Principles of Lasers

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Preface

This book is the result of more than ten years of research and teaching in the field of quantum electronics. The purpose of the book is to introduce the principles of lasers, starting from elementary notions of quantum mechanics and electromagnetism. Because it is an introductory book, an effort has been made to make it self-contained to minimize the need for reference to other works. For the same reason, the references have been limited (whenever possible) either to review papers or to papers of seminal importance.

The organization of the book is based on the fact that a laser can be thought of as consisting of three elements: (i) an active material, (ii) a pumping system, and (iii) a suitable resonator. Accordingly, after an introductory chapter, the next three chapters deal, respectively, with the interaction of radiation with matter, pumping processes, and the theory of passive optical resonators. The concepts introduced in this way are then used in Chapter 5 to develop a theory for the cw and transient behavior of lasers. The theory is developed within the lowest-order approximation, i.e., using the rate-equation approach. This treatment is, in fact, capable of describing most laser characteristics. Obviously, lasers based upon different types of active media have somewhat different characteristics. It is therefore natural that next, Chapter 6 should discuss the characteristic properties of each type of laser. At this point, the reader will have acquired a sufficient understanding of laser operation to go on to a study of the characteristic properties of the output

beam (monochromaticity, coherence, directionality, brightness). These properties are dealt with in Chapter 7, which leads to a discussion of the applications for which the laser is potentially suited (Chapter 8). Finally, in Chapter 9, which can be considered as a supplementary chapter, a more advanced treatment is given to the problem of the interaction of radiation and matter. The approach in Chapter 9 is based on the so-called semiclassical approximation. With this more advanced approach, a better physical insight into laser behavior can be obtained. In particular, this treatment is able to account for new physical phenomena (e.g., the production of π, 2π pulses, photon echo, etc.) which could not be described under the rate-equation approximation. A few appendixes, added for completeness, constitute the final part of the book.

Although every effort has been made to present the subject matter in a coherent way, the book must represent some sort of compromise. In some cases, for instance, only the most basic features of a problem are discussed, with very little detail given. Furthermore, as previously mentioned, the treatment is limited to the semiclassical approximation. It is, therefore, not possible to give a full discussion of important phenomena such as spontaneous emission and laser noise. These limitations are, however, often dictated by the need to give the book a wide appeal. In the author's opinion, in fact, the subject matter presented here constitutes the minimum knowledge required for a correct understanding of laser behavior.

O. Svelto

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1
Introductory Concepts

1.1 Spontaneous and Stimulated Emission, Absorption

Quantum electronics can be defined as that branch of electronics where phenomena of a quantum nature play a fundamental role. This book will deal with a particular aspect of quantum electronics, namely, the physical principles of lasers and their behavior. Before going into a detailed discussion of the subject, it seems appropriate to devote a little space to an explanation, in a very simple way, of the ideas behind the laser.

A laser exploits three fundamental phenomena which occur when an electromagnetic (e.m.) wave interacts with a material, namely, the processes of spontaneous and stimulated emission and the process of absorption.

1.1.1 Spontaneous Emission (Fig. 1.1a)

Let us consider two energy levels, 1 and 2, of some given material, their energies being E_1 and E_2 ($E_1 < E_2$). As far as the following discussion is concerned, the two levels could be any two out of the infinite set of levels possessed by the material. It is convenient, however, to take level 1 to be the ground level. Let us now assume that an atom (or molecule) of the material is initially in level 2. Since $E_2 > E_1$, the atom will tend to decay to level 1. The corresponding energy difference $(E_2 - E_1)$ must therefore be re-

leased by the atom. When this energy is delivered in the form of an e.m. wave, the process will be called spontaneous (or radiative) emission. The frequency ν of the radiated wave is then given by the expression (due to Planck)

$$\nu = (E_2 - E_1)/h \tag{1.1}$$

where h is Planck's constant. Spontaneous emission is therefore characterized by the emission of a photon of energy $h\nu = E_2 - E_1$, when the atom decays from level 2 to level 1 (Fig. 1.1a). Note that radiative emission is just one of the two possible ways for the atom to decay. The decay can also occur in a nonradiative way. In this case the energy difference $E_2 - E_1$ is delivered in some form other than e.m. radiation (e.g., it may go into kinetic energy of the surrounding molecules).

The probability of spontaneous emission can be characterized in the following way: Let us suppose that, at time t, there are N_2 atoms (per unit volume) in level 2. The rate of decay of these atoms due to spontaneous emission, i.e., $(dN_2/dt)_{sp}$, will obviously be proportional to N_2. We can therefore write

$$\left(\frac{dN_2}{dt}\right)_{sp} = -AN_2 \tag{1.2}$$

The coefficient A is called the spontaneous emission probability or the Einstein A coefficient (an expression for A was in fact first obtained by Einstein from thermodynamic considerations). The quantity $\tau_{sp} = 1/A$ is called the spontaneous emission lifetime. The numerical value of A (and τ_{sp}) depends on the particular transition involved.

