James B. Hartle

GRAVITY

An Introduction to Einstein's General Relativity

引力

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An Introduction to Einstein's General Relativity

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IMPORTANT SPACETIMES (geometrized units)

Flat Spacetime

Cartesian Coordinates

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \equiv \eta_{\alpha\beta} dx^\alpha dx^\beta$$

Spatial Spherical Polar Coordinates

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Static, Weak Field Metric

$$ds^2 = -(1 + 2\Phi(x^i))\, dt^2 + (1 - 2\Phi(x^i))(dx^2 + dy^2 + dz^2), \quad (\Phi(x^i) \ll 1).$$

Schwarzschild Geometry

Schwarzschild Coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Eddington–Finkelstein Coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Kruskal–Szekeres Coordinates

$$ds^2 = \frac{32M^3}{r} e^{-r/2M} \left(-dV^2 + dU^2\right) + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Kerr Geometry

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \frac{4Mar\sin^2\theta}{\rho^2} d\phi dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right) \sin^2\theta\, d\phi^2,$$

where

$$a \equiv J/M, \quad \rho^2 \equiv r^2 + a^2\cos^2\theta, \quad \Delta \equiv r^2 - 2Mr + a^2$$

Linearized Plane Gravitational Wave

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + h_{\alpha\beta} dx^\alpha dx^\beta$$

where (rows and columns in t, x, y, z order)

$$h_{\alpha\beta}(t, z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_+(t-z) & f_\times(t-z) & 0 \\ 0 & f_\times(t-z) & -f_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

for a wave propagating in the z-direction.

Friedman–Robertson–Walker Cosmological Models

$$ds^2 = -dt^2 + a^2(t)\left[d\chi^2 + \left\{\begin{matrix} \sin^2\chi \\ \chi^2 \\ \sinh^2\chi \end{matrix}\right\}(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad \left\{\begin{matrix} \text{closed} \\ \text{flat} \\ \text{open} \end{matrix}\right\}.$$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad \begin{pmatrix} k = +1, \text{ closed} \\ k = 0, \text{ flat} \\ k = -1, \text{ open} \end{pmatrix}.$$

THE GEODESIC EQUATION

- Lagrangian for the Geodesic Equation of a test particle

$$L\left(\frac{dx^\alpha}{d\sigma}, x^\alpha\right) = \left(-g_{\alpha\beta}(x)\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma}\right)^{1/2}$$

where σ is an arbitrary parameter along the world line $x^\alpha = x^\alpha(\sigma)$ of the geodesic.

- Geodesic equation for a test particle (coordinate basis)

$$\frac{d^2x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} \quad \text{or} \quad \frac{du^\alpha}{d\tau} = -\Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma$$

where τ is the proper time along the geodesic and $u^\alpha = dx^\alpha/d\tau$ are the coordinate basis components of the four-velocity so that $\mathbf{u} \cdot \mathbf{u} = -1$. The Christoffel symbols $\Gamma^\alpha_{\beta\gamma}$ follow from Lagrange's equations or from the general formula (8.19). The geodesic equation for light rays takes the same form with τ replaced by an affine parameter and $\mathbf{u} \cdot \mathbf{u} = 0$.

- Conserved Quantities

$$\boldsymbol{\xi} \cdot \mathbf{u} = \text{constant}$$

where $\boldsymbol{\xi}$ is a Killing vector, e.g., $\xi^\alpha = (0, 1, 0, 0)$ in a coordinate basis where the metric $g_{\alpha\beta}(x)$ is independent of x^1.

Preface

Einstein's relativistic theory of gravitation—general relativity—will shortly be a century old. At its core is one of the most beautiful and revolutionary conceptions of modern science—the idea that gravity *is* the geometry of four-dimensional curved spacetime. Together with quantum theory, general relativity is one of the two most profound developments of twentieth-century physics.

