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——为《十年校庆论文选编》的出版而作

在我校师生员工欢庆建校十周年的时候，我们编辑出版了《十年校庆论文选编》，它是我校教职员工开展教学、科研、管理服务的科学研究的产物；它是凝聚教职员工辛勤劳动的成果；它是教职员工的智慧结晶；它是群体进步的象征。我们把它作为一件礼物，奉献给师生员工，以表示学报编委会、编辑部对这一节日的祝贺！

学报创建于建校初期，它经历了创办、停刊、复刊、发展的历史过程。现在学报已由不定期到定期，又由半年刊到季刊。今年还筹办学报的社会科学版。这标志着学报是随着学校的发展而发展，随着教学、科研、管理服务水平的提高而健康成长。

现在世界科技发展已有很高的水平，石油化工已成为比较成熟的行业，形成了巨大的社会生产力。无论是国外，还是在我国，石油化工在国民经济发展中占有举足轻重的地位，在一定程度上左右着经济的发展。科学技术是第一生产力，石油化工的科技进步产生于石油化工的发展，而石油化工的发展又推动了科技进步。如：材料科学已能使人们设计分子，制造不同性能的新材料。生物技术开始了通过改变基因来制造新物种的时代，各个科技领域都有长足的进步。现在科学技术在迅速发展，被认为是人类历史上的第三次技术革命。以致出现了以科学技术命名我们的时代，如高分子时代，信息时代，空间时代等等，这是科技工作者的骄傲。据报道，本世纪以来的科技新发现，新发明已远远超过过去二千年的总和。

我校是中国石化总公司所属的一所高等学校，担负着为振兴石化工业，促进石化工业发展，培养人才的重任。当今，教育改革在不断深入，科学技术在迅速发展。作为高校教学、科研、管理服务的园地——学报，它将起到发表教学、科研、管理服务的科技成果，传递科技信息，进行学术交流，传播文化知识的作用。

建校十年是我校发展的十年。这十年中，在石化总公司领导下，辽化公司支持下，我校建设已初具规模，不仅有办专科的经验，也有办本科的条件，将形成专科和本科两个层次的石油化工高等学校的办学体系。我校有360余名从事教学科研的科技队伍，其中有250余名教师工作在教学第一线。今年我校科技开发、科研教育研究呈现了好势头，校内外科研立项二十余项，科研经费几十万元。这些为我们办好学报奠定了基础。科学研究工作者能够把研究完成果写成论文发表在学报上，是科学研究和技术开发、经济发展和科学技术进步的需要，是科学技术工作者自身成长的需要，又是提高教育质量，培养适应“四化”建设人才的需要。

学报这块沃土需要精心耕耘，需要支持、关心、爱护它。几年来，有的科技工作者每年为学报撰写论文四、五篇，甚至更多，成为“丰产”作者，这是它的生命力所在。

我们要深入学习党的十三届四中全会精神，坚持“一个中心，两个基本点”，在高教战线上，为祖国培养更多的符合“四化”需要的合格人才，努力工作。我们一切工作，从育人出发，从提高教育质量着眼，做好各项工作。办好学报，盛开“科技之花”。

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THE FUZZY NONSINGULAR MATRIX

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ABSTRACT

Definitions of the row rank, column rank and Schein rank of a fuzzy matrix are given in[1].

Many papers on fuzzy mathematics established the these problems (as [1]—[6]) and success fully given an algorithem for solving the row rank, the column rank and the Schein rank of any fuzzy matrix.

In this paper we put forward a concept of the fuzzy nonsingular matrix and discusse preliminaly its propertis.

Keywords: The fuzzy nonsingutar matrix, The fuzzy matrix of a full row rank, The fuzzy matrix of a full column rank.

I FUNDAMENTAL CONCEPTS

Let fuzzy matrices $A=(a_{ij})_{m\times n}$, $B=(b_{ij})_{m\times n}$ and $k\in[0,1]$. The sum of the two fuzzy matrices, the scalar product of a number and a fuzzy matrix, and the relation"$\leqslant$"of fuzzy matrices are defined respectively as follows:

$$A+B=(a_{ij}+b_{ij})_{m\times n}=(\max\{a_{ij},b_{ij}\})_{m\times n},$$

$$kA=(ka_{ij})_{m\times n}=(\min\{k,a_{ij}\})_{m\times n},$$

$A\leqslant B$ iff $\forall i,j,\ a_{ij}\leqslant b_{ij}$.