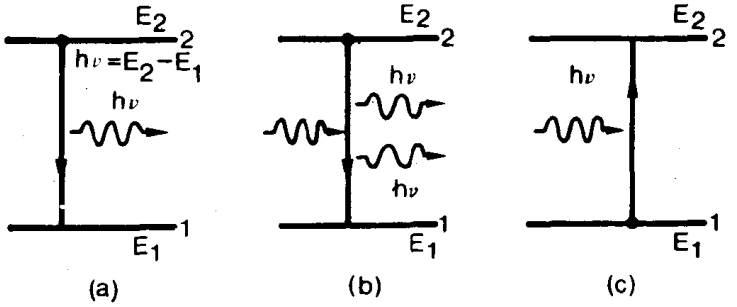

Fig. 1.1. Schematic illustration of the three processes: (a) spontaneous emission; (b) stimulated emission; (c) absorption.

1.1.2 Stimulated Emission (Fig. 1.1b)

Let us again suppose that the atom is found initially in level 2 and that an e.m. wave of frequency ν given by equation (1.1) (i.e., equal to that of the spontaneously emitted wave) is incident on the material. Since this wave has the same frequency as the atomic frequency, there is a finite probability that this wave will force the atom to undergo the transition $2 \rightarrow 1$. In this case the energy difference $E_2 - E_1$ is delivered in the form of an e.m. wave which adds to the incident one. This is the phenomenon of stimulated emission. There is, however, a fundamental distinction between the spontaneous and stimulated emission processes. In the case of spontaneous emission, the atom emits an e.m. wave which has no definite phase relation with that emitted by another atom. Furthermore, the wave can be emitted in any direction. In the case of stimulated emission, since the process is forced by the incident e.m. wave, the emission of any atom adds in phase to that of the incoming wave. This wave also determines the direction of the emitted wave.

In this case, too, we can characterize the process by means of the equation

$$\left(\frac{dN_2}{dt}\right)_{\text{st}} = -W_{21}N_2 \tag{1.3}$$

where $(dN_2/dt)_{\text{st}}$ is the rate at which transitions $2 \rightarrow 1$ occur as a result of stimulated emission and W_{21} is called the stimulated transition probability. Just as in the case of the A coefficient defined by (1.2), the coefficient W_{21} also has the dimension of $(\text{time})^{-1}$. Unlike A, however, W_{21} not only depends on the particular transition but also on the intensity of the incident e.m. wave. More precisely, for a plane e.m. wave, it will be shown that we can write

$$W_{21} = \sigma_{21}F \tag{1.4}$$

where F is the photon flux of the incident wave and σ_{21} is a quantity having the dimensions of area (it is called the stimulated-emission cross section) and depending only on the characteristics of the given transition.

1.1.3 *Absorption (Fig. 1.1c)*

Let us now assume that the atom is initially lying in level 1. If this is the ground level, the atom will remain in this level unless some external stimulus is applied to it. We shall assume, then, that an e.m. wave of frequency ν given again by (1.1) is incident on the materia! In this case there is a finite probability that the atom will be raised to level 2. The energy difference $E_2 - E_1$ required by the atom to undergo the transition is obtained from the energy of the incident e.m. wave. This is the absorption process.

In a similar fashion to (1.3), we can define an absorption rate W_{12} by means of the equation

$$\frac{dN_1}{dt} = -W_{12}N_1 \tag{1.5}$$

where N_1 is the number of atoms (per unit volume) which, at the given time, are lying in level 1. Furthermore, just as in (1.4), we can write

$$W_{12} = \sigma_{12}F \tag{1.6}$$

where σ_{12} is some characteristic area (the absorption cross section) which depends only on the particular transition.

In the preceding sections, the fundamental principles of the processes of spontaneous and stimulated emission as well as that of absorption have been described. In terms of photons, these processes can be described as follows (see Fig. 1.1): (i) In the spontaneous emission process, the atom decays from level 2 to 1 through the emission of a photon. (ii) In the stimulated process, the incident photon stimulates the $2 \rightarrow 1$ transition and we then have two photons (the stimulating plus the stimulated one). (iii) In the absorption process, the incident photon is simply absorbed to produce the $1 \rightarrow 2$ transition. Finally, it should be noted that $\sigma_{12} = \sigma_{21}$, as Einstein showed at the beginning of the century. This shows that the probabilities of stimulated emission and absorption are equal. From now on, therefore, we will write $\sigma_{12} = \sigma_{21} = \sigma$ and σ will be referred to as the transition cross section. The number of atoms per unit volume in some given level will be called the *population* of that level.

