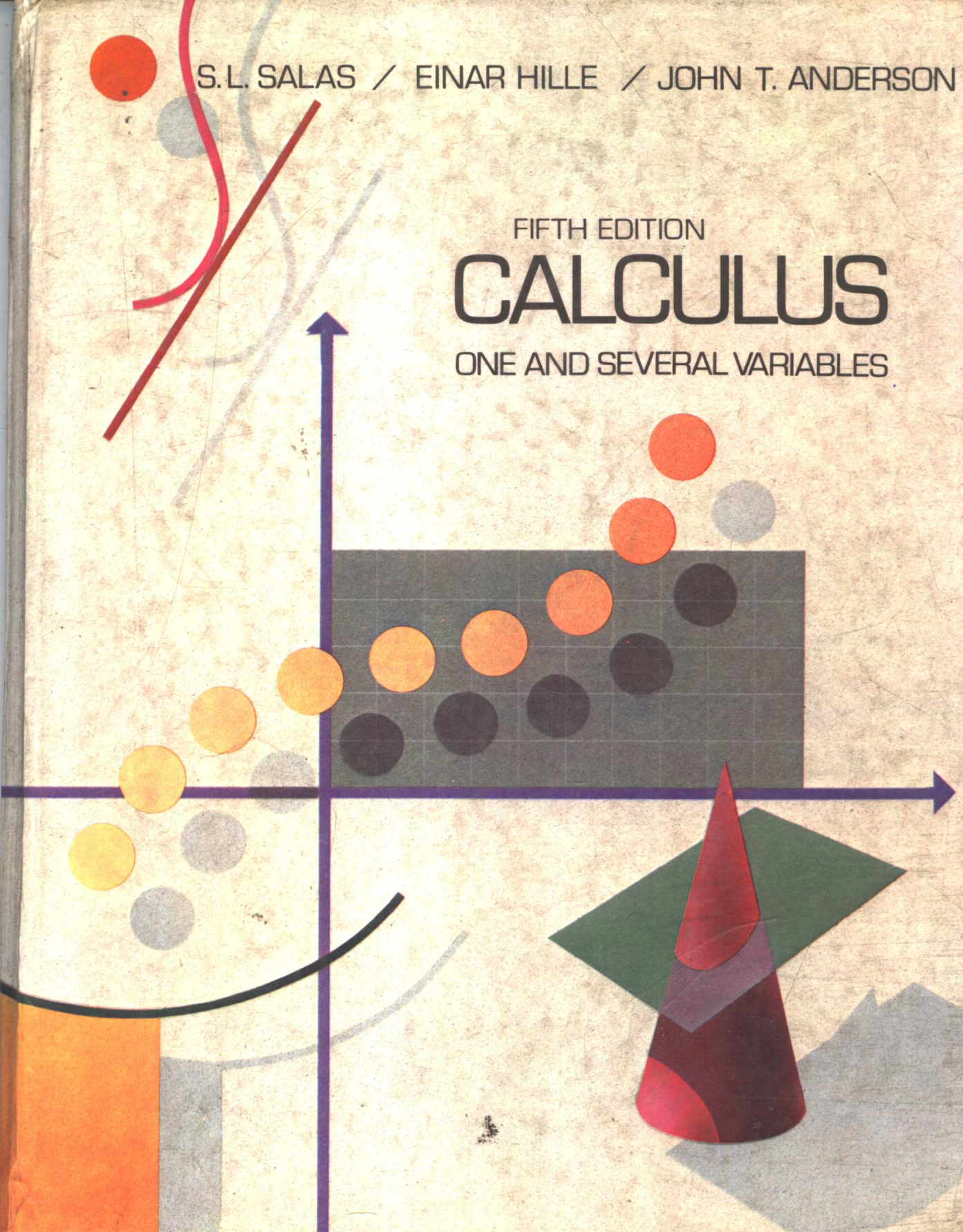


S.L. SALAS / EINAR HILLE / JOHN T. ANDERSON

FIFTH EDITION

CALCULUS

ONE AND SEVERAL VARIABLES



S.L. SALAS EINAR HILLE
JOHN T. ANDERSON

CALCULUS

FIFTH EDITION

ONE AND SEVERAL VARIABLES
WITH ANALYTIC GEOMETRY

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Preface

At the suggestion of several valued reviewers we have made some substantial changes in the organization of the text and in the treatment of certain topics: you will now find an early treatment of the calculus of the trigonometric functions, an increased use of the Riemann definition of the integral, the introduction of several numerical techniques, a relatively early chapter on mathematical modeling, expanded and balanced (odd/even) exercise sets. Other changes: additional detail, suggested procedures for problem solving, revised proofs, and additional examples. All is geared toward providing a complete mathematical treatment that is accessible to students. A detailed description of these and other changes is given on page vii.

