Fuzzy Sets Fuzzy Logic Fuzzy Methods

with Applications

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Preface

The concept of fuzzy sets and its applications has become a battlefield of conflicting opinions, since its steady development during the sixties. On the one hand, in the engineering camp, we observe a euphoric expectancy that this concept will solve with ease many problems which up to now had resisted any useful mathematical modelling or solution. On the other hand, in the mathematics camp we notice a rejecting scepticism with respect to the scientific validity. With such a background we hesitated for a long time to publicize a detailed presentation of this new field of – pure as well as applied – mathematics. Repeated and increasingly frequent inquiries of potential users have finally initiated this book.

The first German version, issued in 1989, was revised and expanded in 1993 in its fourth edition, and it is this which forms the basis for the present English version. Since the early eighties the manuscript was used again and again in courses for different audiences, mainly for engineers, computer scientists, and mathematicians, and underwent several rewritings, before and after each of its published versions. Of course the present version has also profited from this development. So, compared with the last German version, some subsections have been rewritten completely, others are very much revised, all are, at least, checked to correct mistakes and errors of different type and origin. Nevertheless, we are convinced that some errors may have been overlooked or created newly. Hence, we thank the kind reader in advance for any hint to those errors. We owe especial thanks to Mr. Harald Merk for a long list of errata with respect to the fourth German edition, and to Dr. Heinz Voigt for providing us with the special style-file realising some essential details of the actual layout of this book.

The present book introduces the basic notions of fuzzy sets in a mathematically firm manner. But it also treats them in relation to their essential applications. And the principles of such applications are explained too. Many references are included to open the way to the relevant literature for potential users of fuzzy methods, e.g. engineers, scientists, operational researchers, computer scientists and mathematicians, but – we hope – also for people from management science and the humanities. In some places – indicated by smaller type or by a star with the subsection heading – we have added more mathematical remarks or refer to other types of notation or another terminology. These parts can be omitted for a first reading. We did not intend to write a text aimed at mathematicians. Instead, the standard mathematical courses in engineering or management science at the graduate level should suffice to understand the text.

Our notation is quite standard with some tendency toward set theoretically oriented formulations. But this is caused by the very notion of fuzzy set. Often we have to distinguish between crisp and fuzzy sets. Then upper case italics like $A, B, \ldots, M, N, \ldots$ denote fuzzy sets, and upper case calligraphic letters like $\mathcal{A}, \mathcal{B}, \ldots, \mathcal{M}, \ldots, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$ are used for crisp sets.

Formulas are numbered consecutively inside each chapter: formula $(m.n)$ is the n-th formula of chapter m. References to the literature are given by the name(s) of the author(s) or editor(s) and the year of publication.

Finally we thank both our publishing houses, Akademie-Verlag Berlin as part of the VCH Weinheim group, as well as J. Wiley & Sons Ltd., for all their kind understanding and help, bringing the present book into existence. And, last but not least, we are grateful to our wives who, once again, had some hard months with their husbands completely absorbed by some of their book projects.

The Authors

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1 Introduction

1.1 Why Fuzzy Sets?

Traditional mathematical modelling as commonly used in the sciences engineering and economics, refers to standard, classical mathematics. Connected with this mathematical modelling there are usually some rationalizations necessary to transform problems from their intuitive basis into a mathematised form. One aspect of such rationalizations is the transformation of notions which are, to some extent, only vaguely fixed, into clear, crisply determined ones. Thus for example, with chemical processes one may have to specify normal values as well as dangerous ones for temperature or pressure, and for manufacturing metal tools with slide-rest lathes one needs normal speeds for the turning-chisel and standard turning frequencies, and also criteria by which to judge if a turning-chisel has to be considered as worn out. Another aspect of such rationalization is often the assumption that precise data or data with precise error bounds are available at all.

There are many diverse applications for which it is impossible to get relevant data. It may not be possible to measure essential parameters of a process such as the temperature inside a molten glass or the homogeneity of a mixture inside some tank or vessel. It may be that a measurement scale does not exist at all, such as the evaluation of offensive smells, or medical diagnoses by touching/fingering, or evaluating the taste of foods.

In all such situations in which traditional mathematical modelling needs exact notions or precise data this means, mathematically, that one needs suitable sets of objects: numerical data, temperatures, frequencies, states of processes, etc., sets in the traditional mathematical sense of that word.

