NOISE IN ELECTRONIC DEVICES

Papers based on material presented at a conference held by the Electronics Group of The Institute of Physics at the Services Electronics Research Laboratory, Baldock, Hertfordshire on 2-3 October, 1959

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First published 1961

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Catalogue No. 667/4

Printed in Great Britain by F. J. Milner & Sons Ltd., Brentford, Middlesex

Foreword

On 2 and 3 October, 1959, the Electronics Group of The Institute of Physics held a conference on " Noise in Electronic Devices " at the Services Electronics Research Laboratory, Baldock, Hertfordshire.

The papers collected here are based on material presented at the conference.

The contents vary from a concise yet complete account of noise phenomena, to some new considerations of noise in valves, and cover various particular electronic devices in between.

It is hoped that the text will be of use to students and research workers, both as an introductory text and possibly as a work of reference.

C. A. Hogarth

Hon. Secretary, Electronics Group

London
July, 1960.

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The Physical Basis of Noise

By F. J. Hyde, M.Sc., A.M.I.E.E.
University College of North Wales, Bangor

1. INTRODUCTION

"Noise" in relation to electron devices and associated equipments incorporating such devices is a generic term. In practice it will have an acoustic connexion only if there is an audible output. A more generally descriptive term, with which it is intended to be synonymous, is "fluctuations". In the present discussion only spontaneous fluctuations will be considered; i.e. those which are governed by the laws of statistical thermodynamics. Man-made fluctuations, due for example to faulty contacts or radiated interference, are excluded.

The quantities which fluctuate spontaneously on a temporal scale are the numbers of active particles and their momenta or distribution in energy states. These particles may be electrically charged as in thermionic devices or transistors or electrically neutral as in maser materials. In many, although not all cases, a characteristic of the noise process is that the elementary events, which contribute to the noise, occur at random. This means that any particular event in no way influences any future event and the probability that an event occurs in a given time interval is determined only by the length of the interval.

Before dealing in detail with the various mechanisms which contribute to fluctuations in the output current and voltage of electronic devices it is useful to consider briefly a few mathematical facts concerning the specification of noise.

2. MATHEMATICAL BASIS OF NOISE(1-3)

2.1. SINGLE NOISE SOURCE

Consider a quantity which fluctuates with time and has an instantaneous value x as illustrated in Fig. 1. Let its mean value be $\bar{x}$. The value of the instantaneous fluctuation is then

$$\Delta x = x - \bar{x} \tag{2.1}$$

It is evident that the mean value of the fluctuation $\overline{\Delta x}$, will be zero, but its mean square value $\overline{\Delta x^2}$, will not. In fact

$$\overline{\Delta x^2} = \overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 \tag{2.2}$$

so that the mean square value of the fluctuation is equal to the difference between the values of the mean square and the square of the mean of the fluctuating variable. $\overline{\Delta x^2}$ is sometimes called the variance of the fluctuation and $\sigma = (\overline{\Delta x^2})^{\frac{1}{2}}$ its standard deviation.

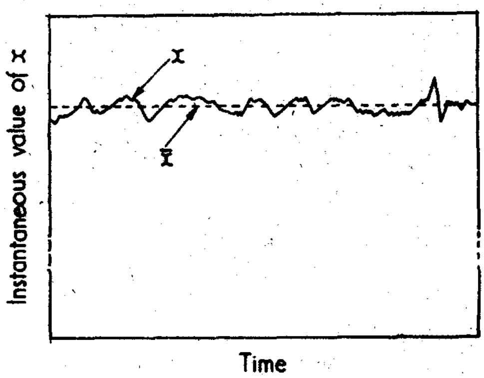

FIG. 1. Temporal fluctuation of a variable x. (Note the magnified scale for x)

2.2. COMBINED EFFECT OF TWO NOISE SOURCES

Consider an output fluctuation Δz which arises from the combined effects of a fluctuation Δx in x and of Δy in y. Since the individual fluctuations are of a microscopic nature it is usual to find that they combine linearly as shown by equation (2.3).

