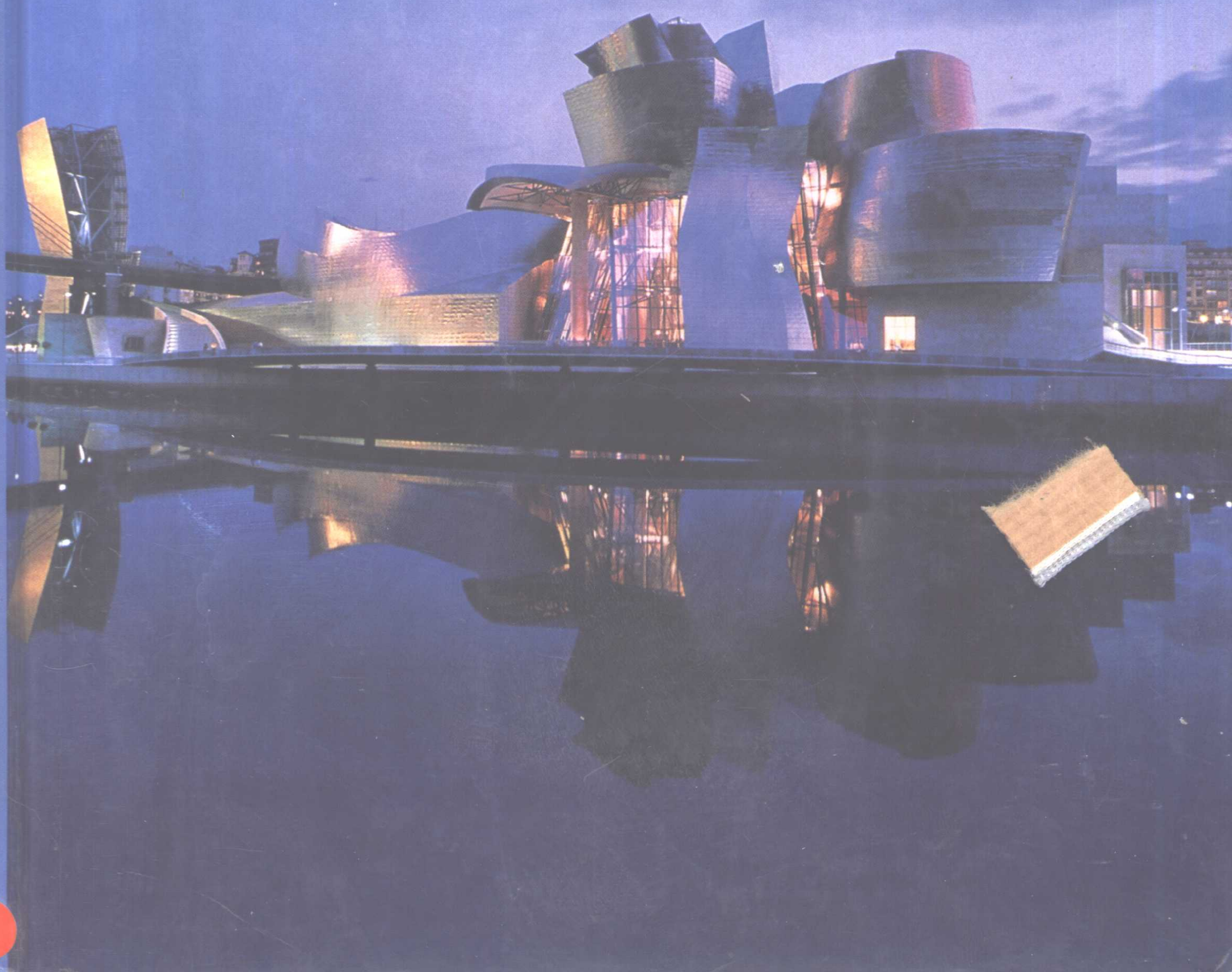
SECOND EDITION

CALCULUS

CONCEPTS AND CONTEXTS

JAMES STEWART



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