

教育部高等教育司推荐
国外优秀信息科学与技术系列教学用书

数字设计

第三版 影印版

Digital Design

Third Edition

■ M. Morris Mano



高等教育出版社
Higher Education Press



Prentice Hall

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前 言

20 世纪末，以计算机和通信技术为代表的信息科学和技术对世界经济、科技、军事、教育和文化等产生了深刻影响。信息科学技术的迅速普及和应用，带动了世界范围信息产业的蓬勃发展，为许多国家带来了丰厚的回报。

进入 21 世纪，尤其随着我国加入 WTO，信息产业的国际竞争将更加激烈。我国信息产业虽然在 20 世纪末取得了迅猛发展，但与发达国家相比，甚至与印度、爱尔兰等国家相比，还有很大差距。国家信息化的发展速度和信息产业的国际竞争能力，最终都将取决于信息科学技术人才的质量和数量。引进国外信息科学和技术优秀教材，在有条件的学校推动开展英语授课或双语教学，是教育部为加快培养大批高质量的信息技术人才采取的一项重要举措。

为此，教育部要求由高等教育出版社首先开展信息科学和技术教材的引进试点工作。同时提出了两点要求，一是要高水平，二是要低价格。在高等教育出版社和信息科学技术引进教材专家组的努力下，经过比较短的时间，第一批引进的 20 多种教材已经陆续出版。这套教材出版后受到了广泛的好评，其中有不少是世界信息科学技术领域著名专家、教授的经典之作和反映信息科学技术最新进展的优秀作品，代表了目前世界信息科学技术教育的一流水平，而且价格也是最优惠的，与国内同类自编教材相当。

这项教材引进工作是在教育部高等教育司和高教社的共同组织下，由国内信息科学技术领域的专家、教授广泛参与，在对大量国外教材进行多次遴选的基础上，参考了国内和国外著名大学相关专业的课程设置进行系统引进的。其中，John Wiley 公司出版的贝尔实验室信息科学研究中心副总裁 Silberschatz 教授的经典著作《操作系统概念》，是我们经过反复谈判，做了很多努力才得以引进的。William Stallings 先生曾编写了在美国深受欢迎的信息科学技术系列教材，其中有多种教材获得过美国教材和学术著作者协会颁发的计算机科学与工程教材奖，这批引进教材中就有他的两本著作。留美中国学者 Jiawei Han 先生的《数据挖掘》是该领域中具有里程碑意义的著作。由达特茅斯学院的 Thomas Cormen 和麻省理工学院、哥伦比亚大学几位学者共同编著的经典著作《算法导论》，在经历了 11 年的锤炼之后于 2001 年

出版了第二版。目前任教于美国 Massachusetts 大学的 James Kurose 教授，曾在美国三所高校先后 10 次获得杰出教师或杰出教学奖，由他主编的《计算机网络》出版后，以其体系新颖、内容先进而倍受欢迎。在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作。这套引进的教材体现了权威性、系统性、先进性和经济性等特点。

教育部也希望国内和国外的出版商积极参与此项工作，共同促进中国信息技术教育和信息产业的发展。我们在与外商的谈判工作中，不仅要坚定不移地引进国外最优秀的教材，而且还要千方百计地将版权转让费降下来，要让引进教材的价格与国内自编教材相当，让广大教师和学生负担得起。中国的教育市场巨大，外国出版公司和国内出版社要通过扩大发行数量取得效益。

在引进教材的同时，我们还应做好消化吸收，注意学习国外先进的教学思想和教学方法，提高自编教材的水平，使我们的教学和教材在内容体系上，在理论与实践的结合上，在培养学生的动手能力上能有较大的突破和创新。

目前，教育部正在全国 35 所高校推动示范性软件学院的建设和实施，这也是加快培养信息科学技术人才的重要举措之一。示范性软件学院要立足于培养具有国际竞争力的实用性软件人才，与国外知名高校或著名企业合作办学，以国内外著名 IT 企业为实践教学基地，聘请国内外知名教授和软件专家授课，还要率先使用引进教材开展教学。

我们希望通过这些举措，能在较短的时间，为我国培养一大批高质量的信息技术人才，提高我国软件人才的国际竞争力，促进我国信息产业的快速发展，加快推动国家信息化进程，进而带动整个国民经济的跨越式发展。

教育部高等教育司

二〇〇二年三月

To My Wife, Children, and Grandchildren



PREFACE

Digital design is concerned with the design of digital electronic circuits. Digital circuits are employed in the design and construction of systems such as digital computers, data communication, digital recording, and many other applications that require digital hardware. This book presents the basic tools for the design of digital circuits and provides the fundamental concepts used in the design of digital systems. It is suitable for use as a textbook in an introductory course in an electrical engineering, computer engineering, or computer science curriculum.