$$\Delta z = a\,\Delta x + b\,\Delta y \tag{2.3}$$

As before $\overline{\Delta z} = 0$ since $\overline{\Delta x}, \overline{\Delta y} = 0$. For the mean square value of the output fluctuation we have

$$\overline{\Delta z^2} = \overline{(a\,\Delta x + b\,\Delta y)^2} = a^2\,\overline{\Delta x^2} + b^2\,\overline{\Delta y^2} + 2ab\,\overline{\Delta x\,\Delta y} \tag{2.4}$$

Before $\overline{\Delta z^2}$ can be calculated from this equation it is necessary to evaluate $\overline{\Delta x\,\Delta y}$. In doing this any functional dependence between x and y must be taken into account. We consider two cases:

(i) x and y completely independent (or uncorrelated); i.e. a given value of x in no way affects the instantaneous value of y. It follows that $\overline{\Delta x\,\Delta y} = 0$ since $\overline{\Delta x}, \overline{\Delta y} = 0$. The averaging can be visualized as being done at a fixed value of x (say) over all values of y or vice versa. Equation (2.4) may now be written

$$\overline{\Delta z^2} = a^2\,\overline{\Delta x^2} + b^2\,\overline{\Delta y^2} \tag{2.5}$$

This is an important result. It shows that completely independent fluctuations add on a mean square basis.

(ii) x and y are to some extent interdependent (correlated); i.e. a given value of x has some influence on the value of y. In these circumstances $\overline{\Delta x\,\Delta y}$ is not equal to zero and must be evaluated for use in the second part of equation (2.4). Alternatively, in accordance with the first part of equation (2.4), the total instantaneous fluctuation may be determined, and its mean square then evaluated.

2.3. THE INSTANTANEOUS VALUE OF A FLUCTUATING QUANTITY

A characteristic of noise is that it is not possible to specify precisely what its instantaneous value will be. Specification is possible only on a statistical basis. Consider a fluctuating quantity x whose instantaneous value is measured repeatedly a great number of times N. Let the range of values of x be divided into a large number of contiguous small intervals dx. Then a "probability density function," $p(x)$, is defined as follows

$$p(x) = \lim_{\substack{dx \to 0 \\ N \to \infty}} \frac{(\text{Number of values of } x \text{ in the interval } dx \text{ at } x)/dx}{\text{Total number of values of } x} \tag{2.6}$$

The probability that a particular measured value lies in an interval of infinitesimal length dx at x is $p(x)\,dx$; i.e. this is the shaded area shown in Fig. 2. Clearly $\int_{-\infty}^{+\infty} p(x)\,dx = 1$. From a knowledge of the probability

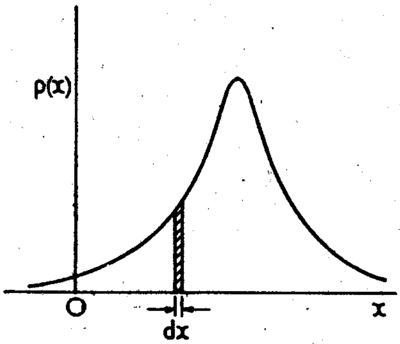

FIG. 2. The probability density function of a fluctuating variable x

density function $p(x)$, it is possible to calculate the mean value $\bar{x}$ of the fluctuating quantity and its mean square value $\overline{x^2}$ as follows

$$\bar{x} = \int_{-\infty}^{+\infty} x\,p(x)\,dx \tag{2.7}$$

$$\overline{x^2} = \int_{-\infty}^{+\infty} x^2\,p(x)\,dx \tag{2.8}$$

In studying noise we are generally more concerned with the value of the fluctuation Δx itself rather than the value of the variable x. In terms of the probability density, the mean square value of the fluctuation is evaluated as

$$\overline{\Delta x^2} = \int_{-\infty}^{+\infty} (x - \bar{x})^2\,p(x)dx \tag{2.9}$$