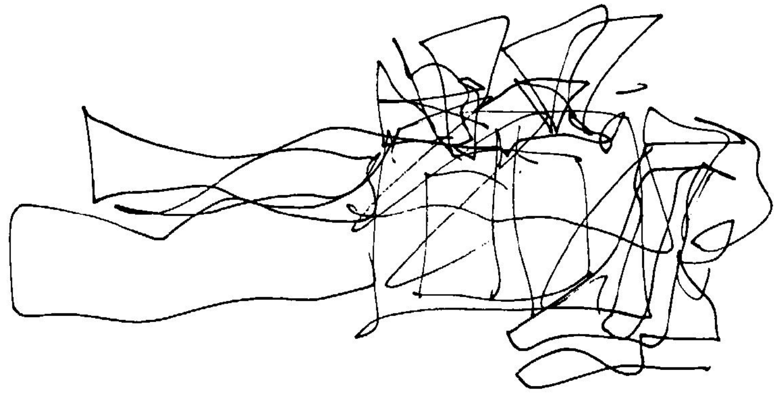
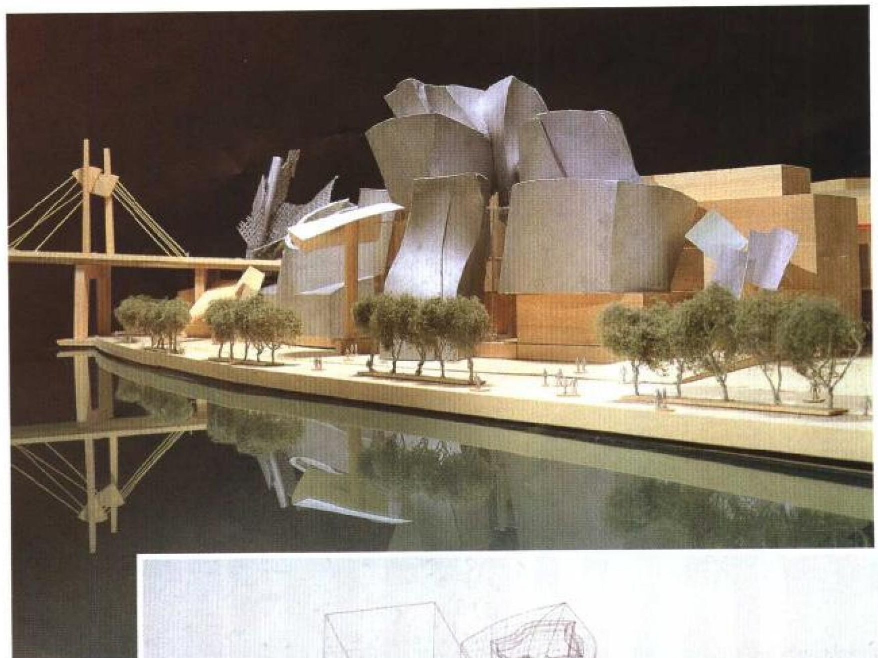
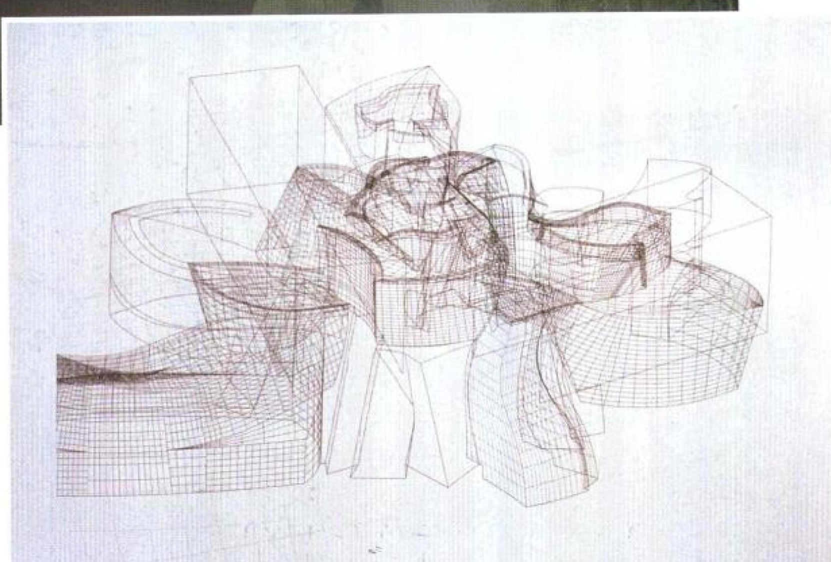