General relativity has been accurately tested in the solar system. It underlies our understanding of the universe on the largest distance scales, and is central to the explanation of such frontier astrophysical phenomena as gravitational collapse, black holes, X-ray sources, neutron stars, active galactic nuclei, gravitational waves, and the big bang. General relativity is the intellectual origin of many ideas in contemporary elementary particle physics and is a necessary prerequisite to understanding theories of the unification of all forces such as string theory.

An introduction to this subject, so basic, so well established, so central to several branches of physics, and so interesting to the lay public is naturally a part of the education of every undergraduate physics major. Yet teaching general relativity at an undergraduate level confronts a basic problem. The logical order of teaching this subject (as for most others) is to assemble the necessary mathematical tools, motivate the basic defining equations, solve the equations, and apply the solutions to physically interesting circumstances. Developing the tools of differential geometry, introducing the Einstein equation, and solving it is an elegant and satisfying story. But it can also be a long one, too long in fact to cover both that and introduce the many contemporary applications in the time that is typically available for an introductory undergraduate course.

Gravity introduces general relativity in a different order. The principles on which it is based are discussed at greater length in Appendix D, but essentially the strategy is the following: The simplest physically relevant solutions of the Einstein equation are presented *first*, without derivation, as spacetimes whose observational consequences are to be explored by the study of the motion of test particles and light rays in them. This brings the student to the physical phenomena as quickly as possible. It is the part of the subject most directly connected to classical mechanics, and requires the minimum of new mathematical ideas. The Einstein equation is introduced later and solved to show how these geometries originate.

A course for junior or senior level physics students based on these principles and the first two parts of this book has been part of the undergraduate curriculum at the University of California, Santa Barbara for over twenty-five years. It works.

Acknowledgments

It will be disappointing if my colleagues in gravitational physics find anything new here. It would mean that they have not studied the classic texts of Landau and Lifshitz (1962), Misner, Thorne and Wheeler (1970), Taylor and Wheeler (1963), Wald (1984), and Weinberg (1972) on which this exposition relies so heavily. I have not acknowledged individual points of indebtedness to these works. I do so generally here.

I am especially grateful to Roger Blandford, Ted Jacobson, Channon Price, Kip Thorne, and Bob Wald, who read early versions of the entire book and provided helpful advice on its overall structure in addition to numerous corrections. The book has benefited from the comments and criticism of my colleagues that have taught from preliminary versions of it over the years. Vernon Barger, Omer Blaes, Doug Eardley, Jerome Gauntlett, Gary Horowitz, Clifford Johnson, Shawn Kolitch, Rob Myers, Thomas Moore, Stan Peale, Channon Price, and Kristin Schleich have my gratitude in this regard. Many colleagues commented constructively on individual chapters. Lars Bildsten, Omer Blaes, Peter D'Eath, Doug Eardley, Wendy Freedman, Daniel Holz, Gary Horowitz, Scott Hughes, Robert Kirshner, Lee Lindblom, Richard Price, Peter Saulson, Bernard Schutz, David Spergel, Joseph Taylor, Michael Turner, Bill Unruh, and Clifford Will have my particular thanks for their help. I am grateful to Eric Adelberger, Neil Ashby, Matt Colless, Francis Everitt, Andrea Ghez, John Hall, Jim Moran, Michael Perryman, Wolfgang Schleich, Tuck Stebbins, Max Tegmark, Dave Tytler, and Jim Williams for assistance with some of the boxes and figures. Instructive exchanges on particular points with Dave Arnett, Peter Bender, Dieter Brill, J. Richard Gott, Jeanne Dickey, Andrew Fabian, Jeremy Gray, Gary Gibbons, Wick Haxton, Gordon Kane, Angela Olinto, and Roger Penrose were useful. In lists this long it is inevitable that I have left somone out. To them I offer my apologies and my hope for another printing.

The help in providing many of the figures is acknowledged in the individual figure credits.