These traditional sets of present day mathematics are named here *crisp sets* to distinguish them from the fuzzy sets which are the central topic of this book. These crisp sets are uniquely characterised by and as the totality of their elements, their numbers.

The members of such a crisp set may be determined by some enumeration or by some characteristic property. Thus one can represent some set $\mathcal{M}$ with elements $a_1, a_2, \ldots, a_{10}$ as

$$\mathcal{M} = \{a_1, a_2, \ldots, a_{10}\} \tag{1.1}$$

or also as

$$\mathcal{M} = \{a_i \mid 1 \leq i \leq 10\}. \tag{1.2}$$

But, if additionally all these elements $a_1, \ldots, a_{10}$ of $\mathcal{M}$ at the same time are objects/members of some more comprehensive class $\mathcal{X}$ of objects then another representation of $\mathcal{M}$ may be preferable: the representation of $\mathcal{M}$ by its characteristic function. This is that one 0-1-valued function $m_{\mathcal{M}} : \mathcal{X} \to \{0, 1\}$ with

$$\forall x \in \mathcal{X} : \quad m_{\mathcal{M}}(x) = \begin{cases} 1, & \text{if } x \in \mathcal{M} \\ 0 & \text{otherwise.} \end{cases} \tag{1.3}$$

The value 1 of this characteristic function thus *marks* the elements of $\mathcal{M}$ among all the objects of $\mathcal{X}$. (Actually sometimes the value 1 is substituted by some other *mark* but that does not change the main idea behind definition (1.3).)

Thus e.g. in a data base listing the employees of some agency the set $\mathcal{M}$ of all employees with an academic education is realised by adjoining in this list $\mathcal{X}$ of names with each member of $\mathcal{M}$, i.e. each employee with academic education, a mark which characterises just this property, that means which characterises exactly the elements of $\mathcal{M}$.

Mathematically it is completely irrelevant if some crisp set $\mathcal{M}$ is determined as (1.1) or as (1.2) or via its characteristic function (1.3). All these representations can be transformed into one another without any difficulties. For fuzzy sets, yet, we shall prefer their representation via – generalised – characteristic functions.

The main concept behind fuzzy sets is most easily grasped if one has in mind that in everyday life, and thus also in a lot of applicational situations for mathematics, one does not directly meet sets with a crisp "borderline", but quite often it seems that there exists something like a gradual transition between membership and non-membership. And this gradual transition cannot be formalised with crisp sets – on the contrary, quite often just these feelings of gradual transition have to be eliminated by the rationalization we mentioned earlier. But often this eliminates a crucial point of the whole problem. Thus the characteristic function shown in Fig. 1.1 is a description of the set of all real numbers > 18. Here the jump at the point $x = 18$ is completely natural. In the same sense this jump is natural if Fig. 1.1 is read as a characterisation of the set $\mathcal{M}_1$ of all ages of people that have reached their majority. This

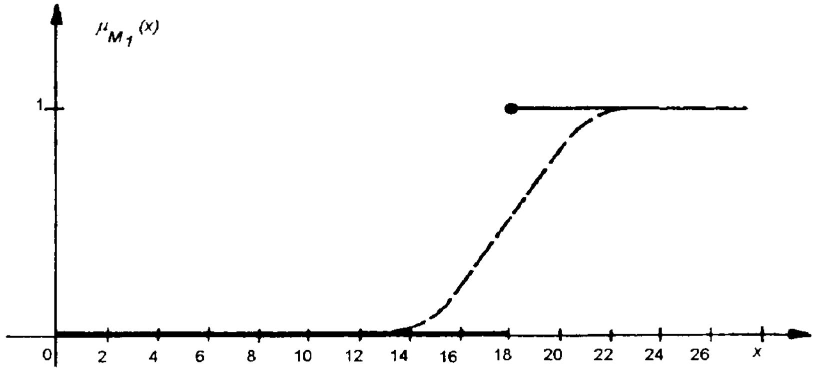

Figure 1.1: Fuzzy vs. crisp set

naturalness of the jump is lost if one intends to understand $\mathcal{M}_1$ as the set of all ages at which a human being is (in the biological sense) full grown, or also if one looks at $\mathcal{M}_1$ as the set of all temperature values of a sufficiently heated room.