Calculus and the Architecture of Curves

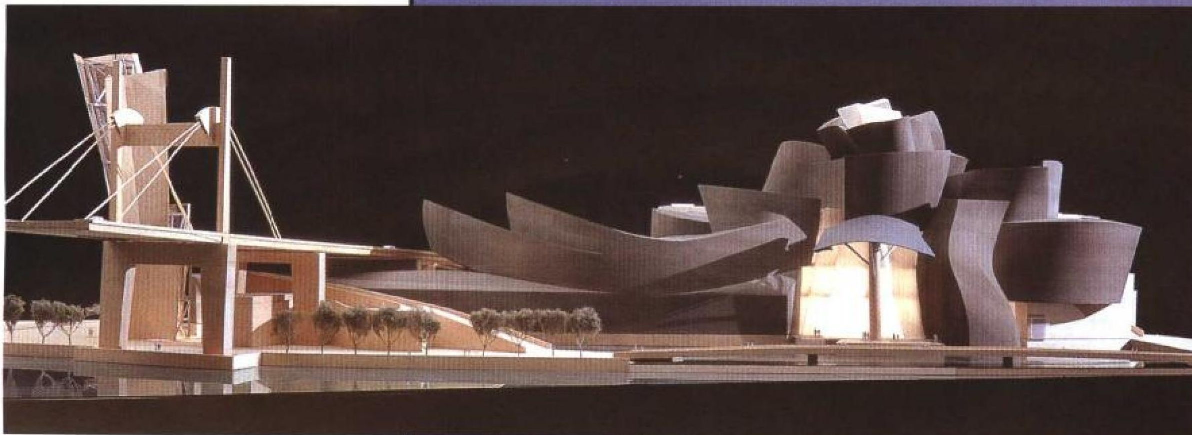


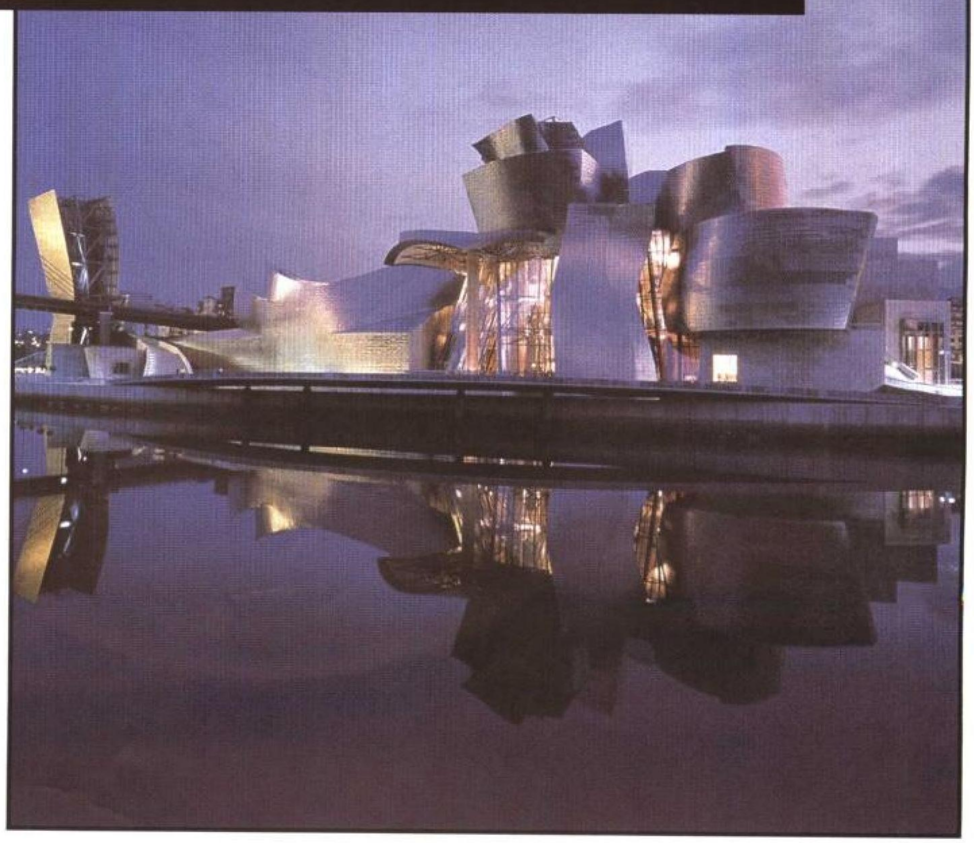


The cover photograph shows the Guggenheim Museum in Bilbao, Spain, designed and built 1991–1997 by Frank Gehry and Associates. With its implied motion and its cluster of titanium-clad components, this is surely the most arresting and original building of our time.