1.2 The Laser Idea

Consider two arbitrary energy levels 1 and 2 of a given material and let N_1 and N_2 be their respective populations. If a plane wave with an intensity corresponding to a photon flux F is traveling along the z direction in the material, the elemental change of this flux due to both the stimulated emission and absorption processes in the shaded region of Fig. 1.2, according to equations (1.3)–(1.6), is given by

$$dF = \sigma F(N_2 - N_1)\, dz \tag{1.7}$$

Equation (1.7) shows that the material behaves as an amplifier (i.e., $dF/dz > 0$) if $N_2 > N_1$, while it behaves as an absorber if $N_2 < N_1$. Now, it is known that, in the case of thermal equilibrium, the energy-level populations are described by Boltzmann statistics. So, if N_1^e and N_2^e are the thermal equilibrium populations of the two levels, we have

$$\frac{N_2^e}{N_1^e} = \exp\left[-\frac{(E_2 - E_1)}{kT} \right] \tag{1.8}$$

where k is Boltzmann's constant and T the absolute temperature of the material. We then see that, for the case of thermal equilibrium, we have $N_2 < N_1$. According to (1.7), the material then acts as an absorber at frequency ν, and this is what happens under ordinary conditions. If, however, a nonequilibrium condition is achieved for which $N_2 > N_1$, then the material will act as an amplifier. In this case we will say that there exists a *population inversion* in the material, by which we mean that the population difference $(N_2 - N_1 > 0)$ is opposite in sign to that which exists under ordinary conditions

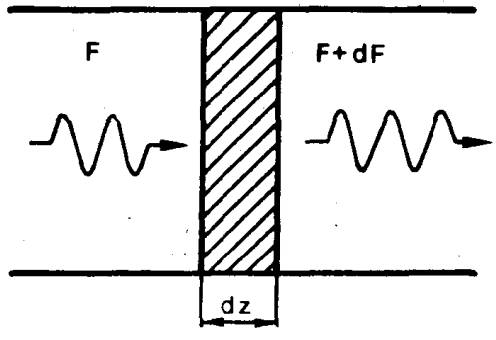

Fig. 1.2. Elemental change dF in the photon flux F for a plane e.m. wave in traveling a distance dz through the material.

($N_2^e - N_1^e < 0$). A material having a population inversion will be called an *active material.*

If the transition frequency $\nu = (E_2 - E_1)/h$ falls in the microwave region, this type of amplifier is called a *maser* amplifier. The word *maser* is an acronym for "*m*icrowave *a*mplification by *s*timulated *e*mission of *r*adiation." If the transition frequency ν falls in the optical region, the amplifier is called a *laser* amplifier. The word laser is again an acronym, obtained by substituting the letter l (*l*ight) for the letter m (*m*icrowave). The word laser is, however, commonly used not only for frequencies of visible light but for any frequency falling in the far- or near-infrared, in the ultraviolet, and even in the x-ray region. In these cases we will talk about infrared, ultraviolet, or x-ray lasers, respectively.

To make an oscillator from an amplifier, it is necessary to introduce a suitable positive feedback. In the microwave range this is done by placing the active material in a resonant cavity having a resonance at the frequency ν. In the case of a laser, the feedback is often obtained by placing the active material between two highly reflecting mirrors (e.g., plane–parallel mirrors, see Fig. 1.3). In this case, a plane e.m. wave traveling in a direction orthogonal to the mirrors will bounce back and forth between the two mirrors and be amplified on each passage through the active material. If one of the two mirrors is made partially transparent, a useful output beam can be extracted. It is important to realize that for both masers and lasers, a certain threshold condition must be fulfilled. In the laser case, for instance, the oscillation will start when the gain of the active material compensates the losses in the laser (e.g., the losses due to output coupling). According to (1.7), the gain per pass in the active material (i.e., the ratio between the output and input photon flux) is $\exp[\sigma(N_2 - N_1)\, l]$, where l is the length of the active material. If the only losses present in the cavity are those due to transmission

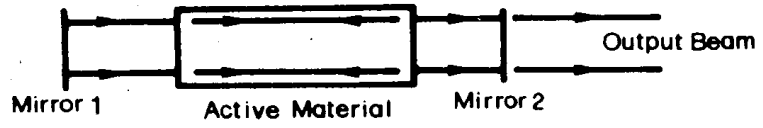

Fig. 1.3. Scheme of a laser.

losses, the threshold will be reached when

$$R_1 R_2 \exp[2\sigma(N_2 - N_1)\, l] = 1 \tag{1.9}$$

where R_1 and R_2 are the power reflectivities of the two mirrors. Equation (1.9) shows that the threshold is reached when the population inversion $N_2 - N_1$ reaches a critical value known as the *critical inversion.* Once the critical inversion is reached, oscillation will build up from the spontaneous emission: The photons which are spontaneously emitted along the cavity axis will, in fact, initiate the amplification process. This is the basis of a laser oscillator, or laser as it is more simply known.

