

## SECTION 7

### STEADY UNIFORM FLOW IN OPEN CHANNELS

Reference to Sec. 3 will show that the conditions specified in the title of this section require that the discharge in the channel be constant with respect to time and that the cross-sectional area remain the same from place to place in the channel. For subcritical flow (p. 8-38), this condition can exist throughout the full length of the channel only if the outlet end of the channel is controlled so that there will be no drawdown or backwater. For supercritical flow, uniform flow can occur throughout the channel only if the water enters the channel at the uniform-flow depth from a pressure chamber and if no obstruction exists at the outlet end of the channel. Strictly speaking, this type of flow can occur only in parallel-walled channels, thus precluding all natural streams. Practically speaking, however, there are often reaches of natural streams in which flow is nearly uniform, and in many cases flow can be considered as steady in rivers for short time intervals.

The principles governing the relationship between depth slope and discharge for uniform flow depend entirely on the rate of energy dissipation due to friction. Consequently, this section deals entirely with this aspect of flow in open channels. However, because the rate of energy dissipation for gradually varied flow (p. 8-36) depends on the same variables as in the case of uniform flow, the material presented here will also be used in Sec. 8. The problems involved in steady nonuniform flow are discussed in Secs. 8 and 9, and unsteady flow in open channels is treated in Secs. 10 and 11.

**Elements of a Cross Section.** The more important elements of cross sections, together with the symbols that will be used to designate them, are as follows:

The *area  $a$*  always means the cross-sectional area of the stream.

The *wetted perimeter*  $p$  is the length of the line of intersection of the plane of the cross section with the wetted surface of the channel, the line  $abc$  (Fig. 7-1).

The *hydraulic radius*  $r = a/p$  is the area divided by the wetted perimeter.

The *depth*  $D$  (Fig. 7-1), if not specified otherwise, refers to the maximum depth of water in the cross section.

The *top width*  $T$  (Fig. 7-1) is the term used to designate the width of cross section at the water surface.

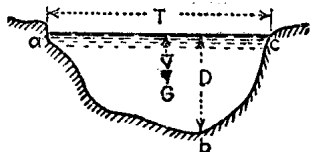


FIG. 7-1. Cross section of open channel.

The *mean depth*  $D_m = a/T$  is the area of cross section divided by the top width. This term will be applied generally to all sections, but it is not literally descriptive of channels with overhanging sides, such as circular conduits flowing more than half full.

The *depth to center of gravity*  $\bar{y}$  (Fig. 7-1) is the depth to the center of gravity of the cross section of the stream.

The above terms and symbols are all used in the discussions and formulas contained in this and the following section. The hydraulic radius enters into formulas involving velocity or discharge. The mean depth occurs in the criterion for indicating when flow in a channel is at critical depth (p. 8-8). The depth to center of gravity of a cross section is employed in determining hydrostatic pressures in problems involving sudden changes in depth of flow.

**Sectional Forms.** Most of the sectional forms used for open channels are shown in Fig. 7-2. It is convenient to have tables which facilitate the determination of numerical values of the elements of a cross section. The equations used in deriving these tables are shown for trapezoidal sections only. Similar equations were derived and used for developing the tables for circular and parabolic sections.

The *trapezoidal section* (Fig. 7-2a) is always used for earth canals. Ordinary earth sections have relatively flat side slopes, usually not steeper than 1:1 in cut and  $1\frac{1}{2}$ :1 in fill. In rock, hardpan, or other indurated material and for lined canals, trapezoidal sections with very steep side slopes are often employed. It is not uncommon to have different side slopes on

the two sides of a canal. Often the uphill side of an earth canal will have a steeper slope than the downhill side.

As indicated in Fig. 7-2a,  $D$  is the maximum depth of water and  $b$  is the bottom width of the canal. The area of the

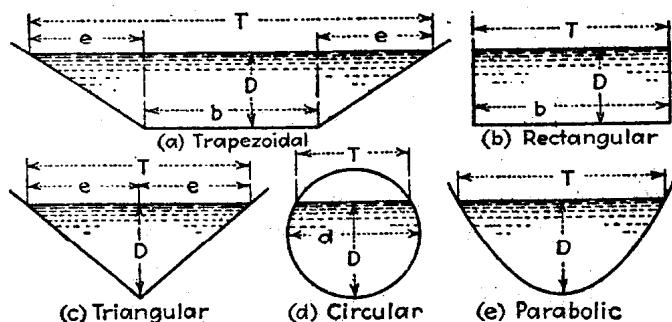


FIG. 7-2. Simple forms of channel sections.