The product of two fuzzy matriees ($A=(a_{ij})_{m\times t}$ and $B=(b_{ij})_{t\times n}$) is defined as follows:

$$AB=(c_{ij})_{m\times n}=\left(\sum_{k=1}^{t} a_{ik}b_{kj}\right)_{m\times n}$$

Under the addition and scalar product the set of all n-ary fuzzy row (column) vectors forms a fuzzy semilinear space, denoted byV_n (V^n).

A vector set$\{X_1, \cdots\cdots, X_t\} \subseteq V_n$ $(V^n)$is independent if and only if there is not $X_i \in \{X_1, \cdots\cdots, X_t\}$ $(i=1, \cdots\cdots, t)$ such that it is represented as a linear combination of elements of $X_1, \cdots\cdots, X_{i-1}, X_{i+1} \cdots\cdots X_t$. If there is some $X_i \in \{X_1, \cdots\cdots, X_t\}$ $(1 \leqslant i \leqslant t)$ such that it is a liniear combination of elements of $X_1, \cdots\cdots, X_{i-1}, X_{i+1}, \cdots\cdots, X_t$, then set$\{X_1, \cdots\cdots, X_t\}$ it said to be dependent.

The set of linear combination of all column vectors of a $m \times n$ fuzzy matrix A is a subspace of V^n, denoted by C (A).

The rank ρ_c (A) of C (A) is number of vectors in minimum generating set of C (A), i. e. Din C (A) $=\rho_c$ (A).

Analogously define the row space R(A) and the row rank $\rho_r(A)$ of A.

A fuzzy matrix A is said to be of rank r if $\rho_r(A)=\rho_c(A)=r$, writed by $\rho(A)$.

The Schein rank $\rho_s(A)$ of a fuzzy matrix A is the least number of rank1 matrices shose sum is A.

II FUZZY NONSINGULAR MATRIX

Definition 2.1 A $m \times n$ fuuzzy matrix A is said to be nonsingular if $\rho_r(A)=m$ and $\rho_c(A)=n$.

Definition 2.2 A fuzzy permutation matrix is a square fuzzy matrix such that every row and every column contain exactly one 1, and other elements are 0.

Definition 2.3 A fuzzy square matrix B is said to be an inverse of a fuzzy square matrix A if $AB=BA=I$, where I is the unit matrix.

By definition for a fuzzy matrix of order $n \times n$, we have that:

Theorem 2.1 A fuzzy matrix A of order $n \times n$ is a nonsingular if and only if all rows of A are linear independent and all columns of A are also linear independent.

Theorem 2.2 A fuzzy matrix A of order $n \times n$ is a nonsingular if and only if $\rho_r(A)=\rho_c(A)=\rho(A)=n$.

Therefore by the theorem 3.2 in paper[7] we give that:

Theorem 2.3 A fuzzy martix A of order $n \times n$ is a nonsingular if and only if $\rho_s(A)=n$.

Proposition 2.1 Let A is a $n \times n$ fuzzy permutation matrix. And let B is an arbitrary $n \times n$ fuzzy matrix. Then $\rho_r(AB)=\rho_r(BA)=\rho_r(B)$, $\rho_c(AB)=\rho_c(BA)=\rho_c(B)$, $\rho_s(AB)=\rho_s(BA)=\rho_s(B)$. If a fuzzy matrix B has a rank, then $\rho(AB)=\rho(BA)=\rho(B)$.

Theorem 2.4 Let A is a pemutation matrix , then A′is an inveres of A.

