparency	357

CHAPTER 16**Introduction to Nonlinear Optics—Second-Harmonic Generation**

		378
16.0	Introduction	378
16.1	The Nonlinear Optical Susceptibility Tensor	379
16.2	The Nonlinear Field Hamiltonian	383
16.3	On the Physical Origins of the Nonlinear Optical Coefficients	384
16.4	The Electromagnetic Formulation of the Nonlinear Interaction	389
16.5	Optical Second-Harmonic Generation	392
16.6	Second-Harmonic Generation with a Depleted Input	398
16.7	Second-Harmonic Generation with Gaussian Beams	400
16.8	Internal Second-Harmonic Generation	402

CHAPTER 17**Parametric Amplification, Oscillation, and Fluorescence**

		407
17.0	Introduction and Lumped Circuit Analog	407
17.1	The Basic Equations of Parametric Amplification	409
17.2	Parametric Oscillation	411
17.3	Power Output and Pump Saturation in Parametric Oscillators	418
17.4	Frequency Turning in Parametric Oscillation	419
17.5	Quantum Mechanical Treatment of Parametric Interactions	421
17.6	Frequency Up-Conversion	425
17.7	Spontaneous Parametric Fluorescence	430

17.8 Backward Parametric Amplification and Oscillation	435
17.9 Squeezed States of the Electromagnetic Field	437

CHAPTER 18

Third-Order Optical Nonlinearities—Stimulated Raman and Brillouin Scattering 453

18.0 Introduction	453
18.1 The Nonlinear Constants	453
18.2 molecular Raman Scattering	457
18.3 Stimulated Molecular Raman Scattering	465
18.4 Electromagnetic Treatment of Stimulated Raman Scattering	469
18.5 Anti-Stokes Scattering	473
18.6 Stimulated Brillouin Scattering	475
18.7 A Classical Treatment of Brillouin Scattering	475
18.8 Self-Focusing of Optical Beams	482

CHAPTER 19

Phase-Conjugate-Optics and Photorefractive Beam Coupling 495

19.0 Introduction	495
19.1 Propagation Through a Distorting Medium	495
19.2 Image Transmission in Fibers	497
19.3 Theory of Phase Conjugation by Four-Wave Mixing	498
19.4 Optical Resonators with Phase-Conjugate Reflectors	506
19.5 The <i>ABCD</i> Formalism of Phase-Conjugate Optical Resonators	507
19.6 Some Practical Applications of Phase Conjugation	510
19.7 Optical Phase Conjugation by Stimulated Nonlinear Scattering	513
19.8 Beam Coupling and Phase Conjugation by the Photorefractive Effect	516

CHAPTER 20

Q-Switching and Mode Locking of Lasers 534

20.0 Introduction	534
20.1 Q-Switching	534
20.2 Mode Locking in Inhomogeneously Broadened Laser Systems	542
20.3 Mode Locking in Homogeneously Broadened Laser Systems	553
20.4 Relaxation Oscillation in Lasers	560
20.5 Passive Mode Locking	565

CHAPTER 21	
Noise and Spectra of Laser Amplifiers and Oscillators	570
21.0 Introduction	570
21.1 Noise in Laser Amplifiers	570
21.2 Spontaneous Emission Noise in Laser Oscillators	577
21.3 Some Mathematical Background	582
21.4 The Laser Equations	584
21.5 The Laser Spectra	586
21.6 The Laser Spectra Experiments	592
21.7 The α Parameter	594
21.8 The Measurement of $(\Delta\nu)_{\text{laser}}$	596
CHAPTER 22	
Guided Wave Optics—Propagation in Optical Fibers	600
22.0 Introduction	600
22.1 The Waveguide Modes	600
22.2 Mode Characteristics of the Planar Waveguide	603
22.3 Coupling Between Guided Modes	606
22.4 The Periodic Waveguide—Distributed Feedback Lasers	608
22.5 The Coupled-Mode Solutions	611
22.6 The Distributed Feedback Laser	615
22.7 Electrooptic Modulation and Mode Coupling in Dielectric Waveguides	623
22.8 Directional Coupling—Supermodes	627
22.9 The Eigenmodes of a Coupled Waveguide System ("Supermodes")	631
22.10 Propagation in Optical Fibers	640
APPENDIX 1	
The Kramers–Kronig Relations	651
APPENDIX 2	
Solid Angle Associated with a Blackbody Mode	653
APPENDIX 3	
The Spontaneous Emission Lifetime for a Vibrational-Rotational Transition in a Linear Molecule	655
APPENDIX 4	
Quantum Mechanical Derivation of Nonlinear Optical Constants	658

xx CONTENTS

A P P E N D I X 5

**The Interaction of An Electron and An Electromagnetic
Field**

663

A P P E N D I X 6

**The Derivation of the Spontaneous Emission Langevin
Fluctuation "Power"**

666

Index

669

CHAPTER I

Basic Theorems and Postulates of Quantum Mechanics

1.0 INTRODUCTION

In this chapter we shall consider some of the basic postulates and theorems of quantum mechanics. These are general and independent of the specific system studied. The application of these results to special problems will be the concern of Chapter 2 and, to a lesser extent, of the rest of the book.