Many of the features in this third edition remain the same as those of the previous editions except for rearrangement of the material or changes in emphasis due to changes in the technology. Combinational circuits are covered in one chapter instead of two, as in the previous edition. The sequential circuit chapter emphasizes design with *D* flip-flops instead of *JK* and *SR* flip-flops. The material on memory and programmable logic are combined in one chapter. Chapter 8 has been revised to include register transfer level (RTL) design procedures.

The main revision in the third edition is the inclusion of sections on Verilog Hardware Description Language (HDL). The HDL material is inserted in separate sections so it can be covered or skipped as desired. The presentation is at a suitable level for beginning students that are learning digital circuits and a hardware description language at the same time.

- Digital circuits are introduced in Chapters 1 through 3 with an introduction to Verilog HDL in Section 3-9.
- Further discussion of HDL occurs in Section 4-11 following the study of combinational circuits.
- Sequential circuits are covered in Chapters 5 and 6 with corresponding HDL examples in Sections 5-5 and 6-6.
- The HDL description of memory is presented in Section 7-2.

- The RTL symbols used in Verilog HDL are introduced in Sections 8-2.
- Examples of HDL descriptions in the RTL and structural levels are provided in Sections 8-5 and 8-8.
- Section 10-10 covers switch-level modeling corresponding to CMOS circuits.
- Section 11-19 supplements the hardware experiments of Chapter 11 with HDL experiments. Now the circuits designed in the laboratory can be checked by means of hardware components and/or by HDL simulation.

The CD-ROM in the back of the book contains the Verilog HDL source code files for the examples in the book and two simulators provided by SynaptiCAD. The first simulator is VeriLogger Pro, a traditional Verilog simulator that can be used to simulate the HDL examples in the book and to verify the solutions of HDL problems. The second is a new type of simulation technology, called an Interactive Simulator. This simulator allows engineers to simulate and analyze design ideas before a complete simulation model or schematic is available. This technology is particularly useful for students, because they can quickly enter Boolean and *D* flip-flop or latch input equations to check equivalency or to experiment with flip-flops and latch designs. Tutorials are available as HTML files in the CD-ROM Flash display and as MS Word files in the SynaptiCAD installed directory under Book Tutorials.

Additional resources are available in a companion Website at <http://www.prenhall.com/mano>. It includes all the Verilog HDL examples from the book for downloading, all of the figures and tables in the book in PDF format, tutorials on the use of the Verilog software in the CD-ROM, and more.

The following is a brief description of the topics that are covered in each chapter with emphasis on the revisions that were made for the third edition.

Chapter 1 presents the various binary systems suitable for representing information in digital systems. The binary number system is explained and binary codes are illustrated. Examples are given for addition and subtraction of signed binary numbers and decimal numbers in BCD.

Chapter 2 introduces the basic postulates of Boolean algebra and shows the correlation between Boolean expressions and their corresponding logic diagrams. All possible logic operations for two variables are investigated and from that, the most useful logic gates used in the design of digital systems are determined. The characteristics of integrated circuit gates are mentioned in this chapter but a more detailed analysis of the electronic circuits of the gates is done in Chapter 10.

Chapter 3 covers the map method for simplifying Boolean expressions. The map method is also used to simplify digital circuits constructed with AND-OR, NAND, or NOR gates. All other possible two-level gate circuits are considered and their method of implementation is explained. Verilog HDL is introduced together with simple gate-level modeling examples.

Chapter 4 outlines the formal procedures for the analysis and design of combinational circuits. Some basic components used in the design of digital systems, such as adders and code converters, are introduced as design examples. Frequently used digital logic functions such as parallel adder and subtractor, decoders, encoders, and multiplexers are explained, and their use in the design of combinational circuits is illustrated. HDL examples are given in the gate-level, dataflow, and behavioral modeling to show the alternative ways available for describing com-

binational circuits in Verilog HDL. The procedure for writing a simple test bench to provide stimulus to an HDL design is presented.

Chapter 5 outlines the formal procedures for the analysis and design of clocked synchronous sequential circuits. The gate structure of several types of flip-flops is presented together with a discussion on the difference between level and edge triggering. Specific examples are used to show the derivation of the state table and state diagram when analyzing a sequential circuit. A number of design examples are presented with emphasis on sequential circuits that use D-type flip-flops. Behavioral modeling in Verilog HDL for sequential circuits is explained. HDL Examples are given to illustrate Mealy and Moore models of sequential circuits.

Chapter 6 deals with various sequential circuits components such as registers, shift registers, and counters. These digital components are the basic building blocks from which more complex digital systems are constructed. HDL descriptions of shift registers and counter are presented.

Chapter 7 deals with random access memory (RAM) and programmable logic devices. Memory decoding and error correction schemes are discussed. Combinational and sequential programmable devices are presented such as ROM, PAL, CPLD, and FPGA.

