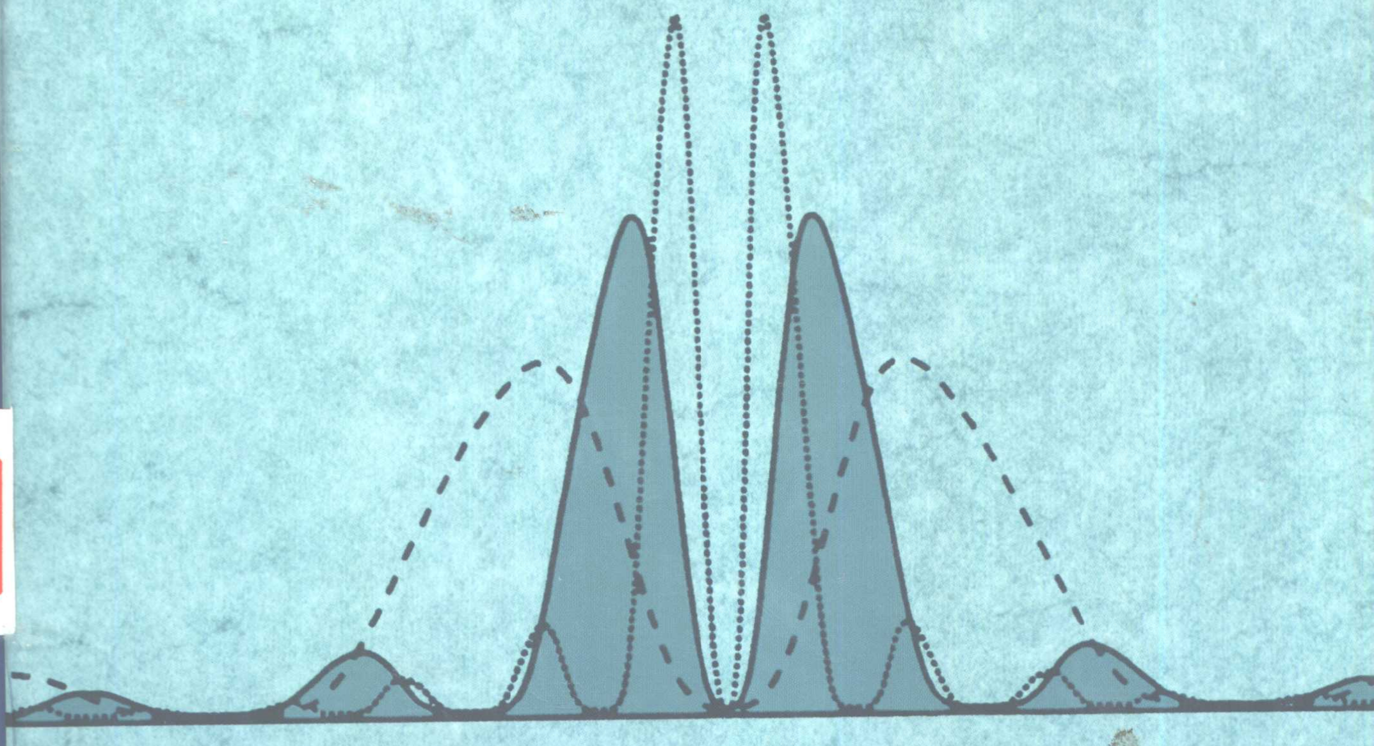
Ali N. Akansu / Richard A. Haddad

Second Edition

MULTIRESOLUTION SIGNAL DECOMPOSITION

Transforms • Subbands • Wavelets



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Transforms, Subbands, and Wavelets

Second Edition

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and

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New Jersey Institute of Technology
Newark, NJ

ACADEMIC PRESS
A Harcourt Science and Technology Company

San Diego San Francisco New York Boston
London Sydney Tokyo

This book is printed on acid-free paper. ∞

ACADEMIC PRESS
A Harcourt Science and Technology Company
525 B Street, Suite 1900, San Diego, CA 92101-4495 USA
http://www.academicpress.com