1.1 THE SCHRÖDINGER WAVE EQUATION

According to quantum mechanics, the behavior of a particle is described by the wavefunction $\psi(\mathbf{r}, t)$, which is a solution of the Schrödinger wave equation

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) \right] \psi = i\hbar \frac{\partial \psi}{\partial t} \quad (1.1-1)$$

$V(\mathbf{r}, t)$ is the potential energy function of the particle and $\hbar = h/2\pi$ where h is Planck's universal constant.

By associating the differential operator $-i\hbar \nabla$ with particle linear momentum \mathbf{p} , that is,

$$\mathbf{p} \rightarrow -i\hbar \nabla \quad (1.1-2)$$

the operator on the left side of (1.1-1) can be associated with the sum of the kinetic and potential energy of the particle.

$$E \rightarrow -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \quad (1.1-3)$$

Statistical Interpretation of the Wavefunction

Consider a very large number of independent spaces with identical potential functions $V(\mathbf{r}, t)$. The motion of a particle in each one of these spaces is described by the same $\psi(\mathbf{r}, t)$. The a-priori probability $P(\mathbf{r}, t) dv$ of finding any given particle inside a volume dv (centered about \mathbf{r}) is taken as the fraction of the particles found, by measurement, to be inside dv at time t . According to quantum mechanics, the probability density $P(\mathbf{r}, t)$ is given by

$$P(\mathbf{r}, t) = \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t) \quad (1.1-4)$$

2 BASIC THEOREMS AND POSTULATES OF QUANTUM MECHANICS

where the asterisk superscript stands for the complex conjugate of the quantity in question.

The statistical interpretation of $(\psi^*\psi)$ is made plausible by showing, as will be done in Section 1.2, that the average motion of a particle as determined from the statistical point of view agrees with its classical counterpart. The final arbiter of the validity of this statement is, of course, the agreement of the results derived using it with experiment.

The first condition resulting from the statistical interpretation is that the total probability of finding the particle somewhere in space is finite and is a constant, that is,

$$\int_{\text{all space}} P(\mathbf{r}, t) dv = \int_{\text{all space}} \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t) dv = \text{constant} \quad (1.1-5)$$

or that

$$\frac{d}{dt} \int_{\text{all space}} \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t) dv = 0 \quad (1.1-6)$$

The proof proceeds as follows

$$\frac{d}{dt} \int_V \psi^*\psi dv = \int_V \frac{\partial}{\partial t} (\psi^*\psi) dv = \int_V \left(\psi \frac{\partial \psi^*}{\partial t} + \frac{\partial \psi}{\partial t} \psi^* \right) dv$$

substituting for $\partial\psi/\partial t$ and $\partial\psi^*/\partial t$ from (1.1-1), the terms involving $V(\mathbf{r}, t)$ drop out, and the result is

$$\frac{i\hbar}{2m} \int_V (\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*) dv$$

Use is now made of Green's theorem

$$\int_V (f \nabla^2 g - g \nabla^2 f) dv = \int_A (f \nabla g - g \nabla f) \cdot \mathbf{n} da \quad (1.1-7)$$

where f and g are two arbitrary scalar functions, A is the surface bounding V , and \mathbf{n} is the unit outward normal vector. This leads to

$$\frac{d}{dt} \int_V (\psi^*\psi) dv = \frac{i\hbar}{2m} \int_A (\psi^* \nabla \psi - \psi \nabla \psi^*) \cdot \mathbf{n} da \quad (1.1-8)$$

If the volume V is that of all space, the admissible solutions of (1.1-1) are indeed those in which the behavior of ψ as $r \rightarrow \infty$ is such that the integration specified by (1.1-8) leads to a zero result. If the volume V is finite, the same result is obtained by choosing ψ so that its value on the bounding surface A leads to a zero result in (1.1-8). This point is discussed further in Problem 1.1.

Having proven (1.1-5), we are free to normalize ψ so that the constant appearing in (1.1-5) is unity. This is consistent with the probabilistic interpretation, since the probability of finding the particle somewhere in all of space is unity, that is,

$$\int_{V=\text{all space}} P(\mathbf{r}, t) dv = \int_V \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t) dv = 1 \quad (1.1-9)$$