As before, *Calculus: One and Several Variables* is available in a single complete volume and in two parts:

Part I—Functions of one variable, differential equations, analytic geometry, sequences and series (Chapters 1–13 of the complete volume);

Part II—Sequences and series, vector calculus, functions of several variables (Chapters 12–19 of the complete volume).

Many people have contributed to this revision. We are particularly indebted to the following mathematicians for their comments and suggestions:

W. W. Comfort, *Wesleyan University*; Hudson Kronk, *SUNY, Binghamton*; Giles Maloof, *Boise State University*; Gordon D. Prichett, *Babson College*; John Saber,

Babson College; Ted Scheick, *Ohio State University*; and Donald Sherbert, *University of Illinois, Urbana*.

Special thanks go to Elaine Rauschal and Philip McCaffrey, who supervised production and to Karin Kincheloe, who supervised the design. The commitment to excellence on the part of Wiley's mathematics editor Carolyn Moore is deeply appreciated. Finally, we are grateful to Charles G. Salas for keeping a careful eye on the development of this revision, from initial draft to finished product.

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A Note From The Publisher. A *Student Solutions Manual* with complete solutions to all the odd-numbered exercises is available. Also available is an *Instructor's Manual*, which includes answers to all exercises together with selected solutions. A *Complex Variables/Differential Equations Supplement*, chapters 20 and 21 of the Fourth Edition, is free to those who wish to cover this material. Finally, there is a *Testing Disk*, for Apple and IBM, which includes a set of test questions for each chapter.

True BASIC Calculus Software by John G. Kemeny (for the Apple Macintosh, Commodore Amiga, and IBM PC, XT, and AT)

True BASIC Calculus Software written by John Kemeny (for the Apple Macintosh, Commodore Amiga, and IBM PC, XT, and AT, and compatibles) integrates very nicely with this text. A study guide prepared by Robert G. Phillips keys the True BASIC software into the text. It is designed for a calculus course in which a personal computer plays an important role as a computational and pedagogical tool. This supplement uses the computer to reexamine many of the examples and exercises in the text. It also contains additional examples and exercises that are intended to be solved using the True BASIC Calculus software.

Although True BASIC is an item sold in individual copies or in quantity, the publisher has arranged for free copies of the software and the study guide to be made available to those instructors who adopt this text.

THE CHANGES

The major changes are highlighted below.

- Trigonometric functions are treated in such a way that the instructor can use them from the outset (Chapter 2 on limits) or postpone the work to as late as Chapter 7 on the natural logarithm without disrupting the flow of the course. Sections 2.5, 3.7, 5.12, and 7.6 are devoted to the trigonometric functions. Exercises and a few examples are given in most of the other intervening sections.
- Exercise sets have been expanded (approximately 1100 new exercises) and balanced (odd/even). The answer section at the back of the text and the *Student Solutions Manual* cover odd-numbered exercises only.
- Chapters 3 and 4 have been significantly rearranged with tangent-line problems and motion problems treated much sooner to provide greater motivation and to permit more variety in the exercise sets.
- Numerical techniques are given much fuller treatment: bisection method (Section 2.6), Newton-Raphson method (Section 3.12), Euler and Runge-Kutta methods (Section 8.9), numerical integration with error estimates (Section 9.9).
- Basic applications of the integral are treated sooner (Chapter 6) and are based on the Riemann definition of the integral. The notion of integral is still introduced with the Darboux definition.
- Chapter 8 is devoted to mathematical modeling and differential equations for the benefit of those instructors who are following the CUPM recommendations and experimenting with UMAP modules or comparable materials in the traditional second semester. The first five sections include topics (exponential growth and harmonic motion) typically covered in Chapter 7 on logarithms and may be treated earlier. Subsequent sections include topics (mechanical vibrations, a predator-prey model, numerical methods) whose coverage can be postponed.
- Additional work with u -substitutions has been provided.
- Integration by parts has been placed with other techniques of integration in Chapter 9.
- Graphing techniques in polar coordinates (Section 11.2) and parametric equations (Section 11.5) are presented at a slower pace in much more detail.
- Series solutions of differential equations are introduced in Section 13.11.
- More explanation and additional hints to some exercises have made it unnecessary to label any material as optional. (Of course, some material can still be omitted.)
- The preliminaries in Chapter 1 are treated at a more relaxed pace. Appendices on radian measure and complex numbers have been added.
- The treatment of max-min problems in several variables has been expanded to include boundary-value problems.
- Suggested procedures for solving certain kinds of problems, such as max-min problems, have been added.
- Chapter summaries with page references to key terms and results are provided.

