QUARKS, LEPTONS & GAUGE FIELDS

2nd edition

夸克、轻子和规范场

第2版

KERSON HUANG

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For Kathryn Camille

PREFACE

According to the current view, the basic building blocks of matter are quarks and leptons, which interact with one another through the intermediaries of Yang-Mills gauge fields (gravity being ignored in this context). This means that the forms of the interactions are completely determined by the algebraic structure of certain internal symmetry groups. Thus, the strong interactions are associated with the group $SU(3)$, and is described by a gauge theory called quantum chromodynamics. The electro-weak interactions, as described by the now standard Weinberg-Salam model, is associated with the group $SU(2) \times U(1)$.

This book is a concise introduction to the physical motivation behind these ideas, and precise mathematical formulation thereof. The goal of the book is to explain why and how the mathematical formalism helps us to understand the relevant observed phenomena. The audience for which this book is written are graduate students in physics who have some knowledge of the experimental parts of particle physics, and an acquaintance with quantum field theory, including Feynman graphs and the notion of renormalization. This book might serve as a text for a one-semester course beyond quantum field theory. The first edition of this book, which came out in 1982, was based on a course I gave at M.I.T., and on lectures I gave in Santiago, Chile, in 1977, and in Beijing, China, in 1979. I am indebted to I. Saavedra for the opportunity to lecture in Chile, to Chang Wen-yu and S.C.C. Ting for the inducement to give the Beijing lecture, and to M. Jacob and K. K. Phua for the encouragement to bring out the first edition.

The main addition to the second edition are Wilson's approach to renormalization, lattice gauge theory, and quark confinement. I am grateful to the many readers who have pointed out errors in the first edition, which I hope have been corrected in this edition.

I owe special thanks to my colleagues at M.I.T., especially A. Guth, R. Jackiw, K. Johnson, and J. Polonyi, from whom I have learned much that is being passed along in this book.

Kerson Huang

Marblehead, Mass.
February 1991

CONTENTS

CHAPTER I

INTRODUCTION

1.1 Particles and Interactions

一尺之棰　日取其半　万世不竭

Take half from a foot-long stick each day;
you will not exhaust it in a million years.

The thought experiment contemplated in this proposition by an ancient Chinese sophist[1] is an apt allegory for what physicists actually do in the laboratory, in their search for the ultimate constituents of matter.

During the three centuries since the birth of physics in the modern sense, we have done about 60 days' worth of "halving" (down to 10^{-16} cm). At around day 30 (at 10^{-8} cm), we encountered the first granular structure of matter—atoms, which appeared at first to be indivisible. As we know, they turned out to be divisible further into electrons and nuclei; and nuclei could in turn be split into nucleons. Now we are at the stage when constituents of the nucleon—quarks—can be confidently identified. Indications are that the subdividing process will continue. The ancient sophist seems to be right so far.

From an experimental point of view, particles are detectable packets of energy and momentum, be they billiard balls, photons, or lambda hyperons. At each stage of our understanding, we designate certain particles as "fundamental", in the sense that they are the most elementary interacting units in our theories. As our experimental knowledge expands, we have often been forced to revise our views. The necessity for such revisions rests with the stringent requirement we place upon our theories: they must, in principle, be able to predict the quantitative results of all possible experiments.

It is fortunate that, at any given stage, we were able to regard certain particles as provisionally fundamental, without jeopardizing the right to change our mind. The reason is that, according to quantum mechanics, it is a good approximation to ignore those quantum states of a system whose excitation energies lie far above the energy range being studied. For example, a nucleus could be treated phenomenologically as a point mass at energies far below 1 MeV. We have discovered many layers of substructure since the era of atomic physics; but it is a remarkable fact that the dynamical principles learned from that era, as synthesized by relativistic local quantum field theory[2], continues to work up to the present stage.

