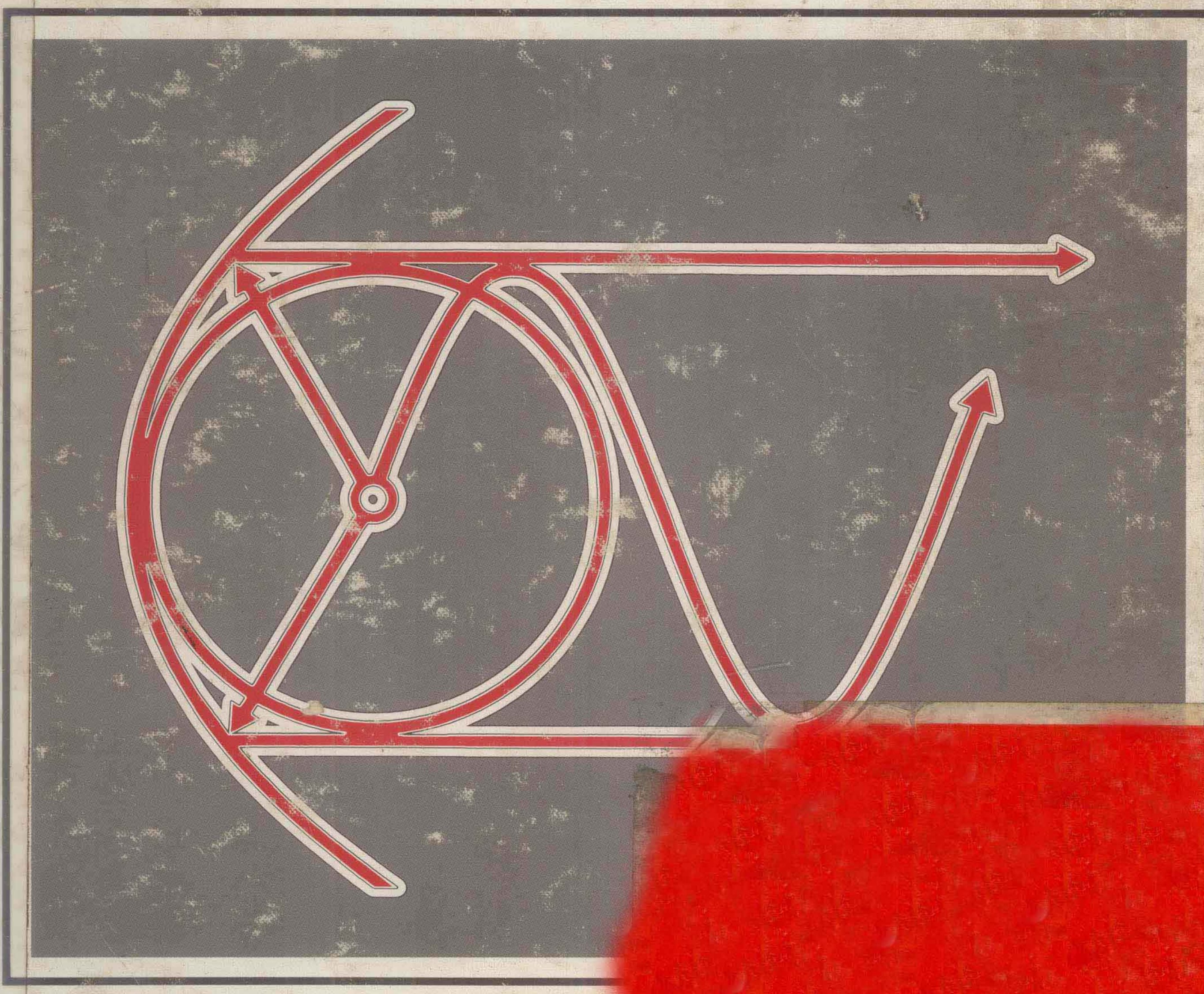
Fundamental Algebra and Trigonometry

Keedy · Bittinger



Fundamental Algebra and Trigonometry

Second Edition

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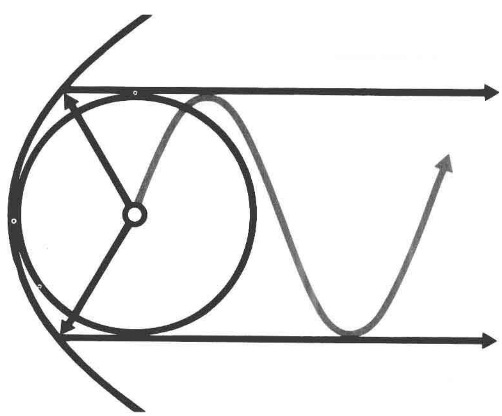

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Preface

There are substantial differences between the first edition of this text and the present second edition. Following is a list of features that characterize this edition.