Chapter 8 deals with the register transfer level (RTL) representation of digital systems. The algorithmic state machine (ASM) chart is introduced. A number of examples demonstrate the use of the ASM chart, RTL representation, and HDL description in the design of digital systems. This chapter is the most important chapter in the book as it prepares the student for more advanced design projects.

Chapter 9 presents formal procedures for the analysis and design of asynchronous sequential circuits. Methods are outlined to show how an asynchronous sequential circuit can be implemented as a combinational circuit with feedback. An alternate implementation is also described that uses SR latches as the storage elements in asynchronous sequential circuits.

Chapter 10 presents the most common integrated circuit digital logic families. The electronic circuits of the common gate in each family is analyzed using electrical circuit theory. A basic knowledge of electronic circuits is necessary to fully understand the material in this chapter. Examples of Verilog switch-level descriptions demonstrate the ability to simulate circuits constructed with MOS and CMOS transistors.

Chapter 11 outlines experiments that can be performed in the laboratory with hardware that is readily available commercially. The operation of the integrated circuits used in the experiments is explained by referring to diagrams of similar components introduced in previous chapters. Each experiment is presented informally and the student is expected to produce the circuit diagram and formulate a procedure for checking the operation of the circuit in the laboratory. The last section supplements the experiments with corresponding HDL experiments. Instead of, or in addition to, the hardware construction, the student can use the Verilog HDL software provided on the CD-ROM to simulate and check the design.

Chapter 12 presents the standard graphic symbols for logic functions recommended by an ANSI/IEEE standard. These graphic symbols have been developed for SSI and MSI components so that the user can recognize each function from the unique graphic symbol assigned. The chapter shows the standard graphic symbols of the integrated circuits used in the laboratory experiments. The various digital components that are represented throughout the book are similar to commercial integrated circuits. However, the text does not mention specific integrated

circuits except in Chapters 11 and 12. The practical application of digital design will be enhanced by doing the suggested experiments in Chapter 11 while studying the theory presented in the text.

Each chapter has a list of references and a set of problems. Answers to selected problems appear in at the end of the book to aid the student and to help the independent reader. A solutions manual is available for the instructor from the publisher.

I would like to thank Charles Kime for introducing me to Verilog. My greatest thanks go to Jack Levine for guiding me and checking the sections, examples, and problem solutions to all Verilog HDL material. Thanks go to Tom Robbins for helping me decide to write the third edition and my editor Eric Frank for his patience throughout the revision. Appreciation goes to Gary Covington and Donna Mitchell for providing the CD-ROM from SynaptiCad. Thanks also to those who reviewed the third edition: Thomas G. Johnson, *California State University*; Umit Uyar, *City University of New York*; Thomas L. Drake, *Clemson University*; and Richard Molyet, *University of Toledo*. Finally, I am grateful to my wife Sandra for encouraging me to pursue this project.

M. MORRIS MANO



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1

Binary Systems

1-1 DIGITAL SYSTEMS

Digital systems have such a prominent role in everyday life that we refer to the present technological period as the digital age. Digital systems are used in communication, business transactions, traffic control, space guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises. We have digital telephones, digital television, digital versatile discs, digital cameras, and of course, digital computers. The most striking property of the digital computer is its generality. It can follow a sequence of instructions, called a program, that operates on given data. The user can specify and change the program or the data according to the specific need. Because of this flexibility, general-purpose digital computers can perform a variety of information processing tasks that range over a wide spectrum of applications.

One characteristic of digital systems is their ability to manipulate discrete elements of information. Any set that is restricted to a finite number of elements contains discrete information. Examples of discrete sets are the 10 decimal digits, the 26 letters of the alphabet, the 52 playing cards, and the 64 squares of a chessboard. Early digital computers were used for numeric computations. In this case, the discrete elements used were the digits. From this application, the term *digital* computer emerged. Discrete elements of information are represented in a digital system by physical quantities called signals. Electrical signals such as voltages and currents are the most common. Electronic devices called transistors predominate in the circuitry that implements these signals. The signals in most present-day electronic digital systems use just two discrete values and are therefore said to be *binary*. A binary digit, called a *bit*, has two values: 0 and 1. Discrete elements of information are represented with groups of bits called *binary codes*. For example, the decimal digits 0 through 9 are represented in a digital system with a code of four bits. By using various techniques, groups of bits can be made

2 Chapter 1 Binary Systems

to represent discrete symbols, which are then used to develop the system in a digital format. Thus, a digital system is a system that manipulates discrete elements of information that is represented internally in binary form.