Proof. Let

$$A=\begin{pmatrix} a_{11}\cdots a_{1n} \\ \cdots\ \cdots\ \cdots \\ a_{n1}\cdots a_{nn}\end{pmatrix},\quad AA'=D=\begin{pmatrix} d_{11}\cdots d_{1n} \\ \cdots\ \cdots\ \cdots \\ d_{n1}\cdots d_{nn}\end{pmatrix}.$$

Since every row of A constains exactly one 1, so that

$$d_{ii}=\sum_{k=1}^{n} a_{ik}\,a_{ik}=1,\quad (i=1,\ \cdots\cdots n).$$

Let in row i of A the t-th element is 1. If $i\neq j$, since in conlumn j of A′the t-th element is not 1. So that

$$d_{ij}=\sum_{k=1}^{n} a_{ik}a_{jk}=0\quad (i\neq j,\ i,\ j=1,\ \cdots\cdots,\ n).$$

Therefore $D=I$. Similarly may prove that $A'A=I$.

Therefore A′is an inverse of A.

Theorem 2.5 A fuzzy matrix A there is an inverse if and only if A is permutation matrix.

Proof. ⇐By the theorem 2.4 we have that

$A'A=AA'=I$.

Therefore A there is an inverse.

⇒If A has an inverse B, $AB=BA=I$. Suppose that A is not a permutation matrix.

(i) If some row of A not only contains one 1, but also contains one non-zero element. Without loss of generality, we let the row 1 of A is $(1,\ a,\ 0\cdots\cdots 0)$, $(0<a\leqslant 1)$. Since $AB=I$, thus the column 1 of B is $(1.\ 0,\ *\cdots*)'$, and the column 2,⋯,the columnn of B are $(0\ 0\ *\cdots*)$ i. e.

$$B=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ * & * & \cdots & * \\ \cdots & \cdots & \cdots & \cdots \\ * & * & \cdots & * \end{pmatrix}$$

So that

$$BA = \begin{pmatrix} * & \cdots & * \\ 0 & \cdots & 0 \\ * & \cdots & * \\ \cdots & \cdots & \cdots \\ * & \cdots & * \end{pmatrix} \neq I.$$

This is contradiction.

(ii) If some row of A not only contains one 1, but also contains non-zero elements, similarly may porve that there exist a contradicton.

(iii) If some row of A contains not 1, then for any B, $AB \neq I$.

Therefore every row of A contains exactly one 1, and other elements are zero.

Analogously may prove every column of A contains exactly 1, and other elements are zero.

Therefore A is a Permutation matrix.

By this theorem we see that a fuzzy matrix with there is an inverse, in fact, is a Boolean matrix.

In fuzzy mathematics "a nonsingular fuzzy sqnare matrix" "andfuzzy matrix with has inverse matrix" are not equivalent concept.

For example

$$A = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.3 \end{pmatrix}$$

is a nonsingular matrix, but it there is not an inverse matrix.

On the contrary we have that

Theorem 2.6 If a fuzzy square matrix there is an inverse matrix, the it is a nonsingular matrix.

In fact, a fuzzy square matrix A there is an inverse if and anly if A is a pemutation matrix. And every pemutation matrix is a full row rank and full column rank. Therefore A is a nonsingular matrix.

Theorem 2.7 Let A is a $n \times n$ fuzzy nonsingular matrix and B is a $n \times n$ fuzzy matrix. If $\min_{i,j}\{a_{ij}\} \geqslant \max_{i,j}\{b_{ij}\}$, then $\rho_r(AB) = \rho_r(BA) = \rho_r(B)$, $\rho_c(AB) = \rho_c(BA) = \rho_c(B)$, $\rho_s(AB) = \rho_s(BA) = \rho_s(B)$. If B has the rank, then $\rho(AB) = \rho(BA) = \rho(B)$.

Proof. Obvious.

Theorem 2.8 Let A is a $m \times n$ nonsingular fuzzy matrix, and let B is a

$n\times t$ fuzzy matrix. Then $\rho_c(AB)\leqslant\rho_c(B)$.

Proof. Let $\rho_c(B)=c$, and let it is $B=(B_1\cdots B_c\cdots B_t)$, Without loss of generality, we suppose $\{B_1\cdots, B_c\}$ is a basis of $C(A)$. Then $B_1\cdots B_t$ are the linear combinations of $B_1\cdots, B_c$. Let $B_i=\alpha_{i1}B_1+\cdots+\alpha_{ic}B_c$ $(i=1,\cdots, t)$

Thus
$$\begin{aligned}AB_i&=A(\alpha_{i1}B_1+\cdots+\alpha_{ic}B_c)\\&=\alpha_{i1}AB_1+\cdots+\alpha_{ic}AB_c\ (i=1,\cdots,t)\end{aligned}$$

That is $AB_1,\cdots, AB_t$ are the linear combinations of $AB_1,\cdots, AB_c$.