trapezoidal section is

$$a = eD + bD \quad (7-1)$$

and letting

$$z = \frac{e}{D} \quad (7-2)$$

and

$$x = \frac{D}{b} \quad (7-3)$$

then

$$a = \left( z + \frac{1}{x} \right) D^2 \quad (7-4)$$

The wetted perimeter is

$$p = b + 2(e^2 + D^2)^{1/2} \quad (7-5)$$

or substituting for  $b$  and  $e$  as above,

$$p = \left[ \frac{1}{x} + 2(z^2 + 1)^{1/2} \right] D \quad (7-6)$$

Then

$$r = \frac{a}{p} = \frac{1/x + z}{[1/x + 2(z^2 + 1)^{1/2}]} D = C_r D \quad (7-7)$$

Values of  $C_r$  as functions of  $x$  and  $z$  are presented in Table 7-1.

The top width is

$$T = b + 2e = \left( \frac{1}{x} + 2z \right) D \quad (7-8)$$

Then the expression for mean depth becomes

$$D_m = \frac{a}{T} = \frac{1/x + z}{1/x + 2z} D = C_m D \quad (7-9)$$

Values of  $C_m$  in terms of  $z$  and  $x$  are given in Table 7-2. The distance down from the water surface to the center of gravity is obtained by taking moments as follows:

$$a\bar{y} = bD \frac{D}{2} + eD \frac{D}{3}$$

Substituting for  $a$ ,  $b$ , and  $e$ , using Eqs. (7-4), (7-3), and (7-2), respectively,

$$\bar{y} = \frac{1/2x + z/3}{z + 1/x} = C_{\bar{y}} D \quad (7-10)$$

Values of  $C_{\bar{y}}$  are given in Table 7-3.

If the slopes of the two sides of the channel are different, an average value of  $z$  used in Eqs. (7-4), (7-9), and (7-10) will give correct values of  $a$ ,  $D_m$ , and  $\bar{y}$ , respectively, but Eq. (7-7), used with an average  $z$ , will not give exact values of  $r$ . For example, if  $D = 5$ ,  $b = 10$ , and  $z = 1$  and  $2$ ; from Table 7-1, using  $z$  (average) = 1.5, then  $r = 3.12$ , while the correct result is 3.10. For smaller differences in  $z$ , the error will be relatively less. The values corresponding to an average  $z$  obtained from Table 7-1 will usually therefore be within 1 per cent of the correct result.

The *rectangular section* and *triangular section* are special cases of the trapezoidal section. The former has  $z = 0$ , and the latter has  $b = 0$ . The rectangular section (Fig. 7-2b) is used for wooden flumes and for various types of lined conduits. Triangular cross sections (Fig. 7-2c) are seldom encountered, but channels of this form have interesting hydraulic properties.

For the rectangular section,  $a = bd$ ,  $r = bd/(b + 2d)$ ,

$$\bar{y} = \frac{D}{2}$$

and  $D_m = D$ . For the triangular section,  $a = zD^2$ ,

$$r = \frac{zD}{2} \sqrt{1 + z^2}$$

$\bar{y} = D/3$ , and  $D_m = D/2$ . In Table 7-1, the first column gives  $C_r$  [Eq. (7-7)] for rectangular sections, and the bottom row gives this factor for triangular sections.

*Circular conduits*, designed to flow partially full under normal operating conditions, are commonly used for sewers and drains and for other purposes where underground channels are required. Semicircular sections are sometimes employed for flumes and lined canals.

As indicated in Fig. 7-2*d*,  $D$  is the maximum depth of a partially filled circular conduit, and  $d$  is the diameter. The area of the section varies as  $C_a d^2$ , and the other elements as  $Cd$ ,  $C$  in each case being a function of  $D/d$ . Tables of values of  $C$  are as follows: for determining the area  $a$ , Table 7-4; for determining the hydraulic radius  $r$ , Table 7-5; for determining the top width  $T$ , Table 7-6; for determining the mean depth  $D_m$ , Table 7-7; for determining the depth to center of gravity  $\bar{y}$ , Table 7-8.

The *parabolic section* (Fig. 7-2*e*) is occasionally used for lined channels, and it approximates the form assumed by many natural streams and old earth canals. The hydraulic features of parabolic channels are interesting. The equation of the section shown in Fig. 7-2*e* in terms of  $x$  and  $y$  coordinates is  $x^2 = ky$ , in which  $k = 10$ . A parabolic section can be defined in terms of the top width  $T$  and maximum depth  $D$ . Then, if  $\frac{1}{2}T$  and  $D$  are substituted, respectively, for  $x$  and  $y$  in the general equation,  $k$  can be determined, and the equation can be used to locate other points of the section.