The problem with these last two interpretations of course does not consist in the fact that this jump in Fig. 1.1 is not placed in the right point at $x = 18$. Rather, the problem is the jump itself. For, looking at the interpretation of being full-grown, this jump indicates that a person after age 18 is (completely) full-grown, and that at each earlier point in time this is not (entirely) the case. The same type of interpretation applies to the case of a sufficiently heated room. Intuitively, instead, one has the feeling that in reality here properties do not change totally "in the moment" but that there really is some *gentle move* from being not full-grown to being full-grown, or from being not sufficiently heated to being so. (Here it is irrelevant that there are additional variations from one person to another; also in looking at just one fixed person these gentle moves appear.)

It is not hard to find a lot of further examples which realise the same effect that intuitively there does not exist some jump point of a suitable scale which divides membership from non-membership for some suitable set, or which marks the (exact) transition from one property to its opposite one. To have examples consider e.g. (over some suitable scale of real numbers) the properties:

– that some cars need little petrol;
– that some computers have sufficient storage capacity;
– that two cars on a lane have sufficient distance between them;

- that the lighting of a desk is too bright;
- that some radiation levels are unhealthy;
- that some roads are slippery.

Of course, as usual each one of these properties defines a set, the set of all objects which have this property.

In each one of these cases intuition tells us that there is a gradual transition from (true, complete) membership in these sets to (true, complete) non-membership in these sets. But, this intuition is just not grasped by the usual "crisp" set. Fuzzy sets, instead, are designed to realise just this specific intuition. Hence they have to realise some gradual transition from membership to non-membership.

Mathematically it is not too difficult to realise this idea. One simply has to adjoin the description of a set by its characteristic function (1.3) to the membership (coding) degrees 0, 1 with additional membership degrees "between" 0 and 1. It has become a kind of standard usage to take all the real numbers between 0 and 1, i.e. the real unit interval

$$\boldsymbol{I} = [0,1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$$

as the set of all these (additional) "generalised" membership degrees. But, other choices are possible – and usually only some concrete application can give any hint whether this choice or another one is the more suitable one.

In Fig. 1.1 there is additionally shown (lineated) a second graph which may be understood as indicating such generalised membership degrees and thus realising such a gradual transition from non-membership 0 to membership 1.

In the same spirit as we connected the idea of fuzzy sets with vaguely determined notions (or sets) one can connect fuzzy sets with fuzzy, i.e. vaguely or imprecisely given data. One way to do this is to look at imprecisely given data as vaguely given sets of measured values. Sometimes one does not even need any specific interpretation: vague data may appear in a natural way as fuzzy sets – e.g. in visual data analysis as grey tone pictures, among others e.g. as two-dimensional pictures of three-dimensional objects. Similarly, planar "fuzzy" data may result from hardness measurements, but also in many other cases (cf. Section 2.2 and Chapter 6).

Finally it has to be mentioned as an empirical fact that very often quite difficult chemical, economical, technological etc. processes can be described relatively briefly, but for practical reasons sufficiently precise in qualitative terms – without any quantitative analysis. To give some examples one may think about: instructions for use, recipes, decision criteria etc. And it seems that there exists a kind of complementarity between the complexity of processes or systems, on the one hand, and the possibilities of their description and numerical treatment solely by traditional (crisp) methods, on the other hand. Thus, with the ever growing complexity of the systems which one intends to use as a model or to control, an ever growing tendency toward an integration of fuzzy notions and methods into the description of such systems has to be expected.

1.2 Development of Theory and Applications

Experiences and reflections of the type we explained in Section 1.1 have been the background ideas which stimulated the American systems engineer L. A. ZADEH since the mid-1960s to initiate and propagate the transition from traditional mathematical modelling in engineering to a new, much more qualitative, "rough" modelling which uses fuzzy sets and fuzzy methods; cf. ZADEH (1965, 1965a, 1969, 1971a, 1973). With these ideas and approaches a door was opened to introduce vague notions in a mathematically sound way, types of notions which had already been discussed in more philosophically-oriented research; cf. GOGUEN (1968/69). A firm mathematical foundation for fuzzy sets can be given within the framework of category theory (cf. GOGUEN (1974) and RODABAUGH/KLEMENT/HÖHLE (1992)) or as a combination of (usual) set theory and many-valued logic; cf. e.g. GILES (1976, 1979), GOTTWALD (1979a, 1981, 1984b, 1993), NOVAK (1986).

