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Modern Many-Particle Physics: Atomic Gases, Nanostructures and Quantum Liquids

2nd Edition

现代多粒子物理

——原子气体、纳米结构和量子液体

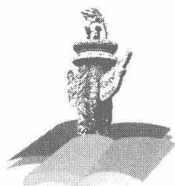
第二版

(影印版)

[意] 利帕里尼 (E. Lipparini) 著



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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学家的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了《中外物理学精品书系》,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,《中外物理学精品书系》力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

《中外物理学精品书系》另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,《中外物理学精品书系》还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子亲身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

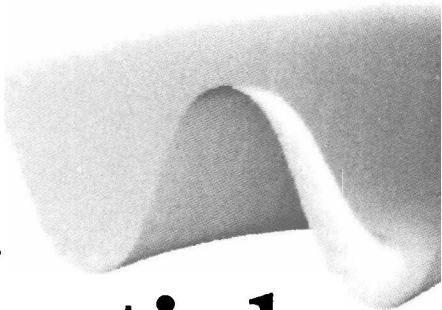
温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套《中外物理学精品书系》的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

《中外物理学精品书系》编委会 主任
中国科学院院士,北京大学教授

王恩哥

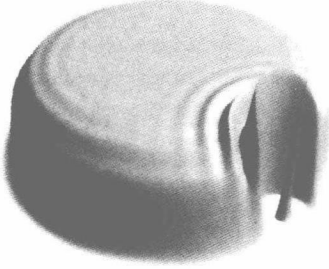
2010年5月于燕园

**Atomic Gases,
Nanostructures and
Quantum Liquids**



Modern Many-Particle Physics

2 n d E d i t i o n



Enrico Lipparini

University of Trento, Italy

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Preface

This book is the fruit of my lectures on “The Theory of Many-Body Systems,” which I have been teaching for many years in the degree course on Physics at the University of Trento. As often happens, the outline of the book came from my students’ notes; in particular, the notes of the students of the academic year 1999–2000, which were extremely useful to me. Chapter 6, on the Monte Carlo methods, is the work of Francesco Pederiva, a research assistant in our department. During the course Francesco, apart from illustrating the method, teaches the students all the computer programs (continually referred to in this book), by means of practical exercises in our computational laboratory. In particular, he teaches the Hartree–Fock, Brueckner–Hartree–Fock, Kohn–Sham and diffusion Monte Carlo programs for the static properties, the RPA, and time-dependent HF and the LSDA for Boson and Fermion finite systems. These programs are available to anyone who is interested in using them.

The book is directed toward students who have taken a conventional course on quantum mechanics and have some basic understanding of condensed matter phenomena. I have often gone into extensive mathematical details, trying to be as clear as possible, and I hope that the reader will be able to rederive many of the formulas presented without too much difficulty.

In the book, even though a lot of space is devoted to the description of the homogeneous systems, such as electron gas in different dimensions, quantum wells in an intense magnetic field, liquid helium and nuclear matter, the most relevant part is dedicated to the study of finite systems. Particular attention is paid to those systems realized recently in laboratories throughout the world: metal clusters, quantum dots and the condensates of cold and dilute atoms in magnetic traps. However, some space is also allotted to the more traditional finite systems, like the helium drops and the nuclei. I have tried to treat all these systems in the most unifying way possible, hoping to bring all the analogies to light. My intention was to narrow the gap between the usual undergraduate lecture course and the literature on these systems presented in scientific journals.

It is important to note that this book takes a “quantum chemist’s” approach to many-body theories. It focuses on methods of getting good numerical approximations to energies and linear response based on approximations to first-principle Hamiltonians. There is another approach to many-body physics that focuses on symmetries and symmetry breaking, quantum field theory and renormalization groups, and aims to extract the emergent features of the many-body systems. This works with “effective” model theories, and does not attempt to do “*ab initio* computations.” These two ways of dealing with many-body systems complement each other, and find common ground in the study of atomic gases, metal clusters, quantum dots and quantum Hall effect systems, which are the main application of the book.