The most outstanding properties of a laser beam are its high degree of (i) directionality, (ii) monochromaticity, (iii) coherence, and (iv) brightness.

(i) Directionality. This property is simply a consequence of the fact that the active material is placed in a resonant cavity such as the plane parallel one of Fig. 1.3. Only an e.m. wave which propagates along the cavity direction (or in a direction very near to it) can be sustained in the cavity.

(ii) Monochromaticity. Without entering into too many details, we can say that this property is due to the following two circumstances: (a) Only an e.m. wave of frequency ν given by (1.1) can be amplified. (b) Since the two-mirror arrangement forms a resonant cavity, oscillation can occur only at the resonant frequencies of this cavity.

(iii) Coherence. To first order one can, for any e.m. wave, introduce two independent concepts of coherence, namely, spatial and temporal coherence:

(a) Spatial Coherence. Let us consider two points P_1 and P_2 which, at time t_0, lie in the same wavefront of some given e.m. wave and let $E_1(t)$ and $E_2(t)$ be the corresponding electric fields at those points. By definition, the phase difference between the two fields at time t_0 is zero. Now, if this difference remains zero at any time t, we will say that there is a perfect coherence between the two points. If this occurs for any two points of the e.m. wavefront we will say that the wave has *perfect spatial coherence.* In practice, for any point P_1 the point P_2 must be within some finite area S around P_1 if we are to have a good phase correlation. In this case we will say that the wave

has a *partial spatial coherence* and for any point P we can introduce a suitably defined coherence area $S(P)$.

(b) Temporal Coherence. We now consider the electric field of the e.m. wave at a given point P, at times t and $t + \tau$. If, for a given time delay τ, the phase difference between the two fields remains the same for any time t, we will say that there is temporal coherence over a time τ. If this occurs for any value of τ, the e.m. wave will be said to have *perfect time coherence.* If this occurs for a time delay τ such that $0 < \tau < \tau_0$, the wave will be said to have *partial temporal coherence*, with a coherence time equal to τ_0. An example of an e.m. wave with a time coherence equal to τ_0 is presented in Fig. 1.4. This shows a sinusoidal electric field undergoing phase jumps at time intervals equal to τ_0. We see that the concept of temporal coherence is directly connected with that of monochromaticity. We will in fact show, although this is already obvious from the example shown in Fig. 1.4, that an e.m. wave with a coherence time τ_0 has a bandwidth $\Delta\nu \simeq 1/\tau_0$.

Before ending this preliminary discussion of coherence, it is worth noting that the two concepts of temporal and spatial coherence are indeed independent of each other. In fact, examples can be given of a wave having perfect spatial coherence but only a limited temporal coherence (or *vice versa*). If in fact the wave shown in Fig. 1.4 were to represent the electric fields at points P_1 and P_2 mentioned earlier, the spatial coherence between these points would be complete while the wave would have a limited temporal coherence.

(iv) Brightness. We define the brightness of a given source of e.m. waves as the power emitted per unit surface area per unit solid angle. To be more precise, let dS be the elemental surface area at a point O of the source (Fig. 1.5). The power dP emitted by dS into

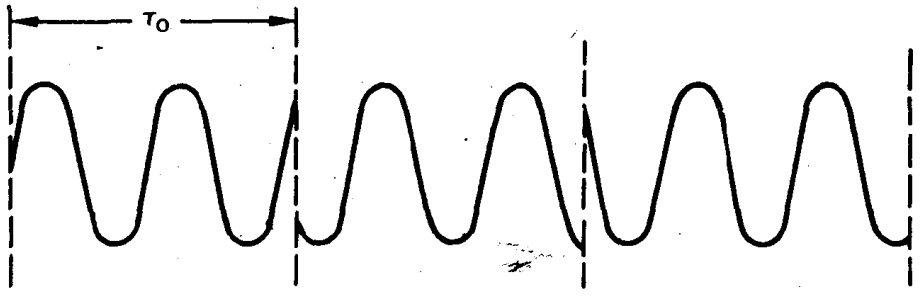

Fig. 1.4. Example of an e.m. wave with a coherence time of approximately τ_0.