

EXOTIC OPTIONS

2nd
Edition

A Guide to Second Generation Options

Peter G. Zhang

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To my father Zuheng Zhang

PREFACE TO THE SECOND EDITION

Nearly one year has elapsed since the first edition of this book came into existence in early 1997. Within the past year, two significant events occurred which are directly related to the derivatives profession. The first was that on October 14, 1997, the Nobel committee gave the 1997 Nobel Prize in Economic Sciences to Professor Robert Merton of Harvard University and Professor Myron Scholes of Stanford University for their work on the development of option pricing theory. The Nobel committee made it clear that had he lived, Fischer Black would have shared the prize. As described in the first edition of this book, most of the models and pricing formulas in this book have been within a Black-Scholes-Merton world which has been central to the development of financial engineering as both a discipline and profession.

The other is the still-going-on financial crisis spreading from East Asia to around the globe. This crisis started with the rapid devaluation of Thailand's bhat early in July, spreaded to the neighboring Southeast Asian countries of Indonesia, Malaysia, Singapore, and Philippines. Because of similar economics structures and foreign exchange rate policies, these countries began their competitive devaluation of their currencies. Within months, the crisis moved North to Hong Kong, Taiwan, and then South Korea. After defending the New Taiwan dollar for one week, the Taiwan central bank also followed the Southeast countries on October 16, by letting its currency float against the US dollar. The devaluation of the New Taiwanese dollar pressured Hong Kong Hang Seng Stock Index down for four consecutively days from October 17 to October 20 with an accumulated 3175 points, or nearly 15%. The tremendous fall of Hong Kong Stock market pushed the US stock markets (the Dow Jones Index dived 554 points on October 27, the largest one-day drop since the Black Monday in October 1987) and stock markets around the world down significantly. Volatilities in both currency and equity markets have increased significantly during the crisis. Derivatives should have good potentiality for wider use, especially in East Asia, as there

still shows no signs that the crisis will tranquilize in the near future as evidenced from the second-round of crisis starting from Indonesia on January 9, 1998.

As stated in the first edition of this book, the innovation process for newer products has slowed down because the concept of vanilla options has been extended in almost every aspect. With only a few new types of exotic options such as “pure vega digital” options (it could be classified as one type of correction option with the measurement asset specified as the implied volatility of another option, see Chapter 15 of this edition for more details), the market has been learning and familiarizing with the existing products as evidenced in the foreign exchange options markets with various types of barrier options, knockouts, range binaries, one-touch bets, and etc. covered in Chapters 10, and 11, and 15, and particularly the use of average rate barriers (these barrier options are special cases of outside Asian-barrier options classified in Section 11.9) [see Nusbaum (1997) for more detailed descriptions of recent uses of exotic options]. At the same time, traders have been improving the ways to hedge various types of exotic options and researchers incorporating volatility surface into pricing many types of exotic options.

Also as stated in the first edition of this book, exotic option have been applied to many exotic underlying markets, especially in the fast growing credit derivatives market. As credit derivatives expand investor universe, popular with different investor classes including insurance companies, mutual funds, pension funds, banks hedge funds, and corporations, they have expanded exponentially in the past few years. Trigger options are among the most popular types of credit options amounted nearly 20% of all the credit derivatives market size or over 40% of all types of credit options (Financial Times, November 22, 1996). The release of a draft of standardized term sheet for credit swaps defining specifically default-event and other important legal terms by ISDA (International Swaps and Derivatives Association) and the launch of JP Morgan’s CreditMetrics package in the first half of 1997 will help this growing market to develop more smoothly with less legal confusion and boost credit derivatives including various types of credit exotic options.

There are some changes in this edition. First of all, a lot of types have been detected and corrected in this edition. Secondly, quite a few errors have been found and corrected in this edition. Thirdly, new materials such as limiting cases of the Black-Scholes model, “pure vega” digital options, outside double-barrier options, trigger compound options, joint quanto options, and some other materials are included in this edition.

Since the publication of the first edition in early 1997, I have received many letters, calls, and e-mails both directly and through the publisher. I have also received many messages through many of my friends and colleagues. All these letters, calls, and messages either pointed out some typos, errors, questions or gave me good suggestions.

I want to take this opportunity to thank all these who have helped me to improve the quality and contents of the book. It would be too long to list the names of all these people. Still, I particularly want to thank Keiji Ohmori, Andrew H. Chen, Lixin Wang, Steven Allan, Bay Way Wee, I. R. Low, A. A. Kotze, Svein Stokke, John Murray, Peng Wei, Cindy Wong, Tim Owens, Gaile Gong, Garry de Jagger, James Xu, Yuko Kawai, Michel Kurek, and many others.