Discrete quantities of information either emerge from the nature of the data being processed or may be quantized from a continuous process. For example, a payroll schedule is an inherently discrete process that contains employee names, social security numbers, weekly salaries, income taxes, and so on. An employee's paycheck is processed using discrete data values such as letters of the alphabet (names), digits (salary), and special symbols (such as \$). On the other hand, a research scientist may observe a continuous process, but record only specific quantities in tabular form. The scientist is thus quantizing his continuous data, making each number in his table a discrete quantity. In many cases, the quantization of a process can be performed automatically by an analog-to-digital converter.

The general-purpose digital computer is the best-known example of a digital system. The major parts of a computer are a memory unit, a central processing unit, and input-output units. The memory unit stores programs as well as input, output, and intermediate data. The central processing unit performs arithmetic and other data processing operations as specified by the program. The program and data prepared by a user are transferred into memory by means of an input device such as a keyboard. An output device, such as a printer, receives the results of the computations and the printed results are presented to the user. A digital computer can accommodate many input and output devices. One very useful device is a communication unit that provides interaction with other users through the Internet. A digital computer is a powerful instrument and can perform not only arithmetic computations, but also logical operations. In addition, it can be programmed to make decisions based on internal and external conditions.

There are fundamental reasons why commercial products are made with digital circuits. Like a digital computer, most digital devices are programmable. By changing the program in a programmable device, the same underlying hardware can be used for many different applications. Dramatic cost reductions in digital devices have come about because of the advances in digital integrated circuit technology. As the number of transistors that can be put on a piece of silicon increases to produce complex functions, the cost per unit decreases and digital devices can be bought at an increasingly reduced price. Equipment built with digital integrated circuits can perform at a speed of hundreds of millions of operations per second. Digital systems can be made to operate with extreme reliability by using error-correcting codes. An example of this is the digital versatile disk (DVD) in which digital information representing video, audio, and other data is recorded without a loss of a single item. Digital information on a DVD is recorded in such a way that by examining the code in each digital sample before it is played back, any error can be automatically identified and corrected.

A digital system is an interconnection of digital modules. To understand the operation of each digital module, it is necessary to have a basic knowledge of digital circuits and their logical function. The first seven chapters of this book present the basic tools of digital design such as logic gate structures, combinational and sequential circuits, and programmable logic devices. Chapter 8 introduces digital design at the register transfer level (RTL). Chapters 9 and 10 deal with asynchronous sequential circuits and the various integrated digital logic families. Chapters 11 and 12 introduce commercial integrated circuits and show how they can be connected in the laboratory to perform experiments with digital circuits.

An important trend in digital design is the use of hardware description language (HDL). HDL resembles a programming language and is suitable for describing digital circuits in textual form. It is used to simulate a digital system to verify its operation before hardware is built in. It is also used in conjunction with logic synthesis tools to automate the design. HDL descriptions of digital circuits are presented throughout the book.

As previously stated, digital systems manipulate discrete quantities of information that are represented in binary form. Operands used for calculations may be expressed in the binary number system. Other discrete elements, including the decimal digits, are represented in binary codes. Data processing is carried out by means of binary logic elements using binary signals. Quantities are stored in binary storage elements. The purpose of this chapter is to introduce the various binary concepts as a frame of reference for further study in the succeeding chapters.

1-2 BINARY NUMBERS

A decimal number such as 7,392 represents a quantity equal to 7 thousands plus 3 hundreds, plus 9 tens, plus 2 units. The thousands, hundreds, etc. are powers of 10 implied by the position of the coefficients. To be more exact, 7,392 should be written as

$$7 \times 10^3 + 3 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

However, the convention is to write only the coefficients and from their position deduce the necessary powers of 10. In general, a number with a decimal point is represented by a series of coefficients as follows:

$$a_5 a_4 a_3 a_2 a_1 a_0 \cdot a_{-1} a_{-2} a_{-3}$$

The a_j coefficients are any of the 10 digits (0, 1, 2, ..., 9), and the subscript value j gives the place value and, hence, the power of 10 by which the coefficient must be multiplied. This can be expressed as

$$10^5 a_5 + 10^4 a_4 + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0 + 10^{-1} a_{-1} + 10^{-2} a_{-2} + 10^{-3} a_{-3}$$

The decimal number system is said to be of *base*, or *radix*, 10 because it uses 10 digits and the coefficients are multiplied by powers of 10. The *binary* system is a different number system. The coefficients of the binary numbers system have only two possible values: 0 or 1. Each coefficient a_j is multiplied by 2^j . For example, the decimal equivalent of the binary number 11010.11 is 26.75, as shown from the multiplication of the coefficients by powers of 2:

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

In general, a number expressed in a base- r system has coefficients multiplied by powers of r :

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \dots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \dots + a_{-m} \cdot r^{-m}$$

The coefficients a_j range in value from 0 to $r - 1$. To distinguish between numbers of different bases, we enclose the coefficients in parentheses and write a subscript equal to the base used (except sometimes for decimal numbers, where the content makes it obvious that it is decimal). An example of a base-5 number is