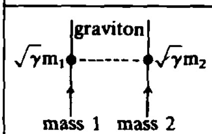
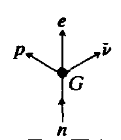
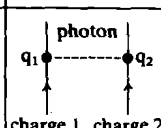
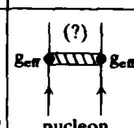
[1] Kungsun Lung (公孙龙), quoted in Chuang Chou, *Chuangtse* (ca. 300 B.C.), chapter 33. (莊子天下篇第三十三).
[2] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964); *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965); C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

Interactions among experimentally observed particles fall into four types of markedly different strengths: gravitational, weak, electromagnetic, and strong interactions. These are briefly reviewed in Table 1.1.

In the current theoretical view, which has come to be known as the "standard model," the weak and electromagnetic interactions are low-energy manifestations of a single unified interaction, and the strong interactions originate in a hidden charge called "color", carried by quarks permanently confined in nucleons and other strongly interacting particles. All these interactions are supposed to be mediated via the exchange of vector mesons with "minimal" coupling, similar to the well-known situation in electrodynamics. We are even in a position to speculate that all the above interactions are really low-energy manifestations of a single "grand unified" interaction, whose simplicity will be directly revealed in experiments only at energies above 10^{17} GeV! Unfortunately, nothing reliable can be said about the microscopic aspects of the gravitational interaction, due to a total lack of experimental information. Important as it may be in an eventual grand synthesis of all the interactions, we will have nothing to say about gravity in this book.

Basic to the theoretical classification of particles is the assumption that physical laws are invariant under Poincaré transformations, i.e., Lorentz transformations and space-time translations. A particle, be it "fundamental" or composite, is defined as a state of a quantum field that transforms under elements of the Poincaré group according to a definite irreducible representation. This implies that a particle has definite mass and spin, and that to each particle is associated an antiparticle of the same mass and spin[3]. The assumption

Table 1.1 THE FOUR TYPES OF INTERACTIONS

Interaction	Gravitational	Weak	Electromagnetic	Strong
Manifestation	Celestial mechanics	β-radioactivity	Everyday world	Nuclear binding
Quantum view				
Static potential	$-\dfrac{\gamma m_1 m_2}{r}$ r = distance between sources	—	$\dfrac{q_1 q_2}{4\pi r}$	$-g_{\text{eff}}\dfrac{e^{-\mu r}}{4\pi r}$ $\dfrac{\hbar}{\mu c} \sim 10^{-13}$ cm
Coupling strength	$\dfrac{\gamma m_p^2}{\hbar c} = 5.76 \times 10^{-36}$ m_p = proton mass	$Gm_p^2 = 1.01 \times 10^{-5}$	$\dfrac{e^2}{4\pi\hbar c} = \dfrac{1}{137.036}$ e = electron charge	$\dfrac{g^2}{4\pi\hbar c} \cong 10$

[3] E. P. Wigner, *Ann. Math.* **40**, 149 (1934).

of microcausality in local quantum field theory further implies a connection between spin and statistics: particles with integer spin are bosons, and those with half-integer spin are fermions[4]. The interactions among particles are required to be invariant under the Poincaré group; this imposes non-trivial conditions on possible local quantum field theories.[5]

In addition to Poincaré invariance, which is a space-time symmetry, there are also internal symmetries having to do with space-time-independent transformations of particle states. The invariance of interactions under internal symmetry groups gives rise to further quantum numbers that label particle states, such as electric charge, baryon number, isospin, etc.

A partial list of known particles, classified according to mass, spin, internal quantum numbers, and the types of interactions they have, is shown in Fig. 1.1.

"Hadrons" denote bosons and fermions having strong interactions, and "leptons" denote fermions without strong interactions[a]. Among the hadrons, "mesons" are bosons with baryon number 0, and "baryons" are fermions with baryon number different from 0[b]. Of all these particles (apart from the photon not shown in Fig. 1.1), only electrons and nucleons are relevant to our everyday experience. One might go a little further and include neutrinos as important catalysts for the generation of solar power, and μ mesons are free gifts from heaven[c]. Everything else is created primarily in high-energy accelerators.

