ENGINEERING EQUATION SOLVER

APPLICATION TO ENGINEERING AND THERMAL ENGINEERING PROBLEMS

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Alpha Science

Engineering Equation Solver

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Preface

The subject, Computational methods in thermal engineering has been introduced to the curriculum of mechanical engineering for final year and first year post graduate students in IIT Kharagpur since 2000. Since then, the subject has been taught with a heavy inclination towards programming, mainly using Fortran programs from available texts (like Numerical Recipes in Fortran). Under this subject, topics like, solution of simultaneous non-linear equation, matrix solution of linear equations, integration, interpolation, solution of differential equations (initial value problems and boundary value problems all of which are linear) by using numerical tools, finite difference methods to solve thermal engineering problems (like fin problems, heat transfer in slabs), stability of several difference schemes and application to computation of boundary layer equations (both hydrodynamic and thermal) are normally covered along with some special topics. I had been teaching the subject for last 8 years following the foot prints of others.

I have seen, during a semester, completing all the above mentioned topics is not a problem for the teacher at all, but the students normally do not do much in tutorials and also in the home work problems simply because the programming skill for solving any kind of problem from the above topic is too demanding and the students do not posses such high level of programming skills. Moreover learning programming to a high level and solving any problem from thermal engineering is not just possible in one semester. So the students were not taking enough interest in the subject. There are two reasons for this. First, they could not acquire such good programming skills in a short time to solve meaningful problems and second, in the examination not a single problem could be asked to solve using a program written by their own hand (may be subroutines can be given or copied to their computer) because not a single student could solve a question in the span of the examination time of 3 hours. So the examination was only conducted for very simple problems where the students could at least solve a situation with 3 equations and 3 unknowns using their calculator. This gave a feeling to the students not to learn programming which can never be tested in an examination system. Normally the motivation among the students is always examination driven and hence they are logical in their own way of not learning programming, therefore the learning of computational methods also suffered.

Of late I realized that the methods which are taught to them are too basic and rather the implementation of that through a program is too difficult for which we can not progress much in teaching thermal engineering problems to them. If we can use a program like EES (Engineering Equation Solver), where the method can be taught and demonstrated easily and moreover a number of complicated problems (not only in thermal engineering but in all fields of engineering) can be solved within a couple of minutes compared to the old method of programming through Fortran, where a simple problem can even take 4 hours to be solved, then the learning of thermal engineering methods can be improved drastically. This I have tried in the class and have seen that all the above topics mentioned in paragraph-1 could be covered quickly and all the students could solve moderate level of complicated problems in a very short time and could get a practical feel of the situation by just plotting the graphs through EES (which is again too easy). The entire class became very vibrant and interest arose among the students to solve any problem of their own whatsoever they met in a book of heat transfer or fluid mechanics. The teaching was not limited to only teaching of methods on a black board, rather all the classes were converted to tutorial classes where I showed new way of solving an equation for a particular class of problem and the students could apply that immediately to a practical problem and get the result very quickly and could see the result of that method also immediately. The students took so much of interest because they knew that the examination would be also taken on similar problems using a computer where any kind of thermal engineering problem can be asked in the examination. They did not have the fear of solving the problem straight on computer in the examination because the practice was done on computer in tutorial classes and also in their home work. Rather each and every student liked the examination process where they answered complicated problems like: *Find the time difference between the ascent and descent of a ball thrown vertically upward in air taking air resistance and buoyancy in to account (of course details of the ball and drag laws were given to them).* Just imagine, if such a problem was to be tested by writing their own program in the examination then could anybody answer it rightly in a span of 3 hours? In fact, problems like the above one were given many in numbers (for example 6 to 7 problems in 2 hours) in an examination where many students could secure very high marks.

The overall appreciation for computational methods in thermal engineering has increased to a very great extent among the students after adopting the new method of teaching through EES. I have compiled all the chapters here the way I have taught in the class and I feel that the overall organization of the book will give a solid feel to the students for solving any kind of problems in real engineering life although details of computational methodologies are not covered in the book for any method that has been shown for any particular problem (because of the fear that details of the methodologies will again drive away the students from reality). I presume that once the students develop a taste to solve problems through computers, they will automatically learn the method behind it if they at all need to do it. So without understanding the need of each student we must not force the education of computational methods to everyone. Just teaching the method without any real solution of problems will keep away students from learning new things and developing new things, which was happening in our old classes where programming was emphasized too much rather than the solution of realistic problems.