1. *Completeness.* Topics have been added to the first edition and the paperback version of this text in an effort to satisfy the needs of various users. As a result, this book has become "topic-wise" very complete. It is hoped that it will now meet the needs of any user, besides being a valuable reference source.

Naturally, no class can cover this entire book in an ordinary freshman course. Topics must be chosen judiciously and some topics will have to be omitted. The following feature is therefore very important.

2. *Flexibility.* There are many ways in which this book can be used. Many topics are optional, and there are numerous paths that one can take through various topics, teaching them in various orders. Some of the possibilities are detailed on pp. vi–vii.

3. *Exercises.* This edition contains many new exercises. In response to comments from users, the authors have added exercises that require something of the student other than an understanding of the immediate objectives of the lesson at hand, yet are not necessarily highly challenging. The challenge exercises of the first edition have been augmented here. Thus, the first exercises in an exercise set are very much like the examples in the text for that section. These exercises are graded in difficulty and are paired. That is, each even-numbered exercise is very much like the one that precedes it. The next exercises (marked ☆) require the student to go beyond the immediate objectives. For the first two types of exercises, answers to the odd-numbered ones are given at the back of the book. The instructor can therefore easily make an assignment that is varied in terms of availability of answers. The challenge exercises are marked ★, and some of them are quite difficult. Answers to these exercises are not in the text, but are given in the answer booklet.

4. *Calculators.* Many exercises, as well as some parts of the development, are designed with the calculator in mind. Although it is perfectly feasible to use this text without a calculator, we have indicated those exercises or sections in which the use of a calculator is recommended by the symbol . A calculator will be found useful in many other places as well.

Most of the calculator exercises in the first edition were much like the other exercises, except that the numbers were more complicated. In this edition the use of the calculator has been made much more comprehensive.

5. *Readability and understandability.* Although readability and understandability are related, they are actually separate features of a text. It is easy to write prose that is easy to read but impossible to understand. Therefore, we discuss these items separately.

With respect to *readability*, we have striven to say what we feel needs to be said, but without excess verbiage. The goal was to make the reading level of this text quite low, without sounding condescending. This book is written *to the student*. Theorems, principles, and procedures are stated for maximum student understanding, yet the tone of the book is still mature.

With respect to *understandability*, the goal was to produce a sequence in which each topic is developed, step by carefully-described step. At each appropriate point, examples are given, sufficient in coverage that the routine part of the homework exercises is thoroughly covered. *Cautions* are frequently given to the student; for example, "Don't make the mistake of thinking that $\sqrt{a^2 + b^2} = \sqrt{a} + \sqrt{b}$."

6. *Functions and transformations.* This text applies the concepts of function, relation, and transformation quite thoroughly. The idea of transformations makes Chapter 3 unique and sets up the study of later material.

Chapter 3 may be a bit long. Users have given us valuable feedback with respect to this chapter. Some have recommended that it be broken up, with some of the topics imbedded in other chapters, while others feel quite positive about teaching the entire chapter. After careful consideration, we have decided to leave the chapter intact. Although the topics can be taught in various orders, there is no *best* order. Moreover, for reference purposes, it is desirable to leave all the material together in its own chapter. Some possible reorderings and omissions of the topics are given on pp. vi–vii.

The concept of transformation helps to simplify topics related to quadratic functions and to graphing. It provides a new approach to solving inequalities with absolute value, and many of the trigonometric identities are rendered very simple. Instructors tell us of cases in which students are afraid that they may not be understanding the material because they grasp the ideas so readily.

7. *Accuracy.* In this edition the text, examples, and answers have been checked thoroughly by several different people, in a conscientious effort to make the book error-free. Although we cannot guarantee that nothing has "slipped through the cracks," we have certainly tried hard to minimize errors and inaccuracies. We will be most grateful to any user who calls our attention to any error that somehow went undetected.

8. *Supplementary materials.* The following supplementary materials are available.

- An Answer Booklet containing the answers to the even-numbered exercises and to the more challenging exercises.
- A Student Solutions Booklet containing worked-out solutions to selected odd-numbered exercises.
- A Test Booklet containing five classroom-ready tests for each chapter and five final examinations.