Two striking features should be mentioned. First, all the leptons appear to be point-like particles, the latest experimental upperbound on their "radii" being 10^{-16} cm.[6] This is particularly remarkable for the τ, which is about twice as heavy as the proton. Secondly, there is a wild proliferation of hadrons. As noted by Hagedorn[7], a plot of the density of hadronic states against mass suggests an exponential growth, as shown in Fig. 1.2.

If this trend continues to asymptotically large masses, there would exist an "ultimate temperature" of about 160 MeV (2×10^{13} K), beyond which no system could be heated[8]. If the growth were faster than exponential, the partition function of statistical mechanics would not exist. Thus, the density of hadronic states seems to be growing at the maximum rate consistent with thermodynamics.

Even if we had not detected experimentally a finite radius for the proton (which we have, at about 10^{-13} cm)[9], the sheer number of the hadrons would make it absurd to suppose that they are all "fundamental". A key to the inner

[a] All observed bosons so far have strong interactions except the photon. Historically, leptons were so named because they were light; but this is no longer true with the discovery of the τ.

[b] The reason that all baryons are fermions, while all mesons are bosons, comes from baryon conservation in relativistic field theory, i.e., fermion fields must occur bilinearly in the Lagrangian, but bosons can occur linearly.

[c] "Who ordered them?" asked I. I. Rabi.

[4] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (W. A. Benjamin, New York, 1964).

[5] N. N. Bogolubov, G. G. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory* (W. A. Benjamin, Reading, Mass., 1975).

[6] D. P. Barber *et al.*, *Phys. Rev. Lett.* **43**, 1915 (1979).

[7] R. Hagedorn, *N. Cim.* **56A**, 1027 (1968).

[8] K. Huang and S. Weinberg, *Phys. Rev. Lett.* **25**, 895 (1970).

[9] R. Hofstadter and R. W. McAllister, *Phys. Rev.* **98**, 217 (1955).