Therefore $\rho_c(AB)\leqslant c=\rho_c(B)$.

Notice that it is uncertain whether $\rho_c(AB)=\rho_c(B)$ or $\rho_c(AB)<\rho_c(B)$.

Example 2.1 Let

$$A=\begin{pmatrix}0.1&0&0\\0&0.2&0\\0&0&0.3\end{pmatrix},\quad B=\begin{pmatrix}1&0.9\\0.9&0.8\\0.8&0.7\end{pmatrix}.$$

Then

$$AB=\begin{pmatrix}0.1&0.1\\0.2&0.2\\0.3&0.3\end{pmatrix}.$$

Obvious A is a nonsingular fuzzy matrix and $1=\rho_c(AB)<2=\rho_c(B)$. If let

$$C=\begin{pmatrix}0.9&0&0\\0&0.8&0\\0&0&0.7\end{pmatrix},\quad D=\begin{pmatrix}0.1&0.2\\0.2&0.3\\0.3&0.4\end{pmatrix}.$$

Then

$$CD=\begin{pmatrix}0.1&0.2\\0.2&0.3\\0.3&0.4\end{pmatrix}.$$

Obvious C is a fuzzy nonsingular matrix and $\rho_c(CD)=\rho_c(D)=2$.

Corollary Let A is a $m\times n$ fuzzy nonsingular matrix, and let B is a $t\times m$ fuzzy matrix. Then $\rho_r(BA)\leqslant\rho_r(B)$.

Let a $m\times n$ fuzzy matrix is

$$A=\begin{pmatrix}X_1\\\vdots\\X_m\end{pmatrix}=(Y_1\cdots Y_n). \tag{2.1}$$

all rows of A are a linear independent if and only if there extist not $X_i \in \{X_1, \cdots X_m, \}$ $(i = 1, \cdots, m)$ such that it is repersented as a linear combination of elements of $\{X_1, \cdots, X_{i-1}, X_{i+1} \cdots, X_m\}$, if and only if equations

$$X_i = x_1X_1 + \cdots + x_{i-1}X_{i-1} + x_{i+1}X_{i+1} + \cdots + x_mX_m \qquad (i = 1, \cdots, m)$$

there are not a solution. If and only if fuzzy relational equations

$$(x_1 \cdots x_{i-1} x_{i+1} \cdots x_m) \begin{pmatrix} X_1 \\ \vdots \\ X_{i-1} \\ X_{i+1} \\ \cdots \\ X_m \end{pmatrix} = X_i \quad (i = 1, \cdots, m) \qquad (2.2)$$

there are not a solution.

Similarly all columns of A is a linear independent if and only if fuzzy ralational equations

$$(Y_1 \cdots Y_{j-1} Y_{j+1} \cdots Y_n) \begin{pmatrix} y_1 \\ \vdots \\ y_{j-1} \\ y_{j+1} \\ \vdots \\ y_n \end{pmatrix} = Y_j \quad (j = 1, \cdots, n) \qquad (2.3)$$

there are not a solution.

In summary we have

Theorem 2.9 Let as A as (2.1). A is a fuzzy nonsingular matrix if and only if fuzzy ralational equations (2.2) there are not a solution, and fuzzy ralational equations (2.3) there are also not a solution.