The area of a parabolic cross section is  $a = \frac{3}{8}TD$ . The hydraulic radius is  $r = C_r D$ , where  $C_r$  is the factor, varying with  $D/T$ , given in Table 7-9. The mean depth  $D_m = \frac{3}{8}D$ , and the depth to center of gravity  $\bar{y} = \frac{3}{8}D$ .

**Most Efficient Channel Section.** Considered purely from the standpoint of the hydraulics of a channel, it can be seen from the Manning equation [Eq. (7-35)] that for a given area the most efficient channel will be the one with the minimum wetted perimeter. Therefore, if in the expression for the wetted perimeter given by Eq. (7-6) the depth is replaced in terms of  $a$ ,  $z$ , and  $x$  from Eq. (7-4),  $p$  can be differentiated first with respect to  $x$  and then with respect to  $z$ . The first operation yields the following expression:

$$\frac{1}{x} = \frac{b}{D} = 2[(z^2 + 1)^{3/2} - z] \quad (7-11)$$

which yields the best ratio  $b/D$  for any  $z$ . The second differentiation gives  $z = 1/\sqrt{3}$ , which, when substituted in Eq.

(7-11), gives  $b/D = 2/\sqrt{3}$ , thus showing that the most efficient trapezoidal section is a half hexagon. The most efficient rectangular section is that for which  $b/D = 2$ , as shown by letting  $z = 0$  in Eq. (7-11). It may be seen from Eq. (7-11) that for flat side slopes, as would be used in earth canals, the most efficient sections have small bottom widths. All cross sections which satisfy Eq. (7-11) have forms such that semi-

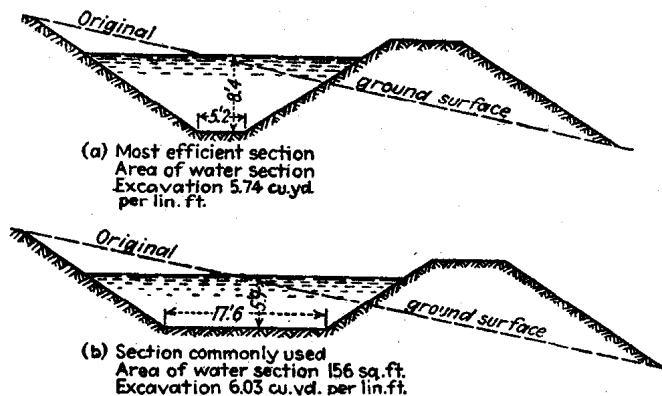


FIG. 7-3. Trapezoidal canal sections. Data for both sections:

$Q = 300$  cu ft per sec

$z = 1.5$  to 1

$n = 0.0225$  Freeboard = 2 ft

$s = 0.000134$

Slope of ground = 20 per cent

Top of embankment = 8 ft

circles can be inscribed in them and the most efficient open-channel cross-sectional shape of all is a semicircle. Steep-sided parabolas also have a high efficiency.

Not only does Eq. (7-11) give the channel with the smallest wetted perimeter for a given area, but it therefore also gives the channel which provides a given discharge with the minimum area. Therefore the section determined from Eq. (7-11) will be the most economical section to build in so far as both excavation and canal lining are concerned. However, earth canals may involve other considerations, such as construction difficulties or maintenance problems, which make it desirable to depart from the ideal section.

Two earth-canal sections with balanced cut and fill are illustrated in Fig. 7-3. Each has the same capacity, 300 sec-ft,

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and all conditions except the ratio of depth to bottom width and area of section are the same for each. Figure 7-3a is the theoretically most efficient section, while Fig. 7-3b has dimensions approximating those commonly employed for canals of this capacity. For the indicated ground slope, 20 per cent, there will be about 5 per cent more excavation in section b than in a, but the former section will be easier to construct and maintain than the latter. For canals in flatter country the difference in excavation for the two sections will be relatively less than that indicated in the figure. As the ground slope increases, the excavation saved by using the more efficient section increases.

**Energy Losses in Open Channels.** As in pipes, the energy losses are of two types, those due to friction and those due to

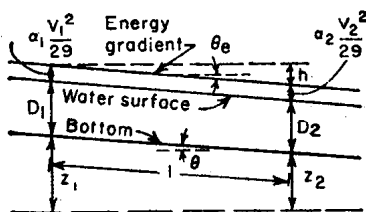


FIG. 7-4. Uniform flow.

sudden changes in the direction and magnitude of the velocity, which are called minor losses. However, because this section of the Handbook deals with uniform flow, only friction losses will be discussed in this section and minor losses will be discussed in Sec. 8.

The mechanics of uniform flow can be illustrated by first considering laminar flow in wide channels. Thereafter turbulent flow will be discussed.