The main objective of this book is to introduce computational methods in such a way that (except for Chapter 1 where we have shown how EES can be applied to any field of engineering) the students should be able to solve any level of problem up to the end of their engineering curriculum. I have chosen to introduce the method through solution of example problems so that the students get interested in solving them quickly rather than being frightened by the details of programming. We are emphasizing on the solution of the thermal engineering problems because we intend that the book can be chiefly used by the final year undergraduate and first year postgraduate students of mechanical engineering. However, the book can be used by any course instructor and student in any engineering field. That is why I have specifically shown varieties of problems in Chapter 1 so that students from any engineering field can get interest from this book. The book definitely is not a substitute to any of the text books in engineering, rather a help to fluid mechanics, thermodynamics, heat transfer and computational methods and may be adopted by course instructors (and students) in their class so that he/she can help the class to solve a variety of problems for complicated and difficult situations which otherwise could not have been done.

Chapter 1 shows varieties of problems from all engineering fields, starting from a simple arithmetic problem to complicated thermodynamics and fluid network problems. Chapter 2 covers integration in detail. All types of integrals are covered, starting from single variable integration to triple and quadruple integration. The use of integration is shown to realistic situations. Chapter 3 is on interpolation where the use of EES is made to solve practical problems from pump characteristics. Chapter 4 is on solution of differential equations where we have shown initial value problems and boundary value problems and the solution of boundary value problems by initial value method. This is a very powerful way of solving boundary value problems. Mostly non-linear equations have been shown as examples and comparisons with analytical solutions are given for almost all the cases. Chapter 5 is on application of initial value problems to thermal engineering. At the beginning, applications to heat transfer problems are shown and comparisons with analytical solutions are made. Solutions on liquid column oscillation and motion of spring mass systems with and without damping are shown. Then problems on fluid mechanics are solved using shooting method, which are very exciting specially, because of the computational ease the students feel while solving these problems. Many problems on initial value method are shown including the Falkner-Skan wedge flow, flow near a rotating infinite disk (Von Karman viscous pump) and boundary layer flow on a flat plate with heat transfer. In almost all the cases validations are shown with other numerical and analytical solutions. Chapter 6 is an introduction to finite difference method where application to heat transfer in circular fin, conical fin and wedge shape fin have been shown along with comparisons from analytical and other numerical solutions. Falkner-Skan problem by finite difference method has been solved to demonstrate the power of non-linear equation solving capability of EES. Towards the end of the chapter solution of many hyperbolic equations has been done with analytical comparisons, where the one dimensional wave equation has been specially introduced. Chapter 7 is an extensive application of partial differential equations to thermal engineering problems and fluid mechanics. In almost all the cases comparisons with analytical solutions have been made and wherever it is not possible to get analytical solutions there we have shown comparison with other numerical solutions. I feel and am confident that at the end of the semester any graduate student (adopting this book) will be able to solve any kind of equations whatever he sees in his day to day life and to a large extent in his research life.

I have faced many criticisms while adopting EES in a course curriculum because seniors feel that the students will not appreciate the basics if they use EES. I do not agree with them because how without the basics a student will at all write the equations (in fact the equations have to emerge from basics only). I rather feel that students will be stronger in applying physics while deducing their equations from basics and then they will turn to EES for a solution. Only when they get a quick solution, they get motivated to solve one more and test their basic conceptions through numerics. If they can not quickly solve a problem or a problem needs too much of effort for a solution then no one will ever try to solve a problem merrily. Then basics will only remain in books and in memory and will never be tested on a real life problem on machine. After going through this book and adopting it for basic learning, if the students and the teachers feel they are benefitted, yet have not forgotten the basics, then that will be best reward and satisfaction to me.

I am indebted to my wife and son who have encouraged me to write this book. They are my critics, support and lovers who make fun of me and my book on EES. If this book is appreciated by the students then my wife will be the happiest lady and can make more humorous joke on me on the dining table.

Sukanta K. Dash

Contents

CHAPTER

1

From Arithmetic to Engineering

INTRODUCTION

There are many varieties of engineering problems which need lot of computer applications for their solutions. Normally a student in the engineering course will use a calculator or a graph sheet or both to solve the problems by lot of trial and error methods or by a straight forward numerical method which is taught to him in the class. If the problem is too complicated then the student will turn to a computer to write a program in Fortran or in C++ and then get the solution. After the solution is obtained, if it is required to plot a graph, then the student either uses a plot routine from Microsoft or from GNU plot. All these efforts are really too time consuming and becomes a burden on the student for which many students hesitate to learn the application part or the computational part of the subject very thoroughly.