Example 2.2 Let

$$A = \begin{pmatrix} 1 & 0.9 & 0.8 \\ 0.9 & 0.8 & 0.7 \\ 0.8 & 0.7 & 0.6 \end{pmatrix}$$

Since

$$(x_2 x_3) \begin{pmatrix} 0.9 & 0.8 & 0.7 \\ 0.8 & 0.7 & 0.6 \end{pmatrix} = (1 \quad 0.9 \quad 0.8),$$

$$(x_1 x_3) \begin{pmatrix} 1 & 0.9 & 0.8 \\ 0.8 & 0.7 & 0.6 \end{pmatrix} = (0.9 \quad 0.8 \quad 0.7)$$

$$(x_1 x_2)\begin{pmatrix} 1 & 0.9 & 0.8 \\ 0.9 & 0.8 & 0.7 \end{pmatrix} = (0.8 \quad 0.7 \quad 0.6)$$

there are not a solution and since

$$\begin{pmatrix} 1 & 0.9 \\ 0.9 & 0.8 \\ 0.8 & 0.7 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.6 \end{pmatrix};$$

$$\begin{pmatrix} 1 & 0.8 \\ 0.9 & 0.7 \\ 0.8 & 0.6 \end{pmatrix}\begin{pmatrix} y_1 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0.9 \\ 0.8 \\ 0.7 \end{pmatrix};$$

$$\begin{pmatrix} 0.9 & 0.8 \\ 0.8 & 0.7 \\ 0.7 & 0.6 \end{pmatrix}\begin{pmatrix} y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.9 \\ 0.7 \end{pmatrix}$$

there are not solution.

Therefore A is a nonsingular fuzzy matrix.

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关于模糊相似优先比方法

王鸿绪　江佩荣

〔摘要〕本文简单介绍了现有的模糊相似优先比方法，进行了一些补充，简化了若干步骤．

关键词：不分明集；相似；优先排队方法

1　现有Fuzzy相似优先比应用方法简介

Fuzzy相似优先比（又作优选比）的方法，在一些实际问题中得到了成功的运用〔1〕〔2〕。我们把现有的方法简介如下：

1.1 确定相似因子

把欲研究的各因子，均称为相似因子。记

$$c=\{c_1\cdots,\ c_m\}$$

为相似因子集。其中c_i(i＝，…，m}为相似因子。

1.2 样本集

样本集设为

$$X=\{x_1,\ \cdots,x_n,x_0\}$$

其中x_0为固定样本，x_1，…，x_n为欲与x_0比较相似程度的样本。样本的数据见表1。

表1　数　据

样本 \ 数据 \ 因子	c_1	c_2	…	…	c_m
x_0	x_0^1	x_0^2	…	…	x_0^m
x_1	x_1^1	x_1^2	…	…	x_1^m
…	⋮	⋮	…	…	⋮
…	⋮	⋮	…	…	⋮
x_n	x_n^1	x_n^2	…	…	x_n^m

表1中x_i^j为样本x_i对第j个相似因子的统计数据。

1.3 绝对值距离

绝对值距离公式为

$$D_i^l = |x_0^l - x_i^l|, \quad (i=1, \cdots n) \qquad (1)$$

公式（1）表示就某个相似因子c_l $(l=1, \cdots m)$ 而言，样本x_i与固定样本x_0的绝对值距离。

1.4 建立Fuzzy相似优先比矩阵

对某一相似因子c_l $(l=1, \cdots, n)$，用公式（1）计算出所有D_i^l $(i=1, \cdots n,)$ 后，由公式

$$\left.\begin{aligned} & r_{ii}^l = 1 \text{（或0）}，\text{当} i=j \text{时} \\ & r_{ij}^l = \frac{D_j^l}{D_i^l + D_j^l}，\text{当} i \neq j \text{时} \\ & r_{ji}^l = 1 - r_{ij}^l，\text{当} i \neq j \text{时} \end{aligned}\right\} \qquad (2)$$

（注：公式（2）中的第三式可由第二式推出：

$$1 - r_{ij}^l = \frac{D_j^l + D_i^l}{D_i^l + D_j^l} - \frac{D_j^l}{D_i^l + D_j^l} = \frac{D_i^l}{D_i^l + D_j^l} = r_{ji}^l$$

但是，开列出第三式，可以简化求Fuzzy相似优先比矩阵的过程）得到 Fuzzy 相似优先比矩阵$R^l = (r_{ij}^l)_{n \times n}$。

1.5 求相似程度

在Fuzzy矩阵R^l中，由大而小地选定λ值，构造λ截矩阵。在λ下降的过程中，首先达到全行除主对角线上的元素外，均为1的那一行所属的样本为与固定样本x_0最相似的样本。按全行均为1出现的先后，可得到就因子C_l而言，各样本与固定样本相似程度的顺序号b_{il} $(i=1, 2, 3, \cdots, n)$，把b_{il}添入表2。