Academic Press
Harcourt Place, 32 Jamestown Road, London NW1 7BY UK

Library of Congress Catalog Number: 99-68565

International Standard Book Number: 0-12-047141-8

Printed in the United States of America
00 01 02 03 04 EB 9 8 7 6 5 4 3 2 1

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Transforms, Subbands, and Wavelets

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Series in Telecommunications

Series Editor
T. Russell Hsing

Bell Communications Research
Morristown, NJ

Multiresolution Signal Decomposition: Transforms, Subbands, and Wavelets

Ali N. Akansu and Richard A. Haddad

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Newark, NJ

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To Bilge and Elizabeth

Preface

Since the first edition of this book in 1992 we have witnessed a flood of books—texts, monographs, and edited volumes describing different aspects of block transforms, multirate filter banks, and wavelets. Some of these have been mathematically precise, designed for the rigorous theoretician, while others sought to interpret work in this arena for engineers and students.

The field is now mature, yet active. The theory is much better understood in the signal processing community, and applications of the multiresolution concept to situations in digital multimedia, communications, and others abound. In the first edition and in the early days of multirate filter banks a prime emphasis was on signal compaction and coding. Today, multiresolution decomposition and time-frequency concepts have opened up new vistas for further development and application. These ideas concerning orthogonal signal analysis and synthesis have led to applications in digital audio broadcasting, digital data hiding and watermarking, wireless and wireline communications, audio and video coding, and many others.

In this edition, we continue to treat block transforms, subband filter banks, and wavelets from a common unifying standpoint. We demonstrate the commonality among these signal analysis and synthesis techniques by showing how the block transform evolves gracefully into the more general multirate subband filter bank, and then by establishing the multiresolution decomposition features common to both the dyadic subband tree structure and the orthonormal wavelet transform. In order to achieve this unification, we have focused mainly on *orthonormal* decompositions and presented a unified and integrated treatment of multiresolution signal decomposition techniques using the property of orthonormality as the unifying theme. (A few exceptions, such as the oversampled Laplacian pyramid and biorthogonal filter banks are also presented because they provide an historical perspective and serve as foils to the critically sampled, orthonormal subband structures we emphasize.)

Our second focus in the first edition was the application of decomposition techniques to signal *compression* and coding. Accordingly, we describe *objective* performance criteria that measure this attribute and then compare the different techniques on this basis. We acknowledge that subjective evaluations of decomposition are important in applications such as image and video processing and coding, machine vision, and pattern recognition. Such aspects are treated adequately in the literature cited and are deemed beyond the scope of this book. A new focus in this edition is the time-frequency properties of signals and decomposition techniques. Accordingly, this text provides tables listing the coefficients of popular block transforms, subband and wavelet filters, and also their time-frequency features and compaction performance for both theoretical signal models and standard test images. In this respect, we have tried to make the book a reference text as well as a didactic monograph.

Our approach is to build from the fundamentals, taking simple representative cases first and then extending these to the next level of generalization. For example, we start with block transforms, extend these to lapped orthogonal transforms, and then show both to be special cases of subband filter structures. We have avoided the theorem-proof approach, preferring to give explanation and derivations emphasizing clarity of concept rather than strict rigor.

Chapter 2 on orthogonal transforms introduces block transforms from a least-squares expansion in orthogonal functions. Signal models and decorrelation and compaction performance measures are then used to evaluate and compare several proposed block and lapped transforms. The biorthogonal signal decomposition is mentioned.

Chapter 3 presents the theory of perfect reconstruction, orthonormal two-band and M-band filter banks with emphasis on the finite impulse response variety. A key contribution here is the time-domain representation of an arbitrary multirate filter bank, from which a variety of special cases emerge—paraunitary, biorthogonal, lattice, LOT, and modulated filter banks. The two-channel, dyadic tree structure then provides a multiresolution link with both the historical Laplacian pyramid and the orthonormal wavelets of Chapter 6. A new feature is the representation of the transmultiplexer as the synthesis/analysis dual of the analysis/synthesis multirate filter bank configuration.