As shown by Eq. (3-11) of Sec. 3, the Bernoulli equation for an open channel takes on the following form:

$$z_1 + D_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + D_2 + \alpha_2 \frac{V_2^2}{2g} + h \quad (7-12)$$

where  $h$  is the energy loss between points 1 and 2 as illustrated in Fig. 7-4. For *uniform flow* the depth and velocity terms are the same at all points, so that Eq. (7-12) reduces to

$$z_1 - z_2 = h \quad (7-13)$$

thus showing that the energy loss  $h$  (Fig. 7-4) is equal to the drop in the bottom or that the energy gradient (e.g.), water surface (w.s.), and bottom are parallel.

If the energy loss per foot of length is called  $s$ , then, for any open channel,

$$s = \frac{h}{l} = \sin \theta. \quad (7-14)$$

where  $\theta$ , is the angle between the energy gradient and a horizontal plane and

$$h = sl \quad (7-15)$$

The relation between the bottom slope  $s_0$ , the angle of the bottom with the horizontal  $\theta$ ,  $l$ , and  $z_1 - z_2$  is expressed by the equations

$$s_0 = \frac{z_1 - z_2}{l \cos \theta} = \tan \theta \quad (7-16)$$

and

$$z_1 - z_2 = s_0 l \cos \theta \quad (7-17)$$

Then, from Eqs. (7-13), (7-15), (7-16), and (7-17), for uniform flow only,

$$s = s_0 \cos \theta = \sin \theta \quad (7-18)$$

When  $\theta$  is very small, as is often the case for open channels,  $\cos \theta \rightarrow 1$  or  $\sin \theta \approx \tan \theta$  and

$$s = s_0 \quad (7-19)$$

Again it should be emphasized that this relation applies only to uniform flow; in nonuniform flow the bottom slope has no relation to  $s$ . In conclusion it may be stated that, under all conditions,  $s$  is the energy loss per foot of length and, for open channels with very small slopes, it may also be defined as the slope of the energy gradient. For uniform flow,  $s$  is also the drop in the channel per foot of length, or  $\sin \theta$ , and for very small slopes; it becomes nearly equal to the slope of the channel,  $s_0$  or  $\tan \theta$ .

**Laminar Flow with a Free Surface.** The law of laminar flow with a free surface for the case of wide rectangular channels may be developed in the same manner as for pipes. Consider the free body of fluid shown in Fig. 7-5, having a width of 1 ft, a length  $l$ , and a height  $D - y$ . The summation of forces in



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the direction of flow gives

$$\tau l = w l (D - y) \sin \theta \quad (7-20)$$

Replacing  $\tau$  with its value from Eq. (1-3), rearranging the terms, and simplifying, the above equation becomes

$$du = \frac{w \sin \theta}{\mu} (D dy - y dy)$$

and integration gives

$$u = \frac{w \sin \theta}{\mu} \left( D y - \frac{y^2}{2} \right) + C \quad (7-21)$$

The value of  $C$  in Eq. (7-21) must be zero to satisfy the condition that  $u = 0$  when  $y = 0$ . The maximum velocity may be obtained by letting  $y = D$ ; then

$$u_{\max} = \frac{w \sin \theta D^2}{2\mu} \quad (7-22)$$

The discharge is obtained from a summation of small elements of discharge, utilizing the value of  $u$  from Eq. (7-21) as follows:

$$Q = \int dQ = \int u dy = \frac{w \sin \theta}{\mu} \int_0^D \left( D y dy - \frac{y^2 dy}{2} \right)$$

from which

$$Q = \frac{w \sin \theta D^3}{3\mu} \quad (7-23)$$

and the average velocity is

$$V = \frac{Q}{a} = \frac{w \sin \theta D^2}{3\mu} \quad (7-24)$$

and

$$\sin \theta = \frac{3\mu V}{w D^2} \quad (7-25)$$

Finally, the Reynolds number  $Dv\rho/\mu$  may be introduced to give

$$\sin \theta = \frac{3V^2}{RgD} \quad (7-26)$$

From Eq. (7-14),

$$s = \frac{h}{l} = \frac{3V^2}{RgD} \quad (7-27)$$

and rearranging terms,

$$h = \frac{6}{R} \frac{l}{D} \frac{V^2}{2g} \quad (7-28)$$

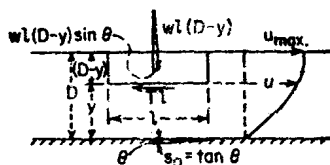


FIG. 7-5. Laminar flow in open channels.