One of the most important distributions in noise theory is the normal or Gaussian distribution

$$p(x) = \frac{1}{\sigma\sqrt{(2\pi)}} \exp\{-(x - \bar{x})^2/2\sigma^2\} \tag{2.10}$$

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where $\sigma = (\overline{\Delta x^2})^{\frac{1}{2}}$. The Gaussian distribution may be considered as that pertaining to systems in which the numbers of independent elementary events, whose effects when summed give the instantaneous value of the variable x, are large. A characteristic of the Gaussian distribution is that no matter how large a value we may choose for x there is a finite probability of it being exceeded in an observation. The probability falls very rapidly at large values of x, however, so that for practical purposes noise peaks exceeding a few times the standard deviation can be ignored. When the numbers of elementary events are not large then the Gaussian distribution is not appropriate. For this or other reasons it may be more appropriate to consider either the binomial distribution (as in the case of division of current between anode and screen of a pentode) or the Poisson distribution (as in the case of thermionic emission or radioactive decay). Both of these distributions, which apply to systems in which the variable can have only integral values, reduce to the Gaussian distribution when the number of elementary events per unit time interval is large.

2.4. THE SPECTRAL DENSITY OF NOISE

Consider for example a time-dependent voltage $v(t)$ between two terminals of an electrical system. When this is a signal voltage of finite extent in time it is possible to represent it by a Fourier integral,

$$v(t) = \int_{-\infty}^{+\infty} F(\omega)\, e^{j\omega t}\, df \qquad (2.11)$$

i.e. to transform its representation from the time domain to the frequency domain.

$F(\omega)\, df$ is interpreted as the complex representation of an infinitesimal component of $v(t)$ containing frequencies in the range df at f. Appropriate values of $F(\omega)$ may be calculated by means of the transformation

$$F(\omega) = \int_{-\infty}^{+8} v(t)\, e^{-j\omega t}\, dt \qquad (2.12)$$

Furthermore, if $Y(\omega)$ is a generalized immittance function associated with the system, for example a transfer admittance, then the transfer current due to $v(t)$ may be determined from

$$i(t) = \int_{-\infty}^{+\infty} Y(\omega)\, F(\omega)\, e^{j\omega t}\, df \qquad (2.13)$$

The above procedure is not possible when $v(t)$ is a *noise* voltage having infinite extent in time because then the integral for $F(\omega)$ does not converge. An alternative procedure is adopted; this is based on the distribution in frequency of the mean square value of the fluctuation. The concept of a *noise power spectral density* $S_v(f)$ is introduced such that

$$\overline{v^2(t)} = \int_0^{\infty} S_v(f)\, df \qquad (2.14)$$

Here the suffix v denotes a "voltage" spectrum. Formally, $S_v(f)$ is defined as follows

$$S_v(f) = \lim_{T\to\infty} \frac{2\,|F(\omega)|^2}{T} \tag{2.15}$$

where the " truncated " Fourier transform $F(\omega)$ is now defined for $v(t)$ between $t = 0$ and $t = T$ as

$$F(\omega) = \int_0^T v(t)\, e^{-j\omega t}\, dt \tag{2.16}$$

In Fig. 3 a typical form of frequency-dependence for $S_v(f)$ is shown. The shaded area represents the amount of the mean square noise voltage of equation (2.14) contained in a narrow band of frequencies df at f. The total area under the curve is equivalent to $\overline{v^2(t)}$.