Students too numerous to mention pointed out mistakes, typos, and arguments that lacked clarity, and I would like to thank Joe Alibrandi, Maria Cranor, Ian Eisenman, Bill Paxton, and Taro Sato for their particular contributions and perspectives.

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The book is dedicated to my wife Mary Jo, for her unstinting support in so many ways, selfless flexibility in the face of deadlines, and boundless tolerance for too-optimistic estimates of when it would be finished.

James Hartle
June, 2002

Figure Credits

Credits for figures in Boxes with multiple figures are listed in the order they appear.

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Figure 1.5, Courtesy of NASA Jet Propulsion Laboratory.

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Figure 13.4, Courtesy of Andrea Ghez, UCLA from data in Ghez, et al., 2002.

Figure 13.5, Courtesy of Chris Carilli and Rick Perley, NRAO

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Box 15.1, Adapted from Thorne et al. 1986.

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Figure 24.7, Computation and graph courtesy of Lee Lindblom, California Institue of Technology.

Organizational Notes

The pedagogical principles that guided the writing of this boo'; are explained in Appendix D. However the following notes may be immediately useful in navigating the text:

- **Boxes:** The boxes contain material that illustrates or expands on the basic material in the text. Sometimes this is a qualitative explanation of a related phenomenon or idea, sometimes a description of a relevant experiment. Sometimes these are expositions that require a knowledge of physics beyond the basic mechanics and special relativity that is assumed in the text. *It is not necessary to understand the boxes to understand the text.*
- **Problems:** The labels on the problems mean the following:

  A = More algebra needed than most problems.
  B = Refers to a discussion in a Box.
  C = More challenging than most problems.
  E = Asks for an order of magnitude estimate in contrast to a calculation.
  N = Requires some computer work.
  P = Requires some aspect of physics outside the prerequites assumed to this text, e.g., electromagnetism.
  S = Straightforward (in the author's opinion.)

  A problem with no labels is just an ordinary problem, referring to the text, of average difficulty, etc.
- ***Mathematica* Programs:** Several *Mathematica* programs are provided for computing curvature quantities for general metrics, orbits, and cosmological models. These can be downloaded from the website below.
- **Website:** A website containing current information about the book can be found at the time of writing at:

  `http://www.aw.com.`

  This includes current errata, notebook files for the *Mathematica* programs, supplementary discussion (Web supplements), some color pictures, and links to other sites that were useful at the time of writing.
- **A few symbols:**

  $\equiv$ defined to be
  $\approx$ approximately equal to
  $\sim$ of order of magnitude
  $\rightarrow$ asymptotically approaches
  $\odot$ the Sun
  $\oplus$ the Earth

COORDINATE AND ORTHONORMAL BASES

- A set $\{\mathbf{e}_{\hat{\alpha}}\}$ of four *orthonormal* basis vectors satisfies

$$\mathbf{e}_{\hat{\alpha}}(x) \cdot \mathbf{e}_{\hat{\beta}}(x) = \eta_{\hat{\alpha}\hat{\beta}}.$$

- A set $\{\mathbf{e}_{\alpha}\}$ of four *coordinate* basis vectors associated with a set of coordinates x^{α} satisfies

$$\mathbf{e}_{\alpha}(x) \cdot \mathbf{e}_{\beta}(x) = g_{\alpha\beta}(x)$$

where the line element has the form $ds^2 = g_{\alpha\beta}(x)dx^{\alpha}dx^{\beta}$.