表2 相 似 程 度 表

样本 \ 相似程度 \ 因子	c_1	$\cdots$	c_m	$\sum_{j=1}^{m} b_{ij}$	总相似程度a_i
x_1	b_{11}	$\cdots$	b_{1m}	$\sum_{j=1}^{m} b_{ij}$	a_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
x_n	b_{n1}	$\cdots$	b_{nm}	$\sum_{j=1}^{m} b_{nj}$	a_n

1.6 求总相似程度

在表2中，把$\sum_{j=1}^{m} b_{ij}$ $(i=1, \cdots, n)$ 列中的数字由小到大排序，得到"总相似程度序号a_i"列，此列中的数字越小，则说明对应的样本与固定样本x_0越相似。

2 对Fuzzy相似优先比方法的若干补充与简化

在上述的Fuzzy相似优先比的应用方法中，尚有些具体问题需要讨论，计算太繁杂的步骤需要简化。本节将进行此项工作。

2.1 相似程度是1的一个约定

对于某相似因子c_l，若存在唯一的样本x_{l_0}（$1\le l_0\le n$），使$x^l_{l_0}=x^l_0$,则$D^l_{l_0}=0$, $D^l_i \neq 0$（$i\in\{1,\cdots,n\}$且$i\neq l_0$），从而

$$0<r^l_{ij}=\frac{D^l_j}{D^l_i+D^l_j}<1,\quad (i,\ j=1,\ \cdots,\ n\text{且}i\neq j,\ ij\neq lo) \tag{3}$$

$$r^l_{l_0j}=\frac{D^l_j}{D^l_{l_0}+D^l_j}=\frac{D^l_j}{D^l_j}=1,\quad (j=1\cdots n\text{且}j\neq lo)$$

$$r^l_{il_0}=1-r^l_{0j}=0,\ (j=1,\ \cdots,n\text{且}j\neq lo)$$

$$r^l_{ii}=1\qquad (i=1,\ \cdots,\ lo,\ \cdots,\ n)$$

故此时相似优先比矩阵为（4）．

$$R^1=\begin{array}{c}\\ x_1\\ x_2\\ \cdots\\ x_{l_0}\\ \cdots\\ x_{n-1}\\ x_n\end{array}\left(\begin{array}{cccccccc} x_1 & x_2 & \cdots & x_{l_0} & \cdots & x_{n-1} & x_n\\ 1 & r^l_{12} & \cdots & 0 & \cdots & r^l_{1n-1} & r^l_{1n}\\ x^l_{21} & 1 & \cdots & 0 & \cdots & r^l_{2n-1} & r^l_{2n}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 1 & 1 & \cdots & 1 & \cdots & 1 & 1\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ r^l_{n-11} & r^l_{x-12} & \cdots & 0 & \cdots & 1 & r^l_{n-1n}\\ r^l_{n1} & r^l_{n2} & \cdots & 0 & \cdots & r^l_{nn-1} & 1\end{array}\right) \tag{4}$$

显而易见，不必再求λ截矩阵，则R^1中第lo行已经首先达到全行（主对角线上的元素除外）均是1．即样本x_{l_0}与x_0最相似．由（3）知（4）中有且仅有一行（第l_0行）首先达到全行均是1，而其它行均不是全行为1（主对角线上元素除外）．即有

命题1 对于某相似因子c_l，若存在唯一的样本x_{l_0}（$1\le lo\le n$），使$x^l_{l_0}=x^l_0$，而其余样本有$x^l_i\neq x^l_0$($i\in\{1,\ \cdots,\ n\}$且$i\neq lo$)，则样本x_{l_0}与固定样本x_0的相似程度是l．

此命题告诉我们：在命题1的条件下，我们不必再把样本x_{l_0}参与计算而可直接得到x_{l_0}与固定样本的相似程度是l．然后只需讨论样本$x_1,\cdots,x_{l_0-1},x_n$，与固定样本x_0的相似程度即可．注意，再向下计算时，相似程度序号应从2开始．