Letting

$$f = \frac{6}{R} \quad (7-29)$$

$$h = f \frac{l}{D} \frac{V^2}{2g} \quad (7-30)$$

In this form the equation resembles closely the Darcy-Weisbach equation for pipes [Eq. (6-19)], and Eq. (7-29) is the counterpart of the similar relationship for pipes given by Eq. (6-20). Because the Manning equation is commonly used for turbulent

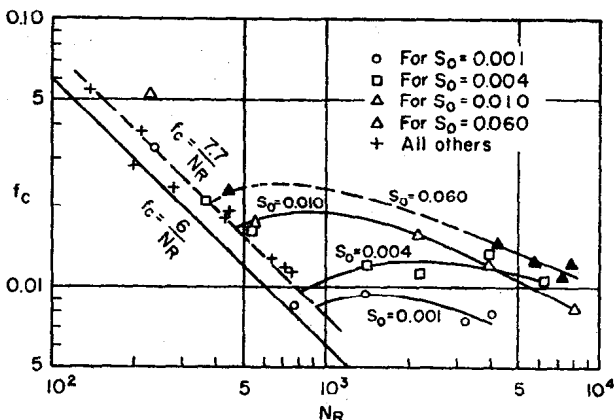


FIG. 7-6.  $f_c$  versus  $N_R$  for the laminar and transition range.

flow in open channels, it is of interest to relate  $f$  to  $n$ . This can be done by equating the value of  $s$  obtained from Eq. (7-30) with its value from the Manning equation [Eq. (7-34)].

$$f \frac{1}{D} \frac{V^2}{2g} = \frac{n^2 V^2}{2.208 r^{4/3}}$$

Because this derivation is for very wide channels,  $r$  may be replaced by  $D$  to obtain the following relationships:

$$f = \frac{2g}{2.21} \frac{n^2}{D^{1/3}} = 29.1 \frac{n^2}{D^{1/3}} \quad (7-31)$$

and

$$n = \frac{f^{1/2} D^{1/6}}{5.4} \quad (7-32)$$

Plotting  $\log f$  against  $\log R$  in accordance with Eq. (7-29) yields a straight line having a slope of  $-1$  as shown in Figs.

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7-6 and 7-7. Numerous tests of laminar flow in wide channels with smooth surfaces have verified Eq. (7-29).<sup>1</sup> However, for rough surfaces, the results follow trends parallel to that of Eq. (7-29) but with a larger value of  $f$ . Such relationships can be represented by the equation

$$f = \frac{C}{R} \quad (7-33)$$

In Fig. 7-6 the value of  $C$  from tests by Woo and Brater<sup>1</sup> for the rough side of masonite is 7.7, and in Fig. 7-7 tests conducted

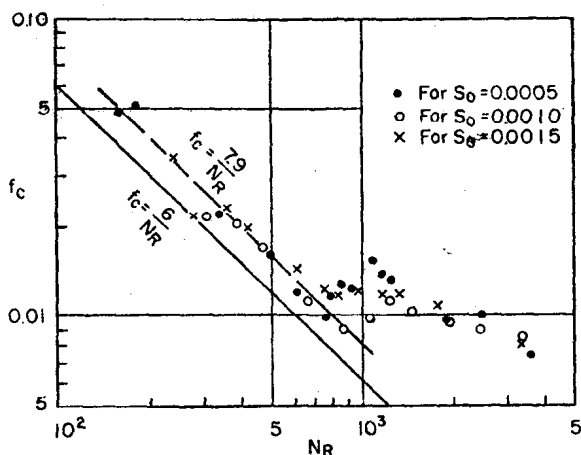


FIG. 7-7. Variation of  $C$  with bottom slope for a sand surface.

at the Waterways Experiment Station<sup>2</sup> on "cement" surfaces yield  $C = 7.9$ . Tests at the University of Michigan<sup>1</sup> with a surface of glued sand having an average size of 0.04 in. gave a value of  $C$  of 9.8 for slopes equal to or less than 0.003. For greater slopes,  $C$  increased consistently with  $s_0$ , reaching a value of 25.0 for a slope of 0.060. Tests at Vicksburg<sup>1,2</sup> with movable

<sup>1</sup> References to research on this topic, together with a more detailed discussion, are presented in D. C. Woo and E. F. Brater, Laminar Flow in Rough Rectangular Channels, *J. Geophys. Res.*, vol. 66, p. 4207, December, 1961.

<sup>2</sup> Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River, *U.S. Waterways Expt. Sta. Paper 17*, January, 1935.

sand surface showed that a sand diameter of 0.0081 in. acted like a smooth surface ( $C = 6.0$ ) but that  $C$  began to increase for a diameter of 0.019 in. All the Vicksburg tests were made with relatively small slopes, so that no variation of  $C$  with slope could be detected.