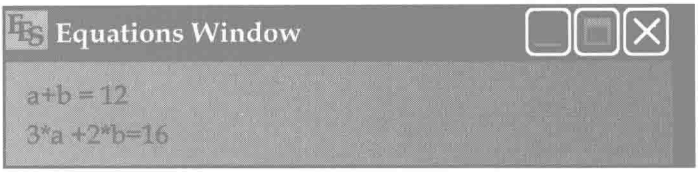
In our opinion if the students use special software to do all these, then there will be lot of motivation left in them to learn engineering problems very thoroughly. One such software is the EES (Engineering Equation Solver) which can be used by the students very efficiently to solve any kind of thermal engineering as well as any engineering problems from day to-day life.

EES learning is very simple and easy. If it is available in your department then you can straight start it (by clicking on the EES icon) or you can download the demo version of EES from ***www.fchart.com*** on to your computer. The demo version will work for 50 equations and 50 unknowns while the registered copy of EES will work for 6000 equations, if you get the commercial version. Now we would take you through varieties of problems in this book so that you would be able to solve many more problems in thermal engineering as well as in many other engineering subjects using EES. In this chapter we will expose you to many classes of problems starting from simple arithmetic to complex pipe network problems where the main objective is to formulate the problem through its equations and then use EES to solve them. Once the equations are ready then EES has the instinct to solve them if they are put in to the equations window. Let's start the first EES problem before we go to any dedicated thermal engineering problems or some mathematical problems in the next chapter.

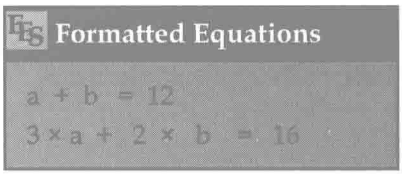
EXAMPLE 1.1

Solution of simple linear equation

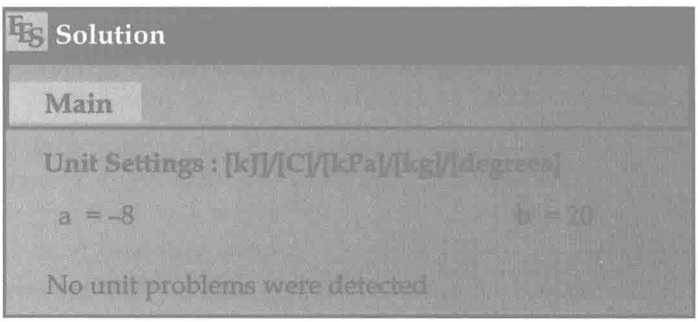
Suppose we want to solve: $a + b = 12$; and $3*a + 2*b = 16$. Then we write this in the equation window of EES as has been shown here in Panel 1.1a. The formatted equation is shown in the Panel 1.1b and the solution is shown in Panel 1.1c. The solution Panel 1.1c shows a message that there is no unit problem encountered in the solution process. This means EES checks for the units automatically for any solution process. We will discuss about this later in this chapter. You must be seeing that this is fairly a simple process to solve any equation. In fact it is very simple, and you have to only write the equations the way you type on a computer and mind that the number of unknowns and the equations must be same so that EES can solve it. In



Panel 1.1(a)



Panel 1.1(b)



Panel 1.1(c)

Panel 1.1 : *Equations window, formatted equations and solution window of Example 1.1*

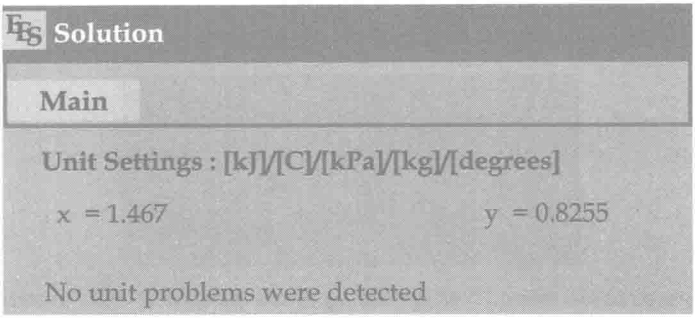
Panel 1.1b you can see the formatted equations which will help you to find out if you have mistyped any of the equations. By this time you must have realized that EES identifies the number of equations and unknowns internally and tries to solve when you press the solve button. EES uses a matrix solver algorithm and also Newton–Raphson method for non-linear problems which we are not going to describe here, because our principal objective is to use the computer for the solution of engineering problems. Interested students can always find relevant literature in the mathematics book about the details of the computing process.