TOPIC SELECTION AND ORDERING

There are numerous topics that can be omitted, depending on the nature of a particular class and/or course. Chapter 6 can easily be omitted entirely, for example, as can several of the later chapters. We will not attempt to list all the topics that can be omitted. Such a list would be unduly long and probably not very helpful. Rather, we shall point out some possibilities that might not be so easy to see on cursory inspection of the book.

Functions and Transformations

Some users consider Chapter 3 a bit long. The following may help such users see how to break it up or omit parts of it.

Sections 3.1, 3.2, and 3.3, on Cartesian products, graphs, and functions, respectively, are the parts of the chapter that are essential in sequence. They should be taught before moving on.

Sections 3.4 and 3.5, on symmetry and transformations, can be postponed until Chapter 4, prior to the material on quadratic functions. Section 3.4 can be partly or entirely omitted without causing much difficulty at this point.

Section 3.7, on inverses of relations, is needed for the material on logarithmic and exponential functions in Chapter 6, and can be postponed until the beginning of that chapter.

Section 3.6, on special classes of functions (including periodic functions), is needed for Chapter 8 on the circular functions and can be taught along with that chapter.

Trigonometry

The trigonometry occurs in successive chapters, but need not necessarily be taught in a block. There are now several different possible tracks through the trigonometry. We detail three such tracks, one with analytic emphasis, the others with triangle emphasis.

Track I (Analytic emphasis)
Proceed through Chapters 8–11 as written, omitting Section 9.6. This track gives a thorough grounding in trigonometry with an initial emphasis on the analytic aspects.

Section 8.1, which is an introduction to triangle trigonometry, could be postponed until the beginning of Chapter 9. Certain topics, such as vectors and polar coordinates, can be omitted.

Track II (Triangle emphasis)

Section 8.1 Chapter 9 (omitting 9.6) Chapter 11	This is triangle trigonometry.
Remainder of Chapter 8 Chapter 10	This is analytic trigonometry.

Track III (Minimal course, triangle emphasis)

Section 8.1 Chapter 9 (omitting 9.6) Chapter 11 (omitting certain optional sections if desired)	This is triangle trigonometry.
Section 9.6 Perhaps Section 8.7 Chapter 10	This is analytic trigonometry.

Note that in the minimal course, most of Chapter 8 is omitted. There are other variations possible. One could teach Chapters 8, 9, 11, and then 10, for example.

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January 1981

M. L. K.
M. L. B.

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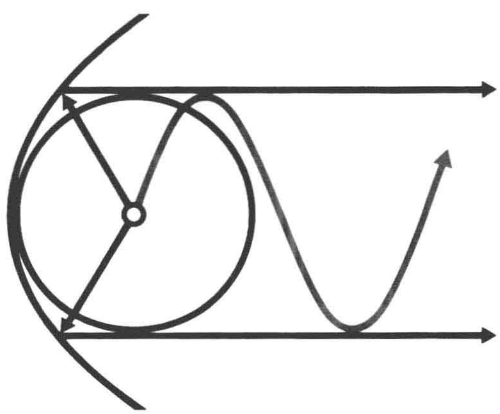
Basic Concepts of Algebra



1.1 THE REAL-NUMBER SYSTEM

Real Numbers

There are various kinds of numbers. Those most used in elementary algebra are the *real numbers.* Later we consider a more comprehensive number system called the *complex numbers.* The real numbers are often pictured in one-to-one correspondence with the points of a line, as in Fig. 1. The positive numbers are shown to the right of 0 and the negative numbers to the left. Zero itself is neither positive nor negative.

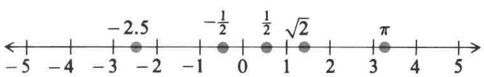


Figure 1

There are several subsystems of the real numbers. They are as follows.

Natural numbers. **Those numbers used for counting: 1, 2, 3, ...**

Whole numbers. **The natural numbers and 0: 0, 1, 2, 3, ...**

Integers. **The whole numbers and their additive inverses: 0, 1, −1, 2, −2, 3, −3, ...**

Rational numbers. **The integers and all quotients of integers (excluding division by 0): $\frac{4}{5}, \frac{-4}{7}, \frac{9}{1}$, 6, −4, 0, $\frac{78}{-5}$, $-\frac{2}{3}$ (can also be named $\frac{-2}{3}$, or $\frac{2}{-3}$).**

Any real number that is not rational is called *irrational.* The rational numbers and the irrational numbers can be described in several ways.

The *rational numbers* are:

1. **Those numbers that can be named with fractional notation a/b, where a and b are integers and $b \neq 0$ (definition);**
2. **Those numbers for which decimal notation either ends or repeats.**

Examples All of these are rational.