- If the coordinate system is *orthogonal* ($g_{\alpha\beta}(x) = 0$ for $\alpha \neq \beta$), the coordinate basis components of an orthonormal basis pointing along the coordinate directions have the form

$$(\mathbf{e}_{\hat{0}})^{\alpha} = [(-g_{00})^{-1/2}, 0, 0, 0], \quad (\mathbf{e}_{\hat{1}})^{\alpha} = [0, (g_{11})^{-1/2}, 0, 0], \quad \text{etc.}$$

USEFUL NUMBERS

Conversion Factors

Velocity of light	$c \equiv 299792458$ m/s $\approx 3 \times 10^{10}$ cm/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-16}$ erg/K $= 8.59 \times 10^{-5}$ eV/K
Second of arc	1 arcsec = 1″ =4.85×10^{-6} rad
Light year	1 ly = 9.46×10^{17} cm
Parsec	1 pc = 3.09×10^{18} cm = 3.26 ly
Electron volt	1 eV = 1.60×10^{-12} erg = 1.16×10^{4} K
Erg (cgs unit of energy)	1 erg = 10^{-7} J
Dyne (cgs unit of force)	1 dyne = 10^{-5} N

Physical Constants

Gravitational constant	$G = 6.67 \times 10^{-8}$ dyn $\cdot$ cm^2/g^2
Stefan–Boltzmann constant	$\sigma = 5.67 \times 10^{-5}$ erg/(cm$^2 \cdot$ s $\cdot$ K^4)
Radiation constant	$a = 7.56 \times 10^{-15}$ erg/(cm$^3 \cdot$ K^4)
Mass of an electron	$m_e = 9.11 \times 10^{-28}$ g
Mass of a proton	$m_p = 1.67 \times 10^{-24}$ g
Planck's constant	$\hbar = 1.05 \times 10^{-27}$ erg $\cdot$ s

ASTRONOMICAL CONSTANTS

Earth

Astronomical unit	$AU = 1.50 \times 10^8$ km
(semimajor axis of Earth's orbit)	$= 1.50 \times 10^{13}$ cm
Mass of the Earth	$M_\oplus = 5.97 \times 10^{27}$ g
	$GM_\oplus/c^2 = 0.443$ cm
Equatorial radius of the Earth	$R_\oplus = 6.38 \times 10^8$ cm $= 6378$ km
Moment of inertia about rotation axis	8.04×10^{44} g $\cdot$ cm^2 $= .331\, M_\oplus R_\oplus^2$
Rotation period	8.62×10^4 s
Angular velocity	$\Omega_\oplus = 7.29 \times 10^{-5}$ rad/s

Sun

Mass of the Sun	$M_\odot = 1.99 \times 10^{33}$ g
	$GM_\odot/c^2 = 1.48$ km
Radius of the Sun	$R_\odot = 6.96 \times 10^{10}$ cm $= 6.96 \times 10^5$ km
Moment of inertia about rotation axis	5.7×10^{53} g $\cdot$ cm^2
Rotation period at Equator	25.5 days
Angular velocity at Equator	2.85×10^{-6} rad/s
Luminosity of the Sun	$L_\odot = 3.85 \times 10^{33}$ erg/s

Moon

Radius of the Moon's orbit (mean)	3.84×10^5 km
Mass of the Moon	$M_{\rm Moon} = 7.35 \times 10^{25}$ g $= M_\oplus/81.3$
Radius of the Moon	$R_{\rm Moon} = 1.74 \times 10^3$ km

Our Galaxy (The Milky Way)

Mass of the Milky Way in visible matter	$\approx 10^{11} M_\odot$
Radius of the luminous Milky Way disk	$\approx 20 - 25$ kpc
Luminosity of the Milky Way	$\approx 4 \times 10^{10}\, L_\odot$

Universe

Hubble Constant	$H_0 \approx (72 \pm 7)[(\text{km/s})/\text{Mpc}]$
	$h \equiv H_0/(100\,[(\text{km/s})/\text{Mpc}]) \approx .7 \pm .1$
Hubble Time	$t_H \equiv H_0^{-1} = 9.78 \times 10^9\, h^{-1}$ yr
Hubble Distance	$d_H \equiv cH_0^{-1} = 2998\, h^{-1}$ Mpc
Critical density	$\rho_c \equiv 3H_0^2/8\pi G = 1.88 \times 10^{-29}\, h^2$ g/cm^3
Temperature of CMB today	$= 2.73$K

Contents