For the benefit of those instructors familiar with the fourth edition of the text we provide a broad overview of the reorganization:

Chapters 3 and 4 are shuffled,
Chapter 6 is the old Chapter 10,
Chapter 7 combines portions of the old
Chapters 6 and 7,

Chapter 8 combines some new material with
portions of the old Chapters 6, 7, and 21,
Chapter 9 is the old Chapter 10,
Chapter 10 is the old Chapter 9.

Here is a list of a few key sections that contain new material or have been revised extensively:

1.9, 2.6, 3.12, 3.5, 3.6, 3.11, 4.7, 6.4, 8.1, 8.8, 8.9, 11.2, 11.5, 13.11, and 17.5.

Finally, Chapter 20 on complex functions and Chapter 21 on the theory and applications of differential equations from the fourth edition are not bound in the text, but are now available as a supplement.

The Greek Alphabet

Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

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1

Introduction

1.1WHAT IS CALCULUS?

To a Roman in the days of the empire a “calculus” was a pebble used in counting and in gambling. Centuries later “calulare” came to mean “to compute,” “to reckon,” “to figure out.” To the mathematician, physical scientist, and social scientist of today calculus is elementary mathematics (algebra, geometry, trigonometry) enhanced by *the limit process*.

Calculus takes ideas from elementary mathematics and extends them to a more general situation. Here are some examples. On the left-hand side you will find an idea from elementary mathematics; on the right, this same idea as extended by calculus.

Elementary Mathematics

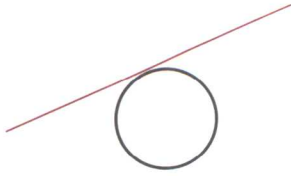
Calculus



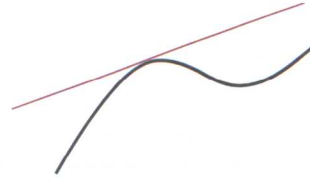
slope of a line
 $y = mx + b$



slope of a curve
 $y = f(x)$



tangent line to
a circle



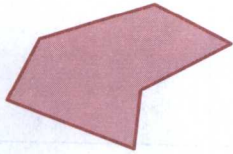
tangent line to a more
general curve

average velocity,
average acceleration

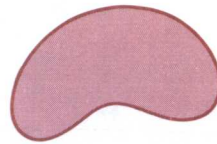
instantaneous velocity,
instantaneous acceleration

distance moved under
a constant velocity

distance moved under
varying velocity



area of a region bounded
by line segments



area of a region bounded
by curves

sum of a finite collection
of numbers

$$a_1 + a_2 + \cdots + a_n$$

sum of an infinite series

$$a_1 + a_2 + \cdots + a_n + \cdots$$

average of a finite
collection of numbers

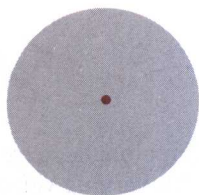
average value of a function
on an interval



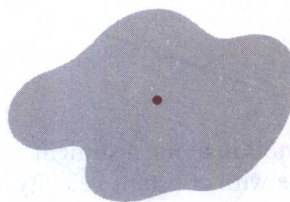
length of a line segment



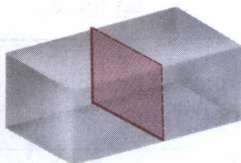
length of a curve



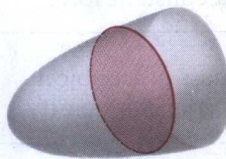
center of a circle



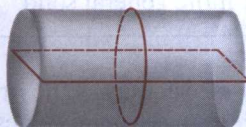
centroid of a region



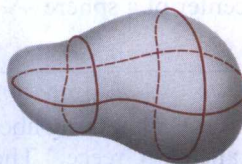
volume of
a rectangular solid



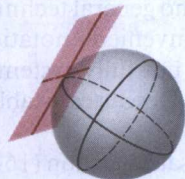
volume of a solid
with a curved boundary



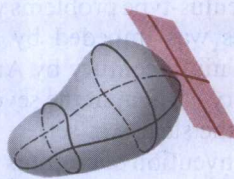
surface area of
a cylinder



surface area of
a more general solid



tangent plane to
a sphere



tangent plane to
a more general surface