Now come back to our equation again. Instead of solving a simple equation like that of Example 1.1 we could have tried to solve a complex one, here is an example:

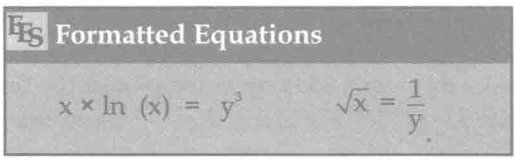
EXAMPLE 1.2

Solution of a non-linear equation

$$x * \ln(x) = y\text{^}3;\ \text{sqrt}(x) = 1/y \tag{1.1}$$



Panel 1.2(a)



Panel 1.2(b)

Panel 1.2 : *Equation and solution window of Example 1.2, Solution of non-linear equation*

Mark the formatted equation window for a better look of the Eqn. (1.1). This helps to a great extent to find out if any mistake is there in the type set of the equations in the equations window. EES assumes an initial guess of 1, 1 for the x and y values to start with and then iterates to get the final value, within a second, as has been shown in the solution window. If you change the initial guess values of x and y then you may get different solutions. Probably for this case only one solution is there.

EXAMPLE 1.3

Non-linear equation with many roots

If we want to solve the following cubic equation for all its roots, then what do we do? We simply type the equation to the EES equation window as such (the way it has been written here). Then we go to the options> variable info> and change the initial guess of x to 1. Then solve by pressing the solve button.

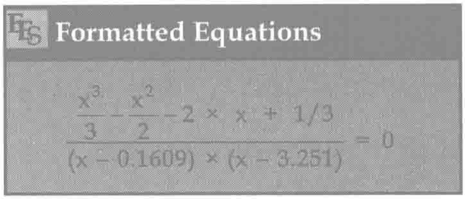
$$\frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} = 0 \tag{1.2}$$

EES will show the value of x to be 0.1609. If we want to get other roots, then we simply divide the Eqn. (1.2) by $(x - 0.1609)$ and again write to the equation window Eqn. (1.3) in the following manner.

$$(x\verb|^|3/3 - x\verb|^|2/2 - 2*x + 1/3)/(x - .1609) = 0 \tag{1.3}$$

We do not have to change the initial guess anymore. Simply press solve button; EES shows the value of x to be 3.251. For the third root of the equation we again divide the above Eqn. (1.3) by $(x - 3.251)$ and write to the equation window Eqn. (1.4)

$$(x\verb|^|3/3 - x\verb|^|2/2 - 2*x + 1/3)/((x - .1609)*(x - 3.251)) = 0 \tag{1.4}$$



Panel 1.3 : *Formatted equation (1.4) appearing in the window of EES formatted equations*

EES will solve for the third value of x and will display $x = -1.912$. If you again divide the equation by $(x + 1.912)$ then you simply get back the same roots again and again. There are no more roots because we have got all the three roots. In the next chapter we will discuss the use of this equation while evaluating area integration. If precise roots are not of consequence, then one can get a feel of the roots by simply looking at the plot of the function, which is shown in Fig. 1.1. To get a plot; write $f = (x\verb|^|3/3 - x\verb|^|2/2 - 2*x + 1/3)$ on the equation window. Open a new parametric table and change the no. of runs to 20 instead of 10. Bring f and x to the "variable in table". Then fill the values of x from –4 to 5 which will be automatically done if you give the first and last value of x. Then solve the table which will generate the parametric table like the way it is shown here in Panel 1.4.

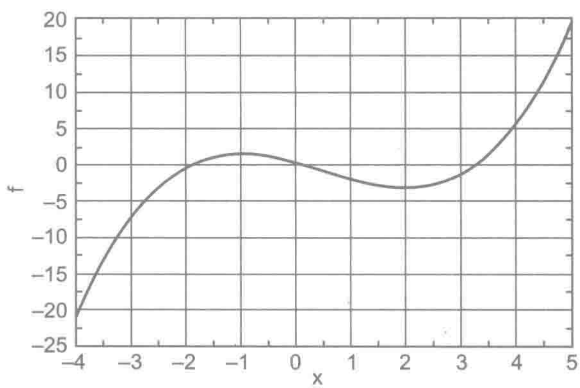


Fig. 1.1 : *Graph of equation (1.2), Roots of the equation can be read from the graph*

In order to get a plot of f vs x, go to "plot window> new plots> $x - y$ plot" and select the x and y-axis variables and press ok for a plot. You will get the plot of f vs x as has been shown in Fig. 1.1. From the plot you can see that there are three roots of f because the f curve crosses