Peter G. Zhang
January 10, 1998 in Tokyo, Japan

PREFACE TO THE FIRST EDITION

In memory of Fisher Black, without whose tremendous contribution to both theory and practice, derivatives research and industry would not have reached the current stage, and certainly, this book which concentrates on a Black-Scholes environment would not have been started.

With derivatives you can have almost any payoff pattern you want. If you can draw it on paper, or describe it in words, someone can design a derivative that gives you that payoff.

Fisher Black (1995)

These days we often come across such terms as exotic options in newspapers, journals, magazines, and many other financial reports. You may wonder, as I did two years ago, what exactly they are. At that time I had just started to work in the financial industry and was much puzzled by the phrase “exotic options”. Such puzzlement left me feeling uneasy as I had previously spent a few years at school studying option pricing theory. I tried to find some systematic sources to reeducate myself, and it turned out to be nearly impossible as there was no systematic source on this subject. Two years later, although the situation has changed somewhat, exotic options still remain mysterious to many people. I have tried to keep a systematic record on this subject, though initially it was not my intention to write a book on exotic options.

My first research paper on this subject was on spread options nearly two years ago. Since then, many other papers have followed. The writing process has been so wonderful that I would never have learned so fast and thoroughly had I not written this book. This book records the accumulation of my knowledge on exotic options, and I would like to share my learning curve with all of you.

To some degree, exotic options are as old as vanilla options. The earliest article on exotic options can be traced to an article titled “Alternative Forms of Options” by Snyder, published in the *Financial Analysts*

Journal. It was in 1969, four years earlier than the establishment of the Chicago Board of Options Exchange (CBOE), the first organized options exchange in the world, and four years earlier than the birth of the seminal work of Black and Scholes, who made the path-breaking contribution to derivatives industry. However, exotic options became somewhat popular only from the late 1970s and have experienced significant growth in the past decade or so. The primary motivations driving the recent innovations of derivatives are cost-reduction and special customer needs such as off-balance sheet opportunities, tax considerations, and so on.

The study of exotic options is indispensable not only for their own active and important trading but also because they provide easy and efficient building blocks for other more complicated financial derivatives. In order not to inundate many readers, I try to spend a significant amount of time in almost every chapter on how to use the pricing formulas and how to apply them in practice.

A series of events in the derivatives industry since 1994, Orange County, Kidder Peabody, Procter & Gamble, Gibson Greetings, Askin Capital, and so on, has attracted a lot of attention in the financial industry as well as among the general public. These events created calls for transparency of special-purpose derivatives activities. One of the objects of this book is to provide a convenient source of information for exotic options and thus to improve the transparency of the market. I try to provide as complete a source as possible on this subject. In each chapter I try to introduce one type of exotic options, what it can achieve, and how to price and use it.

We will concentrate on a Black-Scholes environment throughout this book for the purpose of transparency and easy comparisons with vanilla options, because the Black-Scholes model is best known. In Section 4.3 of Chapter 4, we provide a derivation of the Black-Scholes formula using the method to solve the related partial differential equation. And in Section 4.4 of Chapter 4, we provide an intuitive and concise method to derive the Black-Scholes formula. The method shown in Section 4.4 to derive the Black-Scholes formula is the same method we use to price essentially all exotic options in this book.

It is well-known in physics that energy can neither be created nor destroyed, it can only be transferred from one form to another. Risk, or more specifically, financial risk, is such a complicated subject that many researchers and financial institutions have been struggling to find ways to measure it. Yet intuition suggests that risk, like energy, can neither be created nor destroyed: it is inherent within the financial system. If we consider standard options as vehicles to transfer risk between two parties (writers

and buyers), then exotic options are vehicles tailored or specially-designed to transfer risk between them. These tailored vehicles are more effective in risk-transferring and at the same time they require more training to drive. Therefore, exotic options are also called risk management products.

Although there has been continuing concern in recent years about the future of the derivatives industry — especially in 1994 as a result of the losses in the market — the general trend is still promising. This is because risk will be better understood with new technology and new studies. Improved understanding will lead investors and institutions to decide what kind of risk they have to tolerate and what they will have to eliminate. This trend will keep institutions not only using most of their existing vehicles but also creating more to meet their increasingly specific needs.