对于某相似因子C_l，若存在互不相同的样本$x_{l_1},\cdots,x_{l_t}$（$l_1,\cdots,l_t\in\{1,\cdots n\}$），使得

$$x^l_{l_1}=x^l_{l_2}=\cdots=x^l_{l_t}=x^l_0 \tag{5}$$

其余样本设为$x_{l_{t+1}},\cdots,x_{l_n}$（$l_1,\ l_2,\ \cdots,\ l_t,l_{t+1}\cdots l_n$恰为$1,\cdots,n$的一个排列），且$x^l_i\neq x^l_0$（$i=l_{t+1},\cdots,\cdots l_n$），则

$$D^l_{l_1}=D^l_{l_2}=\cdots=D^l_{l_t}=0 \tag{6}$$

$$D^l_i=0\ \ (i=l_{t+1},\cdots,l_n) \tag{7}$$

此时，直接由公式（2）计算，就会出现$r^l_{ij}=\frac{0}{0}$的现象。例如，由（6）有

$$r^l_{i_1\ i_2}=\frac{D^l_{i_2}}{D^l_{i_1}+D^l_{i_2}}=\frac{0}{0}$$

对于这种情况，直接运用Fuzzy相似优先比的方法，就会出现障碍。

我们看样本$x_{11},x_{12},\cdots x_{1t}$。由（6）、（7）知此t个样本满足命题l的条件，故样本x_{11}与x_0的相似程度是1。同样可证样本x_{12}，…，x_{1t}与x_0的相似程度均是1．此即

命题2 对于某相似因子c_l，若存在互不相同的样本x_1，…，$(1\le l_1, \cdots, \le n)$，使（5）式成立，而其余样本均不等于$x_0$，则样本$x_{11},\cdots$，$x_{1t}$与固定样本$x_0$的相似程度是1．

所以在命题2的条件下，我们不必再把样本$x_{11},\cdots,x_{1t}$参与计算而直接得到这t个样本与固定样本的相似程度均是1．对其余样本则采用1.4的步骤进行相似程度分析，不过也要注意，再得到的相似程度序号也应从2开始。

2.2 （步骤1.5的简化求法）求相似程度的简化方法

在实际应用中我们会发现，要完成步骤1.5，需要计算出大量的λ截矩阵．（例如，文献〔1〕中表2，需要写出百余个λ截矩阵方可获得．）对如此大量的工作，能否简化运算呢？我们下面就来讨论这个问题。

我们仔细分析一下步骤5°不难得到下述命题：

命题3 把Fuzzy相似优先比矩阵$R^l=(r_{ij})^l_{n\times n}$的每一行(主对角线上元素除外)中的最小者，列在$R^l$相应行的右边，组成min列。在此列中数字最大的那行对应的样本与固定样本x_0最相似，在对应样本的下方记上相似序号1．若在min列中数字最大者有多个，则在对应样本的下方都记上相似序号1．然后去掉已标出相似序号的行和列，再用上述方法得到相似序号为2的样本。反复进行下去，即可得到全部样本与固定样本x_0的相似程度序号。

我们通过例题来熟悉这种简化方法。

例1 矩阵$R^{(i)}$如下：

$$\underset{\sim}{R}^{(1)}=\begin{array}{c|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \min \\ \hline 1 & 1 & 0.32 & 0.54 & 0.19 & 0.52 & 0.50 & 0.19 \\ 2 & 0.68 & 1 & 0.71 & 0.33 & 0.70 & 0.68 & 0.33 \\ 3 & 0.46 & 0.29 & 1 & 0.17 & 0.48 & 0.46 & 0.17 \\ 4 & 0.81 & 0.67 & 0.83 & 1 & 0.82 & 0.81 & \underline{0.67} \\ 5 & 0.48 & 0.30 & 0.52 & 0.18 & 1 & 0.48 & 0.18 \\ 6 & 0.50 & 0.32 & 0.54 & 0.19 & 0.52 & 1 & 0.19 \\ \hline b_i & & & & 1 & & & \end{array} \qquad (8)$$