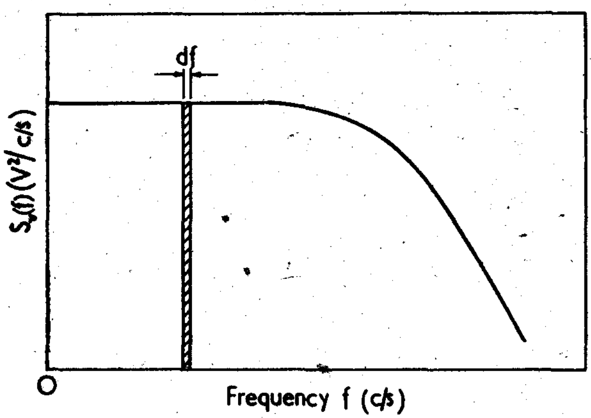

FIG. 3. A typical distribution of noise spectral density

The response of electrical systems to noise may be calculated using a method analogous to that for small signals, which was illustrated by equation (2.13). For example, if $Y(\omega)$ is the admittance between two terminals then

$$S_i(f) = \int_0^\infty S_v(f)\, |\, Y(\omega)\,|^2\, df \tag{2.17}$$

where the suffix i denotes a " current " spectrum.

As was made evident in section 2.2 it is necessary to be able to deal not only with the mean values of the " self-products " of fluctuating quantities, as in equation (2.14), but also with " cross-products ". Consider a fluctuating voltage $v(t)$ which is the sum of two separate but not independent fluctuations $v_1(t)$ and $v_2(t)$; namely

$$v(t) = v_1(t) + v_2(t) \tag{2.18}$$

and let F_v, F_{v_1} and F_{v_2} be the Fourier transforms of the corresponding truncated processes over time range T as defined by equation (2.16), then†

†When a complex number is written in polar form (say) as $|F(v)|e^{j\theta}$ and then multiplied by its complex conjugate $|(F(v)|e^{j\theta})^* = |F(v)|e^{-j\theta}$, the product is $|F(v)|^2$.

$$S_v(f) = \frac{2 \lim}{T\to\infty} \frac{|F_v|^2}{T} = \frac{2 \lim}{T\to\infty} \frac{(F_{v_1}+F_{v_2})(F_{v_1}^*+F_{v_2}^*)}{T}$$

$$= S_{v_1}(f) + S_{v_1v_2}(f) + S_{v_2v_1}(f) + S_{v_2}(f) \quad (2.19)$$

where the cross-spectra are defined by

$$S_{v_1v_2}(f) = \frac{2 \lim}{T\to\infty} \frac{F_{v_1}^* F_{v_2}}{T} \quad (2.20)$$

$$S_{v_2v_1}(f) = \frac{2 \lim}{T\to\infty} \frac{F_{v_2}^* F_{v_1}}{T} \quad (2.21)$$

Here quantities marked with an asterisk are the complex conjugates of the corresponding unmarked quantities. We note that

$$S_{v_1v_2}(f) = S^*_{v_2v_1}(f) \quad (2.22)$$

so that equation (2.19) may be rewritten as

$$S_v(f) = S_{v_1}(f) + S_{v_2}(f) + 2\,Re\,\{S_{v_1v_2}(f)\} \quad (2.23)$$

where *Re* signifies " the real part of".
We consider now two powerful methods for determining $S(f)$, which are based on well-established theorems.

2.4.1 *Carson's theorem*

Consider a fluctuating variable $i(t)$ whose magnitude is the sum of the effects of a large number of identical events occurring at a random rate λ per second. Then Carson's theorem is

$$S_i(f) = 2\,\lambda\,|F(\omega)|^2 \quad (2.24)$$

where $F(\omega)$ is the Fourier transform of each of the individual events. An example of the use of the theorem is in the derivation of the classical shot-noise formula for a temperature-limited electron stream emitted by a thermionic cathode (see section 3.2). The " event " is the emission of an electron by the cathode; its " effect " is the current pulse induced in the external circuit while it is in transit to the anode; $F(\omega)$ is the Fourier transform of this pulse.