Data plotted in Figs. 7-6 and 7-7 show clearly how the relationship between  $f$  and  $R$  changes when turbulence begins. It may be noted from Fig. 7-6 that the slope of the channel affects the point at which turbulence begins, there being a tendency for turbulent flow to begin at lower values of  $R$  for steeper slopes. A review of published data<sup>1</sup> indicates that laminar flow ceases at values of  $R$  ranging from 400 to 900. These values appear to be consistent with those of pipes flowing full, because the depth term used in the Reynolds number is approximately equivalent to the hydraulic radius, and for a pipe flowing full,  $r = d/4$ . Consequently, if values of  $R$  for pipes were based on  $r$  instead of  $d$ , they would be one-fourth as large and therefore the transition into turbulent flow would begin in about the same range of values of  $R$  as shown here.

The relationships derived and discussed here are for rectangular channels which are sufficiently wide so that the effects of the side wall will be negligible. Based on an analytical expression for laminar flow in rectangular conduits first presented by Boussinesq,<sup>2</sup> it can be shown<sup>3</sup> that the theoretical value of  $C$  in Eq. (7-33) varies with the ratio of the width to the depth,  $b/D$ , according to the following tabulation:

$b/D$	$C$
$\infty$	6.0
50	6.2
10	6.8
5	8.0

Laminar flow in channels other than rectangular has been investigated by Straub, Silberman, and Nelson.<sup>4</sup>

**Turbulent Flow in Open Channels.** It is quite probable that future research and analysis will lead to the development of

<sup>1</sup> Woo and Brater, *op. cit.*

<sup>2</sup> M. J. Boussinesq, *Memoire sur l'influence des frottements dans les mouvements réguliers des fluides*, J. Math. Pures et Appl., ser. II, xiii, p. 377, 1868.

<sup>3</sup> Woo and Brater, *op. cit.*

<sup>4</sup> L. G. Straub, E. Silberman, and H. C. Nelson, *Open-channel Flow at Small Reynolds Numbers*, *Trans. ASCE*, vol. 123, p. 685, 1958.

methods of determining energy losses in open channels in the transitional and turbulent ranges similar to those now being used for pipes. However, because of the varying depth and the variety of shapes used for open channels, the problem is exceedingly difficult. Fortunately, most practical problems fall in the fully turbulent range, where the Manning equation gives satisfactory results. The historical relation of the Manning equation to the Chezy equation, together with the significance of the numerical value 1.486, was discussed in the section on pipes (pp. 6-12-6-17).

The Manning equation as usually written is

$$V = \frac{1.486}{n} r^{2/3} s^{1/2} \quad (7-34)$$

where  $r$  is the hydraulic radius, defined on page 7-2, and  $s$  is defined on page 7-8. If both sides of Eq. (7-34) are multiplied by the area and at the same time  $r$  is replaced with  $a/p$ , the Manning equation becomes

$$Q = \frac{1.486}{n} \frac{a^{2/3}}{p^{2/3}} s^{1/2} \quad (7-35)$$

This equation is cumbersome to use, and when solving for a channel dimension with  $Q$  and  $s$  known, it requires a trial solution. Consequently, it is convenient to arrange the equation in such a form that dimensionless constants can be provided in tabular form for rapid solution. The method of arranging the equation will be shown only for trapezoidal channels. Similar procedures were employed in deriving the tables for circular and parabolic channels.

*Trapezoidal Channels.* The values of  $a$  and  $p$  from Eqs. (7-4) and (7-6), respectively, are substituted into Eq. (7-35). Then

$$Q = \frac{1.486(z + 1/x)^{3/2}}{[1/x + 2(z^2 + 1)^{1/2}]^{3/2}} \frac{D^{2/3} s^{1/2}}{n} \quad (7-36)$$

and letting

$$K = \frac{1.486(z + 1/x)^{3/2}}{[1/x + 2(z^2 + 1)^{1/2}]^{3/2}} \quad (7-37)$$

the Manning equation can be written

$$Q = \frac{KD^{2/3} s^{1/2}}{n} \quad (7-38)$$

where  $K = f(z, x)$ ; or, replacing  $D$  in Eq. (7-38) with  $bx$  from Eq. (7-3),

$$Q = \frac{Kx^{3/2}b^{3/2}s^{1/2}}{n}$$

or

$$Q = \frac{K'b^{3/2}s^{1/2}}{n} \quad (7-39)$$

where  $K' = Kx^{3/2} = f_1(z, x)$ .

Tabulations of data relating  $K$  and  $K'$  to  $z$  and  $x$  are presented in Tables 7-10 and 7-11, respectively. They cover all symmetrical trapezoidal channels, including rectangular and triangular channels. They permit the direct solution for  $D$  or  $b$  if the other is known, thus eliminating a solution by trial.