1. $\frac{5}{16} = 0.3125$ Ending (terminating) decimal.
2. $-\frac{8}{7} = -1.142857142857\ldots = -1.\overline{142857}$ Repeating decimal. The bar indicates the repeating part.
3. $\frac{3}{11} = 0.2727\ldots = 0.\overline{27}$ Repeating decimal.

The *irrational numbers* are:

1. **Those real numbers that are not rational (definition);**
2. **Those real numbers that cannot be named with fractional notation a/b, where a and b are integers and $b \neq 0$;**

3. Those real numbers for which decimal notation does not end and does not repeat.

There are many irrational numbers. For example, $\sqrt{2}$ is irrational. We can find rational numbers a/b for which $(a/b)\cdot(a/b)$ is close to 2, but we cannot find such a number a/b for which $(a/b)\cdot(a/b)$ is exactly 2.

Unless a whole number is a perfect square its square root is irrational. For example, $\sqrt{9}$ and $\sqrt{25}$ are rational, but all of the following are irrational:

$$\sqrt{3}, \qquad -\sqrt{14}, \qquad \sqrt{45}.$$

There are also many irrational numbers that cannot be obtained by taking square roots. The number π is an example.* Decimal notation for π does not end and does not repeat.

Examples All of these are irrational.

4. $\pi = 3.1415926535\ldots$ Numeral does not repeat.

5. $-1.101001000100001000001\ldots$ Numeral does not repeat.

6. $\sqrt[3]{2} = 1.25992105\ldots$ Numeral does not repeat.

In Example 5 there is a pattern, but it is not a repeating pattern.

Algebra and Properties of Real Numbers

In arithmetic we use numbers, performing calculations to obtain certain answers. In algebra we use arithmetic symbolism, but in addition we use symbols to represent unknown numbers. We do calculations and manipulate symbols, on the basis of the properties of numbers, which we review now. Algebra is thus an extension of arithmetic and a more powerful tool for solving problems.

Addition

Assuming that addition of nonnegative real numbers poses no problem, let us review how the definition of addition is extended to include the negative numbers. Recall first that the absolute value of a nonnegative number is that number itself. To get the absolute value of a negative number, change its sign (make it positive). The absolute value of a number a is denoted $|a|$. Thus, $|3| = 3$ and $|-7.4| = 7.4$.

---

* $\frac{22}{7}$ and 3.14 are only rational approximations to the irrational number π.

1. **To add two negative numbers, add their absolute values (the sum is negative).**
2. **To add a negative number and a positive number, find the difference of their absolute values. The result will have the sign of the summand with the larger absolute value. If the absolute values are the same, the sum is 0.**

For example,

$$-5 + (-6) = -11, \qquad -\frac{3}{4} + \left(-\frac{7}{8}\right) = -\frac{13}{8}, \qquad 8 + (-5) = 3,$$

$$-5 + 3 = -2, \qquad 8.6 + (-4.2) = 4.4, \qquad \pi + (-\pi) = 0,$$

$$-\frac{9}{5} + \frac{3}{5} = -\frac{6}{5}, \qquad -\sqrt{3} + (-4\sqrt{3}) = -5\sqrt{3}.$$

Properties of Real Numbers Under Addition

We now list, for review, the fundamental properties of real numbers under addition. These are properties on which algebraic manipulations are based, especially when symbols for unknown numbers are used.

Commutativity. **For any real numbers a and b, $a + b = b + a$. (The *order* in which numbers are added does not affect the sum.)**

Associativity. **For any real numbers a, b, and c, $a + (b + c) = (a + b) + c$. (When only additions are involved, parentheses for grouping purposes may be placed as we please without affecting the sum.)**

Identity. **There exists a unique real number 0 such that for any real number a, $a + 0 = 0 + a = a$. (Adding 0 to any number gives that same number.)**

Inverses. **For every real number a, there exists a unique number, denoted $-a$, for which $-a + a = a + (-a) = 0$.**

In connection with additive inverses, we caution the reader about one of the most misunderstood ideas in elementary algebra: It is common to read an expression such as $-x$ as "negative x." This can be confusing, because $-x$ may be positive, negative, or zero, depending on the value of x. The symbol "−," used in this way, indicates an *additive inverse;* somewhat unfortunately, the same symbol may also indicate a negative number, as in −5, or it may indicate subtraction, as in $3 - x$.

Caution: **An initial − sign, as in $-x$ or $-(x^2 + 3x - 2)$ should always be interpreted as meaning "the additive inverse of." The entire expres-**