The highly complex structures that Frank Gehry designs would be impossible to build without the computer. The CATIA software that his architects and engineers use to produce the computer models is based on principles of calculus—fitting curves by matching tangent lines, making sure the curvature isn't too large, and controlling parametric surfaces. "Consequently," says Gehry, "we have a lot of freedom. I can play with shapes."

The process starts with Gehry's initial sketches, which are translated into a succession of physical models. (More than 200 different physical models were constructed during the design of the Bilbao museum, first with basic wooden blocks and then evolving into more sculptural forms.) Then an engineer uses a digitizer to record the coordinates of a series of points on a physical model. The digitized points are fed into a





computer and the CATIA software is used to link these points with smooth curves. (It joins curves so that their tangent lines coincide; you can use the same idea to design the shapes of letters in the Laboratory Project on page 236 of this book.) The architect has considerable freedom in creating these curves, guided by displays of the curve, its derivative, and its curvature. Then the curves are connected to each other by a parametric surface, and again the architect can do so in many possible ways with the guidance of displays of the geometric characteristics of the surface.

The CATIA model is then used to produce another physical model, which, in turn, suggests modifications and leads to additional computer and physical models.

The CATIA program was developed in France by Dassault Systèmes, originally for designing airplanes, and was subsequently employed in the automotive industry. Frank Gehry, because of his complex sculptural shapes, is the first to use it in architecture. It helps him answer his question, "How wiggly can you get and still make a building?"

To Connie Jirovsky and Gary Ostedt

Preface

When the first edition of this book appeared four years ago, a heated debate about calculus reform was taking place. Such issues as the use of technology, the relevance of rigor, and the role of discovery versus that of drill were causing deep splits in mathematics departments. Since then the rhetoric has calmed down somewhat as reformers and traditionalists have realized that they have a common goal: to enable students to understand and appreciate calculus.

The first edition was intended to be a synthesis of reform and traditional approaches to calculus instruction. In this second edition I continue to follow that path by emphasizing conceptual understanding through visual, numerical, and algebraic approaches.

The principal way in which this book differs from my more traditional calculus textbooks is that it is more streamlined. For instance, there is no complete chapter on techniques of integration; I don't prove as many theorems (see the discussion on rigor on page xi); and the material on transcendental functions and on parametric equations is interwoven throughout the book instead of being treated in separate chapters. Instructors who prefer fuller coverage of traditional calculus topics should look at my books *Calculus, Fourth Edition* and *Calculus: Early Transcendentals, Fourth Edition.*

Changes in the Second Edition

- The data in examples and exercises have been updated to be more timely.
- Several new examples have been added. For instance, I added the new Example 1 in Section 5.4 (page 381) because students have a tough time grasping the idea of a function defined by an integral with a variable limit of integration. I think it helps to look at Examples 1 and 2 before considering the Fundamental Theorem of Calculus.
- Extra steps have been provided in some of the existing examples.
- More than 25% of the exercises in each chapter are new.
- Three new projects have been added. The one on page 198 asks students to design a roller coaster so the track is smooth at transition points. The project on page 472, the idea for which I thank Larry Riddle, is actually a contest in which the winning curve has the smallest arc length (within a certain class of curves).
- A CD called *Tools for Enriching Calculus* (TEC) is included with every copy of the second edition. See the description on page xi.
- Chapter 1 has been reorganized. Instead of the full section on modeling in the first edition, I have moved some of this material into Section 1.2 and split the old 1.2 into two sections. The vast majority of users liked the coverage of parametric curves in Chapter 1, but for the convenience of those who prefer to defer parametric equations I have moved this material to the last section of Chapter 1.
- I have added a new (optional) section (5.7) called Additional Techniques of Integration. The idea is not to provide encyclopedic coverage, but rather to give a *brief* treatment of the simplest trigonometric integrals (enough to deal with the simplest cases of trigonometric substitution) as well as simple cases of partial fractions.