The computation of gradually varied flow profiles requires the solution for  $s$  in the Manning equation [Eq. (7-49)]. This solution of the Manning equation involves the term  $(1/K')^2$ . Consequently, a table relating  $(1/K')^2$  to  $x$  and  $z$  is presented, Table 7-12. The eight-thirds and three-eighths powers of numbers may be obtained from Tables 7-19 and 7-20, respectively.

For *circular channels* flowing part full (Fig. 7-2d), the discharge factor for use in formula (7-38) is

$$K = \frac{1.486 \left( \frac{360 - \theta}{360} \frac{\pi}{4} + \frac{1}{8} \sin \theta \right)^{3/2}}{x^{3/2} \left( \frac{360 - \theta}{360} \pi \right)^{3/2}} \quad (7-40)$$

where  $x = D/d$  = ratio of depth of water to diameter of channel and  $\theta$  is the angle between the radii subtending the water surface. Since  $\theta$  is a function of  $x$ , there is in reality only one variable in the right-hand member of this equation. Table 7-13 contains values of  $K$  for different values of  $D/d$ .

By replacing  $D$  with  $xd$ , the following equation is obtained for circular sections:

$$Q = \frac{K'd^{3/2}s^{1/2}}{n} \quad (7-41)$$

where  $K' = f(x)$ . Values of  $K'$  are given in Table 7-14.

The corresponding discharge factor for *parabolic channels* (Fig. 7-2e) is

$$K = \frac{1.2}{x \left[ \sqrt{16x^2 + 1} + \frac{1}{4x} \log_e (16x^2 + 1 + 4x) \right]^{3/2}} \quad (7-42)$$

where  $x = D/T$  = ratio of depth of water to top width of channel. Table 7-15 contains values of  $K$  corresponding to different values of  $D/T$ .

By replacing  $D$  with  $xT$ , the following form of the Manning equation is obtained for parabolic sections:

$$Q = \frac{K' T^{3/8} s^{1/2}}{n} \quad (7-43)$$

where  $K' = f(x)$ . Values of  $K'$  are given in Table 7-16.

**Solution of Problems by the Manning Equation.** For channels of irregular shape or for forms not included in Fig. 7-2, Eqs. (7-34) and (7-35) must be used. Solutions are facilitated by the use of tables. Values of  $r$  are given in Table 7-1. Table 7-17 gives the square roots of decimal numbers, and the two-thirds powers of numbers may be obtained from Table 7-18.

The discharge of channels having any of the sectional forms shown in Fig. 7-2 can be determined from Eq. (7-38):

$$Q = \frac{KD^{3/8} s^{1/2}}{n} \quad (7-38)$$

or from the following equation:

$$Q = \frac{K'}{n} (b, d, \text{ or } T)^{3/8} s^{1/2} \quad (7-44)$$

Transposed into other forms, these formulas are, respectively,

$$D = \left( \frac{Qn}{Ks^{1/2}} \right)^{3/8} \quad (7-45)$$

$$s = \left( \frac{Qn}{KD^{3/8}} \right)^2 \quad (7-46)$$

$$K = \frac{Qn}{D^{3/8} s^{1/2}} \quad (7-47)$$

$$b, d, \text{ or } T = \left( \frac{Qn}{K's^{1/2}} \right)^{3/8} \quad (7-48)$$

$$s = \left( \frac{Qn}{K'b^{3/8}, d^{3/8}, \text{ or } T^{3/8}} \right)^2 \quad (7-49)$$

$$K' = \frac{Qn}{b^{3/8}, d^{3/8}, \text{ or } T^{3/8} s^{1/2}} \quad (7-50)$$

The above formulas provide for a simple and direct solution of problems involving discharge. They are to be solved with the aid of Tables 7-10 to 7-16. It is not necessary to determine either the area or the hydraulic radius since both are included in the discharge factor. To use these formulas, excepting (7-47) and (7-50),  $x = D/b$  or  $D/d$  or  $D/T$  must be known and  $K$  or  $K'$  can then be taken from the appropriate table at the end of this section.

If discharge is the quantity sought, it can be determined from either formula (7-38) or (7-44), each formula providing an independent check on the other. If  $x$  is known or assumed,  $D$  can be obtained from formula (7-45), or if preferred,  $b$ ,  $d$ , or  $T$  can be obtained from formula (7-48), the results by the two formulas checking each other. Either formula (7-46) or (7-49) will be found convenient for computing  $s$ . For determining  $s$  in rectangular and trapezoidal channels by formula (7-49), Table 7-12, giving values of  $(1/K')^2$ , will be most convenient. The use of this table for computations involving nonuniform flow is described on page 8-39. If the depth at which water will flow in a channel of given dimensions is required, it can best be obtained with the aid of formula (7-50) by determining  $K'$  and then from the appropriate table selecting the corresponding value of  $x$ . Then  $D = xb$ ,  $xd$ , or  $xT$ , depending on the channel form. Similarly, if  $D$  is known and the other dimension unknown, it can be determined by the use of formula (7-47). Values of the eight-thirds powers and three-eighths powers of numbers are given in Tables 7-19 and 7-20, respectively.