When the individual events are not identical, equation (2.24) is modified as follows

$$S_i(f) = 2\,\lambda\,\overline{|F(\omega)|^2} \quad (2.25)$$

where $\overline{|F(\omega)|^2}$ is $\int_0^\infty |F_\tau(\omega)|^2 g(\tau)\,d\tau$; $g(\tau)\,d\tau$ is the number of events occurring per second with a characteristic time between τ and $\tau + d\tau$ and for which the Fourier transform is $F_\tau(\omega)$. An example of the use of this modified formula is in the derivation of the noise of a gas discharge.

2.4.2. *The Wiener-Khintchine theorem*

The autocorrelation function (sometimes called the autocovariance) of a time-dependent quantity $x(t)$ is defined as

$$R(s) = \overline{x(t)\, x(t+s)} = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\, x(t+s)\, dt \qquad (2.26)$$

The Wiener-Khintchine theorem specifies $S_x(f)$ in terms of the autocorrelation function as follows

$$S_x(f) = 4\int_0^\infty R(s) \cos \omega s.\, ds \qquad (2.27)$$

so that if $R(s)$ is known for the physical process in question, $S_x(f)$ can be readily determined. An example of the use of this theorem is given in section 3.3 in which the generation-recombination noise of a semiconductor filament is calculated.

3. PHYSICAL SOURCES OF NOISE

It was stated in the introduction that noise is a result of spontaneous fluctuations in the number and energy of the active particles in an electronic device. The use of the word particles presupposes a classical model; that is one for which $hf/kT \ll 1$ or $f \ll 2\times10^{10}\, T\, s^{-1}$. Here $h = 6{\cdot}62\times10^{-34}$ Js is Planck's constant, $k = 1{\cdot}38\times10^{-23}$ J/deg is Boltzmann's constant and T is the absolute temperature. For most of the conditions experienced in electron devices and systems a classical model is adequate. In devices such as masers, however, which operate near the absolute zero of temperature a classical model is not appropriate.

3.1. JOHNSON NOISE [4]

This is alternatively known as " thermal noise ", which is not a good term since most noise may be said to be of " thermal " origin, or Nyquist noise[5] after the man who first derived the expression for its spectral density on a theoretical basis. Johnson showed that a resistor having a.c. resistance R at frequency f, which was in thermal equilibrium with its surroundings, developed at its terminals a noise voltage whose spectral density was given by the formula

$$S_v = 4kTR \qquad (3.1)$$

An equivalent expression for the spectral density of the short-circuit noise current between the terminals is

$$S_i = 4kTG \qquad (3.2)$$

where G is the a.c. conductance at frequency f. These expressions also apply[6] to any complex, passive two-terminal network which is all at the same temperature T.

In such a case R is the resistive component of the impedance and G the conductive component of the admittance.

Equations (3.1) and (3.2) relate the electrical manifestation of the thermal energy of the current carriers (electrons, holes, ions) to the atoms of the material as a whole, through the dissipative mechanism represented by R and G. These equations are of a general nature and apply to any dissipative medium which is in thermal equilibrium. As may be expected therefore, the proof of these formulae rests on arguments of thermodynamics and statistical mechanics. Proofs have, however, been given based on particular models of conduction-electron distribution and scattering. In a resistor for example,[7] the noise is considered to arise from the summation of the effects of the short current pulses of the free electrons as they travel between collisions, each pulse having a flat (or white) noise spectral density.

The information contained in equations (3.1) and (3.2) may be expressed in another way. We may consider a resistor as a source of noise power. the maximum available noise power P from the resistor will be absorbed by a matched load and its spectral density S_p will be

$$S_p = kT \tag{3.3}$$

The Johnson noise power available from a resistance is therefore independent of the resistance and proportional to its absolute temperature.

Classical Johnson noise is well-established both theoretically and experimentally. It is not surprising therefore that the value of its spectral density is often used as a reference standard when specifying the spectral densities of other types of noise. In this connexion the concepts of noise temperature ratio, noise figure and equivalent input noise temperature have been successively introduced. These are defined in sections 5.3 and 5.4.