I am indebted to many of my colleagues in the Physics Department of Trento for discussions and remarks. Specifically, I’m grateful to G. Bachelet, D.M. Brink, S. Giorgini, F. Iachello, W. Leidemann, R. Leonardi, F. Pederiva, G. Orlandini, S. Stringari, M. Traini, G. Vilianni and A. Vitturi. Many aspects of the book were clarified during my stays in Barcelona, Paris and Palma de Mallorca, where I had the occasion to discuss many subjects with M. Barranco, A. Emperador, M. Pi, X. Campi, N. Van Giai, D. Vautherin, Ll. Serra and A. Puente, as well as during the frequent visits to our department by my friends A. Richter and K. Takayanagi.

Thanks are also due to Irene Diamond, for the English translation of the book.

This book has cost me a great investment in time, which recently has kept me from other research projects and, above all, from my family. It is dedicated to my wife, Giovanna, and to my children, Fiorenza, Filippo and Luigi. Filippo has been of enormous help in editing the figures.

Enrico Lipparini
January 2003

Preface to the Second Edition

In this edition the main changes are a new chapter on the spin-orbit coupling in semiconductor heterostructures and a considerable expansion of the chapters dealing with trapped atomic gases, density functional calculations, current response to an electromagnetic field, and the Brueckner-Hartree-Fock and Monte Carlo approaches.

The spin-orbit (SO) interaction in nanostructures has prompted intense activity in recent years since it is an essential mechanism for most spintronic devices. In fact, it links the spin and charge dynamics, opening up the possibility of spin control through an electric field. Indeed, recent experimental and theoretical investigations have shown that the SO coupling affects charge transport, far-infrared absorption, and electronic spin precession in a magnetic field, besides giving rise to the spin-Hall effect. All these topics are analyzed in the new Chapter 6 of this edition.

After the first experimental realization of Bose-Einstein condensation in dilute atomic gases, the field of ultracold gases has become a rapidly growing one. In the last few years a considerable amount of experimental and theoretical work has focused on ultracold Fermi gases. The description of the ground state and excited state properties of these systems has been added in many new sections of the book.

The illustration of density functional calculations in quantum wires and molecules has been subjected to much more detailed examination than before. Particular attention has been given to the description of noncollinear local spin density approximation calculations in nanostructures in the presence of SO interaction.

The sections illustrating current response to an electric field have been expanded to give a detailed description of the conductivity problem, with particular emphasis on Landauer conductance, magnetoconductivity and spin-Hall conductivity. A section on the problem of Hall conductivity in graphene has been added.

The Monte Carlo chapter has been revised and expanded to include numerical applications to trapped Fermi gases and many-nucleon systems. A similar revision

and expansion has been carried out for the chapter dealing with the Brueckner–Hartree–Fock theory.

Apart from the above main additions and expansions, the remainder of the book has undergone slight revisions and corrections.

Enrico Lipparini

June 2007

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Chapter 1

The Independent-Particle Model

1.1 Introduction

The main purpose of many-body theory in nonrelativistic quantum mechanics is the study of the properties of the solutions to Schrödinger's equation with the Hamiltonian

$$H = \sum_{i=1}^N \left(\frac{p_i^2}{2m} + v_{\text{ext}}(\mathbf{r}_i) \right) + \sum_{i < j=1}^N v(\mathbf{r}_{ij}), \quad (1.1)$$

which describes a system composed of N identical particles, which interact with an external field through a one-body potential $v_{\text{ext}}(\mathbf{r}_i)$, and among themselves through a two-body potential $v(\mathbf{r}_{ij})$.

The simplest model case is when the Hamiltonian contains only one-body terms, and is referred to as the independent-particle model:

$$H_0 = \sum_{i=1}^N \left(\frac{p_i^2}{2m} + v_{\text{ext}}(\mathbf{r}_i) \right). \quad (1.2)$$

In this approximation the eigenfunctions of H_0 may be written as the product of single-particle wave functions, each of which satisfies the equation ($\hbar = 1$)

$$\left(-\frac{\nabla^2}{2m} + v_{\text{ext}}(\mathbf{r}) \right) \varphi_k(\mathbf{r}, \sigma) = \varepsilon_k \varphi_k(\mathbf{r}, \sigma), \quad (1.3)$$

where k indicates the set of quantum numbers that characterize the single-particle state, and \mathbf{r} and σ are the position and spin variables, respectively. A further variable is introduced in nuclear physics — the isospin τ . In what follows we will indicate as x the variable set \mathbf{r}, σ, τ . For example, for electrons in a central external field,

$$\varphi_k(x) = \varphi_{n,\ell,m}(\mathbf{r}) \chi_{m_s}(\sigma). \quad (1.4)$$