- I have rewritten Section 9.2 to give more prominence to the geometric description of vectors.
- As before, sigma notation is introduced briefly in Sections 5.1 and 5.2. In this edition, fuller coverage is provided in the new Appendix F, for those who need a more thorough review.

Features

Conceptual Exercises The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first couple of exercises in Sections 2.2. 2.4, 2.5, 5.3, 8.2, 11.2, and 11.3.) Similarly, review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables (see Exercises 1–3 in Section 2.7, Exercises 31–38 in Section 2.8, Exercises 1–2 in Section 10.2, Exercises 27, 30, and 31 in Section 10.3, Exercises 9–14 in Section 11.1, Exercises 3–4 in Section 11.7, Exercises 13–14 in Section 13.2, and Exercises 1, 2, 11, and 23 in Section 13.3). Another type of exercise uses verbal description to test conceptual understanding (see Exercise 8 in Section 2.4; Exercise 48 in Section 2.8; Exercises 5, 9, and 10 in Section 2.10; and Exercise 53 in Section 5.10). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 30, 33, and 34 in Section 2.5 and Exercise 2 in Section 7.5).

Pages 108, 128, 139, 377, 580, 765, 776
Pages 155, 169–170
Pages 716, 724, 757–758
Pages 818, 934, 943–944
Pages 129, 170
Pages 179, 437
Pages 140, 548

Real-World Data My assistants and I have spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. See, for instance, Figures 1, 11, and 12 in Section 1.1 (seismograms from the Northridge earthquake), Figure 5 in Section 5.3 (San Francisco power consumption), Exercise 12 in Section 5.1 (velocity of the space shuttle *Endeavour*), Example 5 in Section 5.9 (data traffic on Internet links), Example 3 in Section 9.6 (wave heights), Exercises 1–2 in Section 11.1 (wind-chill index, heat index), Exercises 1–2 in Section 11.6 (Hurricane Donna contour map), and Example 4 in Section 12.1 (Colorado snowfall).

Pages 11, 15
Pages 376, 356
Pages 423, 686
Pages 756–757
Pages 808, 845

Projects One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project after Section 7.3 asks whether a ball thrown upward takes longer to reach its maximum height or to fall back to its original height. (The answer might surprise you.) *Laboratory Projects* involve technology; the project following Section 3.5 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or cover optional topics (hyperbolic functions) or encourage discovery through pattern recognition (see the project following Section 5.8).

Page 530
Page 236
Page 415

Rigor I include fewer proofs than in my more traditional books, but I think it is still worthwhile to expose students to the idea of proof and to make a clear distinction between a proof and a plausibility argument. The important thing, I think, is to show how to deduce something that seems less obvious from something that seems more obvious. A good example is the use of the Mean Value Theorem to prove the Evaluation Theorem (Part 2 of the Fundamental Theorem of Calculus). I have chosen, on the other hand, not to prove the convergence tests but rather to argue intuitively that they are true.

Technology The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. I assume that the student has access to either a graphing calculator or a computer algebra system. The icon indicates an exercise that definitely requires the use of such technology, but that is not to say that a graphing device can't be used on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-89/92) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

Tools for Enriching™ Calculus
TEC

The CD-ROM called TEC included with every copy of this book is a companion to the text and is intended to enrich and complement its contents. Developed by Harvey Keynes at the University of Minnesota, TEC uses a discovery and exploratory approach. In sections of the book where technology is particularly appropriate, marginal icons direct students to TEC modules that provide a laboratory environment in which they can explore the topic in different ways and at different levels. Instructors can choose to become involved at several different levels, ranging from simply encouraging students to use the modules for independent exploration, to assigning specific exercises from those included with each module, or to creating additional exercises, labs, and projects that make use of the modules.

TEC also includes *homework hints* for representative exercises (usually odd-numbered) in every section of the text, indicated by printing the exercise number in red. These hints are usually presented in the form of questions and try to imitate an effective teaching assistant by functioning as a silent tutor. They are constructed so as not to reveal any more of the actual solution than is minimally necessary to make further progress.