This book is organized as follows. Part I includes two chapters. Chapter 1 gives a bird's eye view of exotic options, and Chapter 2 reviews option pricing theory — the arbitrage-free principle, which will be used to price all options. Part II reviews vanilla options in two chapters. Chapter 3 reviews various aspects of vanilla options, the extensions of the Black-Scholes option pricing formula, modern Greeks, implied volatilities and so on. Chapter 4 reviews the methods to price American options. The review is necessary because most of the terminology used in describing exotic options are from vanilla options. Comparisons between each type of exotic option and its corresponding vanilla option are helpful for us to grasp the characteristics of exotic options. Those with good understanding of vanilla options may skip Chapters 2 to 4 without losing any integrity.

Besides a few chapters which review theories on vanilla options (Chapters 3 and 4), introduce an approximation method (Chapter 6), or point out some limitations of existing methods in pricing correlation options (Chapter 28), each of the other chapters is designed to cover one kind of exotic options. Part III introduces and prices one of the most popular exotic options — path-dependent options — in eight chapters. Chapter 5 introduces geometric Asian options and finds closed-form solutions for them. Chapter 6 illustrates how to approximate arithmetic Asian options with their corresponding geometric Asian options. Using the general mean which includes all existing averages as special cases, we find a linear approximation for any arithmetic average with its corresponding geometric average. This result is used to approximate many other kinds of exotic options in this book. Chapter 7 extends standard Asian options with equal weights to all observations to flexible Asian options which allow uneven weights to different observations in the average. Chapter 8 introduces forward-start options, and Chapter 9 analyzes one-clique options.

Chapter 10 studies vanilla or standard barrier options. We first describe the difficulty in pricing barrier options and then derive the conditional density functions necessary to price all eight types of barrier options. Closed-form solutions are obtained for all eight types of vanilla barrier options including knockout barrier options with time-dependent, or deferrable, rebates. Because of the great variety of barrier options, we study nonstandard or exotic barrier options in Chapter 11. We study time-dependent barrier options, forward-start barrier options, earlier-ending barrier options, window-barrier options, outside barrier options, Asian barrier options, dual-barrier options, and so on. The most interesting feature of our analysis in this chapter is that we provide unified closed-form pricing formulas for all eight types of earlier-ending as well as outside barrier options. These unified pricing formulas include all the pricing formulas of vanilla barrier options as special cases. The unified pricing formulas make it much easier to analyse Greeks. Chapter 12 studies lookback options including floating strike, fixed strike, lookback options, and partial lookback options.

Part IV covers fifteen popular correlation options in sixteen chapters. The order of the chapters largely follows the complexity of the products. Chapter 13 introduces and prices exchange options. Chapter 14 discusses options paying the best/worst of two risky assets and cash. Chapter 15 reviews standard digital options and introduces and prices correlation digital options which include standard digital options as special cases. Chapter 16 studies quotient options or options written on the ratio of two asset prices. Chapter 17 covers product options and prices foreign equity options with domestic strikes using the product option pricing formula. Chapter 18 introduces foreign equity options. Chapter 19 studies equity-linked foreign exchange options. Chapter 20 prices quanto options. Chapter 21 prices rainbow options on two or more than two underlying assets. Chapter 22 discusses both simple spread options and multiple spread options. Chapter 23 covers options written on the spread between the two rainbows or between the maximum and minimum of two asset prices. Chapter 24 discusses dual-strike options or options with two underlying assets and two different strike prices. Chapter 27 prices basket options and discusses basket digital options. Chapter 28 points out the limitations of using constant correlation coefficients in pricing all correlation options in Part IV and tries to estimate the errors of constant correlation coefficients, when they are non-deterministic.

Part V covers exotic options not covered in Part III and Part IV. Chapter 29 describes package options or portfolios of vanilla options, their underlying assets, and cash. Chapter 30 studies nonlinear payoff options.

Chapter 31 prices compound options and discusses how to use the compound option pricing formula to price American options. Chapter 32 studies chooser's options or "as-you-like-options". Chapter 33 describes contingent premium options or pay-later options. Chapter 34 describes many other kinds of exotic options such as Bermuda options, installment options, and so on.

Part VI covers the hedging of exotic options and their further development. Chapter 35 briefly describes the popular methods in hedging most kinds of exotic options and the difficulties. Chapter 36 discusses possible directions in the future development of exotic options and concludes the book.

We have designed questions and exercises at the end of each chapter. The questions are designed for readers to review the important concepts in each chapter, and the exercises are designed to provide some hands-on experiences on how to calculate option prices and other related measures such as sensitivities. Most of the exercises are straightforward applications of the contents in the corresponding chapters. There are, however, a few exercises in each chapter marked with "*" which require more mathematical training. These exercises are mainly designed for analysts or Ph.D. students.

Peter G. Zhang
August 31, 1995 in New York

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