These ideas make a lot of non-traditional applications accessible. The best indication is the broad range of successful applications which have been realised since the end of the 1980's. Early, but still interesting surveys of the then actual state of the art have been given e.g. in GAINES (1976), ZIMMERMANN (1979) and GOTTWALD (1981). The book DUBOIS/PRADE (1980) is a standard reference which collects most of the essential results up to about 1978. In fact, many modern introductory, as well as review, texts are available, among them: ZIMMERMANN (1985, 1987), NOVAK (1989), PEDRYCZ (1989).

In the first decade, application-oriented papers in the fuzzy field had been mainly concerned with theoretical studies toward possible applications and sometimes with real applications on the laboratory scale. The most fruitful idea initially was the concept of fuzzy control initiated by the pioneering approach of MAMDANI, cf. Section 4.2 and e.g. MAMDANI/ASSILIAN (1975). The starting point here was just a qualitative "algorithmic" description of control behaviour. The first application realised the automatic control of a steam engine/boiler combination in the laboratory. But soon the control of a cement kiln was realized and sold on the market; cf. HOLMBLAD/ØSTERGAARD (1982). Since then a lot of true applications have been realised, even to partially controlling the behaviour of consumer goods; cf. Section 4.5. But also further topics have successfully been treated, as e.g. problems of classifications, pattern recognition, database management, modelling of chemical processes, operations research etc. Some typical results are presented e.g. in KANDEL (1982), CARLSSON (1984), SCHMUCKER (1984), BOCKLISCH (1987), SMITHSON (1987), D'AMBROSIO (1989), MIYAMOTO (1990), ROMMELFANGER (1994).

Often a more specific problem inside such approaches is the numerical treatment of fuzzy numerical data. But this field of fuzzy arithmetic has its own mathematical interest; cf. KAUFMAN/GUPTA (1985). Another topic more mathematically than applicationally-oriented is the field of equations and system of equations which involve fuzzy relations; cf. PEDRYCZ (1989) DINOLA/SESSA/PEDRYCZ/SANCHEZ (1989).

The highly innovative papers by ZADEH (1973, 1975, 1978, 1978a) opened new applicational areas and initiated the extension of fuzzy methods and ideas toward knowledge representation and artificial intelligence. Also in these fields the main aim is toward an almost direct representation and inferential treatment of vague as well as qualitative information, and thus toward some aspects of natural language modelling. Combined with these ideas one has an interpretation of fuzzy sets as "elastic constraints" for the values of suitable variables and of their membership degrees as indicating a degree of possibility that some possible value of a variable is its actual value. (Here one has to be careful not to read these possibility degrees in probabilistic terms; cf. Sections 3.3 and 5.3.)

These ideas intend to open ways for almost direct storage of and inference with natural language information as well as to quite flexible and user-friendly organised man-machine dialogue systems. The greatest interest of course is in a full integration of these ideas into methodologies of expert system design. The whole field is most often referred to as a *fuzzy logic* or *approximate reasoning*. And it is the central topic of a lot of actual research work; cf. among many other sources: ZEMANKOVA-LEECH/KANDEL (1984), DUBOIS/PRADE (1985), GOODMAN/NGUYEN (1985), O'HIGGINS HALL/KANDEL (1986), PRADE/NEGOITA (1986), DE BESSONET (1991), KRUSE/SCHWECKE/HEINSOHN (1991), SOMBÉ (1991). And ZADEH (1987) is a collection of all the pioneering papers of this highly influential author.

The actual trends in fuzzy sets applications and in the use of all these fuzzy methods center around fuzzy control and the use of fuzzy information in knowledge bases and expert systems. Fuzzy control is almost a highly standard application now, its main problems usually arise out of the intended application. However, for fuzzy logic and approximate reasoning a lot of theoretical problems are still open.

A huge number of research papers on fuzzy topics have appeared in conference proceedings and contributed volumes. The international journal *Fuzzy Sets and Systems* is the oldest journal specially devoted to the fuzzy field. It also is the official publication of the IFSA, the *International Fuzzy Sys-*

tems Association. The main results are often also presented in the *International Journal of Approximate Reasoning*, the *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, the *IEEE Transactions on Fuzzy Systems* as well as in further journals not mainly devoted to fuzzy topics. The recent literature in the fuzzy field is regularly listed in the *Fuzzy Sets and Systems* journal.