Problem Solving Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles at the end of Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Focus on Problem Solving,* which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from David Hilbert: "A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts." When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

Page 88

Pages 185, 261, 340, 442, 502, 560, 643, 703, 745, 836, 914, 989

Content

A Preview of Calculus The book begins with an overview of the subject and includes a list of questions to motivate the study of calculus.

Chapter 1
Functions and Models

From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view. Parametric curves are introduced in the first chapter, partly so that curves can be drawn easily, with technology, whenever needed throughout the text. This early placement also enables inverse functions to be graphed in the first chapter, tangents to parametric curves to be treated in Section 3.5, and graphing such curves to be covered in Section 4.4.

Page 75

Pages 230, 296

Chapter 2
Limits and Derivatives

The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. (The precise ε, δ definition of a limit is provided in Appendix D for those who wish to cover it.) It is important not to rush through Sections 2.7–2.10, which deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts. Section 2.10 foreshadows, in an intuitive way and without differentiation formulas, the material on shapes of curves that is studied in greater depth in Chapter 4.

Pages 150–180

Page 175

Chapter 3
Differentiation Rules

All the basic functions are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings. Optional topics (hyperbolic functions, an early introduction to Taylor polynomials) are explored in Discovery and Laboratory Projects.

Chapter 4
Applications of Differentiation

The basic facts concerning extreme values and shapes of curves are derived using the Mean Value Theorem as the starting point. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.

Page 279

Chapter 5
Integrals

The area problem and the distance problem serve to motivate the definite integral. I have decided to make the definition of an integral easier to understand by using subintervals of equal width. Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables. There is no separate chapter on techniques of integration, but substitution and parts are covered here and other methods are treated briefly. Partial fractions are given full treatment in Appendix G. The use of computer algebra systems is discussed in Section 5.8.

Pages 411–413

Chapter 6
Applications of Integration

General methods, not formulas, are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm. Some instructors like to cover polar coordinates (Appendix H) here. Others prefer to defer this topic to when it is needed in third semester (with Section 9.7 or just before Section 12.4).

Chapter 7
Differential Equations

Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler's method are studied before separable equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. Predator-prey models are used to illustrate systems of differential equations.

Chapter 8
Infinite Sequences and Series

Tests for the convergence of series are considered briefly, with intuitive rather than formal justifications. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

Chapter 9
Vectors and the Geometry of Space

The dot product and cross product of vectors are given geometric definitions, motivated by work and torque, before the algebraic expressions are deduced. To facilitate the discussion of surfaces, functions of two variables and their graphs are introduced here.

Chapter 10
Vector Functions

The calculus of vector functions is used to prove Kepler's First Law of planetary motion, with the proofs of the other laws left as a project. In keeping with the introduction of parametric curves in Chapter 1, parametric surfaces are introduced as soon as possible, namely, in this chapter. I think an early familiarity with such surfaces is desirable, especially with the capability of computers to produce their graphs. Then tangent planes and areas of parametric surfaces can be discussed in Sections 11.4 and 12.6.

Chapter 11
Partial Derivatives

Page 766

Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. Directional derivatives are estimated from contour maps of temperature, pressure, and snowfall.

Chapter 12
Multiple Integrals

Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, areas of parametric surfaces, volumes of hyperspheres, and the volume of intersection of three cylinders.

Chapter 13
Vector Fields

Page 985

Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem are emphasized.

Ancillaries

Calculus: Concepts and Contexts, Second Edition is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The table on page xiv lists ancillaries available for instructors and students.

▲ INSTRUCTOR ▲

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Online resources include chapter-specific hypercontents and quizzes.

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JAMES STEWART

To the Student

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences —not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix J. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon ⊞ indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. (Section 1.4 discusses the use of these graphing devices and some of the pitfalls that you may encounter.) But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-89/92) are required. You will also encounter the symbol ⊘, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

The icon JTC indicates a reference to the CD-ROM *Journey Through*™ *Calculus*. The symbols in the margin refer you to the location in *Journey* where a concept is introduced through an interactive exploration or animation.