Recently a physical standard of Johnson noise has been set up[8] for the frequency range 0 to 1000 Mc/s. The resistor, of approximately 60Ω is of film type and its temperature may be varied using a furnace. The available noise power output is a linear function of temperature up to 1300°C.

3.1.1 *Quantum-mechanical considerations*

Equation (3.1) represents a transcription to circuit form of black body radiation, which originates from accelerated particles in excited states. This may be shown directly[9] by considering a resistor whose resistance is matched to the radiation resistance of an aerial which is immersed in black-body radiation at temperature T. On the classical model, using the Rayleigh-Jeans approximation for the energy density in the radiation field, equation (3.1) results. If instead the Planck law of quantized radiation is used for the energy density, as is more generally appropriate, then the following expression arises for $S_v(f)$

$$S_v(f) = 4hfR \frac{1}{e^{hf/kT} - 1} \tag{3.4}$$

This of course reduces to equation (3.1) for $hf/kT \ll 1$.

A further quantum-mechanical correction to Johnson's formula must be made to take account of the fact that even in the ground state at the absolute zero of temperature there is a residual zero-point energy $E_o = hf/2$, with which spontaneous emission of radiation is associated. The complete expression for the spectral density of the fundamental "thermal" noise is then

$$S_v(f) = 4hfR\left(\frac{1}{2} + \frac{1}{e^{hf/kT} - 1}\right) \tag{3.5}$$

A rigorous derivation of this formula has been carried out by Ekstein and Rostoker[10].

It should be stressed that the Johnson noise formulae apply to thermal equilibrium conditions. Active devices do not operate in such conditions. Usually there will be current flowing as in valves and transistors.

Nonetheless, if the applied electrical fields are such that the velocity distribution of the current carriers is unchanged apart from the linear addition of " drift " to " thermal " velocities, then Johnson noise as described above will still arise. It will generally be augmented by noise which is associated with applied fields.

An interesting example of the significance of Johnson noise is in the field of parametric amplification (see paper by Butcher), in which a " pumped " variation of semiconductor diode depletion-layer capacitance is used to promote signal gain. Pure reactive elements do not themselves behave as Johnson noise sources, since they are non-dissipative. Practical diodes always possess a finite resistance effectively in series with the depletion-layer capacitance, however, and this does contribute Johnson noise. Such noise may contribute significantly to the total noise of the amplifier. Johnson noise is also of importance in transistors (see paper by Hibberd). There are always dissipative connexions between the terminals and the active regions. In particular there is an ohmic resistance $r_{bb'}$ in series with the base lead: in high frequency applications the Johnson noise of $r_{bb'}$ can be very significant.

3.2. SHOT NOISE[11]

Schottky[12] carried out the first theoretical study of the fluctuations which arise in the anode current of a temperature-limited thermionic diode, on the assumption that the emission of electrons from the cathode is a random process. He showed that the resulting fluctuations in the instantaneous number of electrons in transit produced a noise current in the anode circuit, whose spectral density is given by

$$S_i = 2e\overline{I} \tag{3.6}$$

Here e is the electronic charge and I is the anode current. The analysis was restricted to frequencies considerably less than the reciprocal of the electron transit time. Equation (3.6) is well-established experimentally.

It may be pointed out here that there is an additional source of noise in a temperature-limited electron beam. This arises because, in addition to the emission of electrons being a random process, the velocities will also fluctuate with a Maxwellian distribution about the mean value. For frequencies low compared with the reciprocal of the transit time it was shown by Rack[13] that the spectral density S_u of the velocity fluctuations in the beam is given by

$$S_u = \frac{ekT_c}{mI}(4-\pi) \tag{3.7}$$

Where T_c is the cathode temperature and m the electron rest mass.

The derivation of $S_i(f)$ for a temperature-limited thermionic diode having planar geometry affords a good example of the use of Carson's theorem[14]. It is assumed that the emission of electrons is a random process. To simplify the problem it is also assumed that all electrons leave the cathode with zero velocity. For temperature-limited operation there

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