A more general representation of the discharge equation, embracing both formulas (7-38) and (7-44), is

$$Q = \frac{F}{n} L^{2.48} \quad (7-51)$$

where  $F$  is either  $K$  or  $K'$ , and  $L$  is a linear dimension, either  $D$ ,  $b$ ,  $d$ , or  $T$ . The sectional form of the channel, together with the type of problem, indicates the dimension to be used in the formula and the table from which  $F$  is to be selected.

Formula (7-51) also can be written

$$L = \left( \frac{Qn}{F} \right)^{3/8} \frac{1}{s^{3/16}} \quad (7-52)$$

Using this formula, the author has published in a separate volume a table which gives the dimension  $L$  in feet corresponding to different rates of loss of head for values of  $Qn/F$  that cover the entire range of conditions likely to be encountered in engineering practice. Values of  $F$  ( $K$  and  $K'$ , pp. 7-13 to 7-15) for the different channel forms and a simple description of methods to be employed in the solution of various open-channel problems are also included.

**Roughness Coefficients.** Values of  $n$  to be used in the Manning equation are given in the following table. Many of



Values of  $n$  to Be Used with the Manning Equation

Surface	Best	Good	Fair	Bad
Uncoated cast-iron pipe.....	0.012	0.013	0.014	0.015
Coated cast-iron pipe.....	0.011	0.012*	0.013*	
Commercial wrought-iron pipe, black..	0.012	0.013	0.014	0.015
Commercial wrought-iron pipe, galva- nized.....	0.013	0.014	0.015	0.017
Smooth brass and glass pipe.....	0.009	0.010	0.011	0.013
Smooth lockbar and welded "OD" pipe	0.010	0.011*	0.013*	
Riveted and spiral steel pipe.....	0.013	0.015*	0.017*	
Vitrified sewer pipe.....	{ 0.010 0.011 }	0.013*	0.015	0.017
Common clay drainage tile.....	0.011	0.012*	0.014*	0.017
Glazed brickwork.....	0.011	0.012	0.013*	0.015
Brick in cement mortar; brick sewers..	0.012	0.013	0.015*	0.017
Neat cement surfaces.....	0.010	0.011	0.012	0.013
Cement mortar surfaces.....	0.011	0.012	0.013*	0.015
Concrete pipe.....	0.012	0.013	0.015*	0.016
Wood stave pipe.....	0.010	0.011	0.012	0.013
Plank Flumes:				
Planed.....	0.010	0.012*	0.013	0.014
Unplaned.....	0.011	0.013*	0.014	0.015
With battens.....	0.012	0.015*	0.016	
Concrete-lined channels.....	0.012	0.014*	0.016*	0.018
Cement-rubble surface.....	0.017	0.020	0.025	0.030
Dry-rubble surface.....	0.025	0.030	0.033	0.035
Dressed-ashlar surface.....	0.013	0.014	0.015	0.017
Semicircular metal flumes, smooth..	0.011	0.012	0.013	0.015
Semicircular metal flumes, corrugated..	0.0225	0.025	0.0275	0.030
Canals and Ditches:				
Earth, straight and uniform.....	0.017	0.020	0.0225*	0.025
Rock cuts, smooth and uniform.....	0.025	0.030	0.033*	0.035
Rock cuts, jagged and irregular.....	0.035	0.040	0.045	
Winding sluggish canals.....	0.0225	0.025*	0.0275	0.030
Dredged earth channels.....	0.025	0.0275*	0.030	0.033
Canals with rough stony beds, weeds on earth banks.....	0.025	0.030	0.035*	0.040
Earth bottom, rubble sides.....	0.028	0.030*	0.033*	0.035
Natural Stream Channels:				
(1) Clean, straight bank, full stage, no rifts or deep pools.....	0.025	0.0275	0.030	0.033
(2) Same as (1), but some weeds and stones.....	0.030	0.033	0.035	0.040
(3) Winding, some pools and shoals, clean.....	0.033	0.035	0.040	0.045
(4) Same as (3), lower stages, more ineffective slope and sections.....	0.040	0.045	0.050	0.055
(5) Same as (3), some weeds and stones.....	0.035	0.040	0.045	0.050
(6) Same as (4), stony sections.....	0.045	0.050	0.055	0.060
(7) Sluggish river reaches, rather weedy or with very deep pools.....	0.050	0.060	0.070	0.080
(8) Very weedy reaches.....	0.075	0.100	0.125	0.150

\* Values commonly used in designing.