The CD-ROM *Tools for Enriching*™ *Calculus*, which is included with this textbook, is referred to by means of the symbol **TEC**. It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful. TEC also provides *Homework Hints* for representative exercises that are indicated by printing the exercise number in red: **23**. These homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

TRIGONOMETRIC FORMS

63. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

64. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

65. $\int \tan^2 u\,du = \tan u - u + C$

66. $\int \cot^2 u\,du = -\cot u - u + C$

67. $\int \sin^3 u\,du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$

68. $\int \cos^3 u\,du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$

69. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

70. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

71. $\int \sec^3 u\,du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

72. $\int \csc^3 u\,du = -\frac{1}{2}\csc u\cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

73. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$

75. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1}u - \int \tan^{n-2}u\,du$

76. $\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1}u - \int \cot^{n-2}u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\tan u\sec^{n-2}u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du$

78. $\int \csc^n u\,du = \frac{-1}{n-1}\cot u\csc^{n-2}u + \frac{n-2}{n-1}\int \csc^{n-2}u\,du$

79. $\int \sin au\sin bu\,du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80. $\int \cos au\cos bu\,du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81. $\int \sin au\cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u\sin u\,du = \sin u - u\cos u + C$

83. $\int u\cos u\,du = \cos u + u\sin u + C$

84. $\int u^n\sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

85. $\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$

86. $\int \sin^n u\cos^m u\,du = -\frac{\sin^{n-1}u\cos^{m+1}u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}u\cos^m u\,du$

$= \frac{\sin^{n+1}u\cos^{m-1}u}{n+m} + \frac{m-1}{n+m}\int \sin^n u\cos^{m-2}u\,du$

INVERSE TRIGONOMETRIC FORMS

87. $\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$

88. $\int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + C$

89. $\int \tan^{-1}u\,du = u\tan^{-1}u - \frac{1}{2}\ln(1+u^2) + C$

90. $\int u\sin^{-1}u\,du = \frac{2u^2-1}{4}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{4} + C$

91. $\int u\cos^{-1}u\,du = \frac{2u^2-1}{4}\cos^{-1}u - \frac{u\sqrt{1-u^2}}{4} + C$

92. $\int u\tan^{-1}u\,du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$

93. $\int u^n\sin^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\sin^{-1}u - \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

94. $\int u^n\cos^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1}u + \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

95. $\int u^n\tan^{-1}u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1}u - \int \frac{u^{n+1}\,du}{1+u^2}\right], \quad n \neq -1$

TABLE OF INTEGRALS

EXPONENTIAL AND LOGARITHMIC FORMS

96. $\displaystyle\int ue^{au}\,du = \frac{1}{a^2}(au - 1)e^{au} + C$

97. $\displaystyle\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\displaystyle\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2 + b^2}(a\sin bu - b\cos bu) + C$

99. $\displaystyle\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2 + b^2}(a\cos bu + b\sin bu) + C$

100. $\displaystyle\int \ln u\,du = u\ln u - u + C$

101. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\displaystyle\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

HYPERBOLIC FORMS

103. $\displaystyle\int \sinh u\,du = \cosh u + C$

104. $\displaystyle\int \cosh u\,du = \sinh u + C$

105. $\displaystyle\int \tanh u\,du = \ln\cosh u + C$

106. $\displaystyle\int \coth u\,du = \ln|\sinh u| + C$

107. $\displaystyle\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$

108. $\displaystyle\int \operatorname{csch} u\,du = \ln|\tanh \tfrac{1}{2}u| + C$

109. $\displaystyle\int \operatorname{sech}^2 u\,du = \tanh u + C$

110. $\displaystyle\int \operatorname{csch}^2 u\,du = -\coth u + C$

111. $\displaystyle\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

112. $\displaystyle\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

FORMS INVOLVING $\sqrt{2au - u^2}$, $a > 0$

113. $\displaystyle\int \sqrt{2au - u^2}\,du = \frac{u - a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

114. $\displaystyle\int u\sqrt{2au - u^2}\,du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

115. $\displaystyle\int \frac{\sqrt{2au - u^2}}{u}\,du = \sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

116. $\displaystyle\int \frac{\sqrt{2au - u^2}}{u^2}\,du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C$

117. $\displaystyle\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$

118. $\displaystyle\int \frac{u\,du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a - u}{a}\right) + C$

119. $\displaystyle\int \frac{u^2\,du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2}\sqrt{2au - u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a - u}{a}\right) + C$

120. $\displaystyle\int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$

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