Chapter 4 deals with specific filter banks and evaluates their objective performance. This chapter relates the theory of signal decomposition techniques presented in the text with the applications. It provides a unified performance evaluation of block transforms, subband decomposition, and wavelet filters from a signal processing and coding point of view. The topic of optimal filter banks presented in this chapter deals with solutions based on practical considerations in image coding. The chapter closes with the modeling and optimum design of quantized filter banks.

Chapter 5 on time-frequency (T-F) focuses on joint time-frequency properties of signals and the localization features of decomposition tools. There is a discussion of techniques for synthesizing signals and block transforms with desirable T-F properties and describes applications to compaction and interference excision in spread spectrum communications.

Chapter 6 presents the basic theory of the orthonormal and biorthogonal wavelet transforms and demonstrates their connection to the orthonormal dyadic subband tree of Chapter 3. Again, our interest is in the linkage to the multiresolution subband tree structure, rather than with specific applications of wavelet transforms.

Chapter 7 is a review of recent applications of these techniques to image coding, and to communications applications such as discrete multitone (DMT) modulation, and orthogonal spread spectrum user codes. This chapter links the riches of linear orthogonal transform theory to the popular and emerging transform applications. It is expected that this linkage might spark ideas for new applications that benefit from these signal processing tools in the future.

This book is intended for graduate students and R&D practitioners who have a working knowledge of linear system theory and Fourier analysis, some linear algebra, random signals and processes, and an introductory course in digital signal processing. A set of problems is included for instructional purposes.

For classroom presentation, an instructor may present the material in the text in three packets:

(1) Chapters 2 and 5 on block transforms and time-frequency methods
(2) Chapters 3 and 4 on theory and design of multirate filter banks
(3) Chapters 6 and 7 on wavelets and transform applications

As expected, a book of this kind would be impossible without the cooperation of colleagues in the field. The paper preprints, reports, and private communications they provided helped to improve significantly the quality and timeliness of the book. We acknowledge the generous help of N. Sezgin and A. Bircan for some figures. Dr. T. Russell Hsing of Bellcore was instrumental in introducing us to Academic Press. It has been a pleasure to work with Dr. Zvi Ruder during this project. Dr. Eric Viscito was very kind to review Chapter 3. The comments and suggestions of our former and current graduate students helped to improve the quality of this book. In particular, we enjoyed the stimulating discussions and interactions with H. Caglar, A. Benyassine, M. Tazebay, X. Lin, N. Uzun, K. Park, K. Kwak, and J.C. Horng. We thank them all. Lastly, we appreciate and thank

our families for their understanding, support, and extraordinary patience during the preparation of this book.

Ali N. Akansu
Richard A. Haddad
April 2000

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Chapter 1

Introduction

1.1 Introduction

In the first edition of this book, published in 1992, we stated our goals as threefold:
(1) To present orthonormal signal decomposition techniques—transforms, subbands, and wavelets—from a unified framework and point of view.
(2) To develop the interrelationships among decomposition methods in both time and frequency domains and to define common features.
(3) To evaluate and critique proposed decomposition strategies from a compression coding standpoint using measures appropriate to image processing.
The emphasis then was signal coding in an analysis/synthesis structure or codec. As the field matured and new insights were gained, we expanded our vistas to communications systems and other applications where objectives other than compression are vital — as for example, interference excision in CDMA spread spectrum systems. We can also represent certain communications systems such as TDMA, FDMA, and CDMA as synthesis/analysis structures, i.e., the conceptual dual of the compression codec. This duality enables one to view all these systems from one unified framework.

The Fourier transform and its extensions have historically been the prime vehicle for signal analysis and representation. Since the early 1970s, block transforms with real basis functions, particularly the discrete cosine transform (DCT), have been studied extensively for transform coding applications. The availability of simple fast transform algorithms and good signal coding performance made the DCT the standard signal decomposition technique, particularly for image and video. The international standard image-video coding algorithms, i.e., CCITT