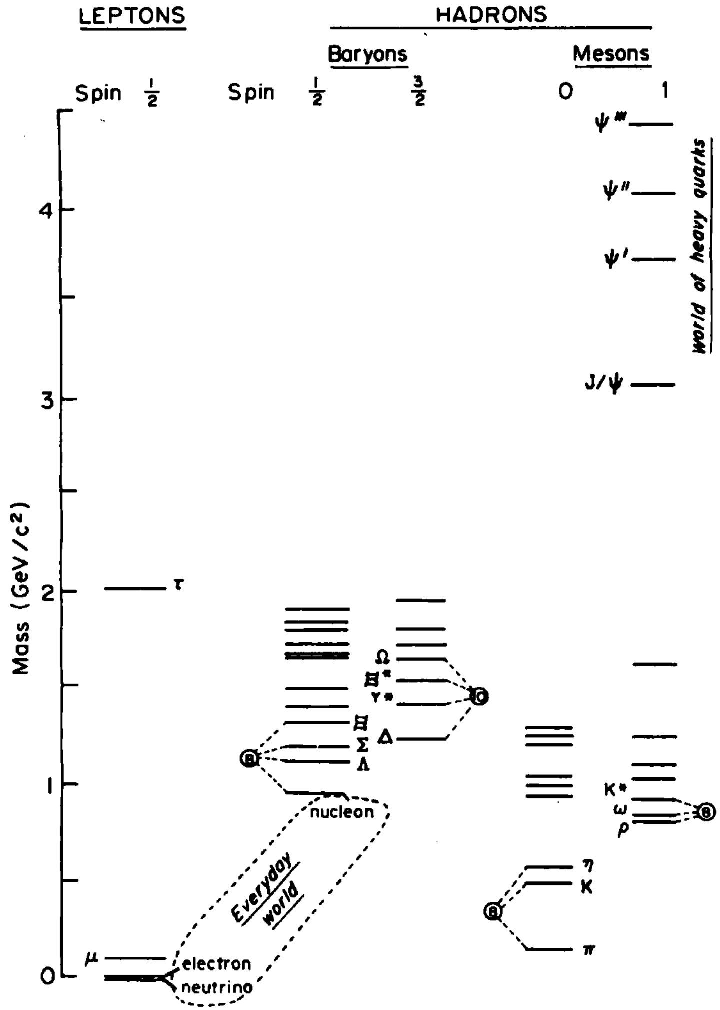


Fig. 1.1 Particle mass spectrum

structure of hadrons are the multiplet structures (e.g., **8** and **10** in Fig. 1.1) identifiable with irreducible representations of an internal symmetry group $SU(3)$. This is the first lead to the notion of quarks as hadronic constituents, namely, they form a fundamental representation of $SU(3)$. A more detailed discussion of the evidence for quarks and their interactions will be given in Chapter 2.

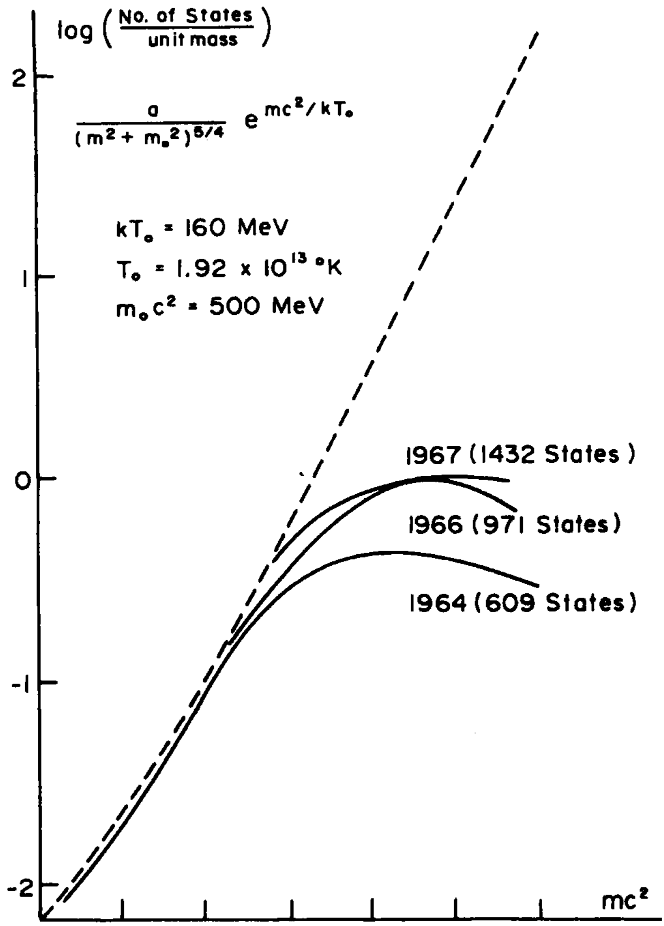


Fig. 1.2 Number of hadronic states as function of mass

1.2 Gauge Theories of Interactions

In the standard model, all the interactions are derived from a "gauge principle" similar to that in electromagnetism. We recall that the coupling of the electro-magnetic field A^μ to a charged matter field ψ can be derived through the following prescription: replace $\partial^\mu\psi$ in the matter Lagrangian by the covariant derivative $(\partial^\mu + ieA^\mu)\psi$, where e is the electric charge of ψ. Before we "turn on" the coupling (i.e., for $e = 0$), the matter Lagrangian must be invariant under constant phase changes of ψ, called "global gauge transformations". What the prescription does is to enlarge this symmetry to a "local gauge invariance", i.e., invariance under arbitrary space-time dependent phase changes of ψ (correlated with corresponding gauge transformations of A^μ)[d]. The original global gauge invariance implies the existence of a conserved matter current j^μ, and the prescription leads to an interaction of the form $ej^\mu A_\mu$, in conformity with Maxwell's theory. Under the usual assumptions of canonical field theory, the prescription is unique, and is called the "gauge principle".

We may restate the gauge principle as follows. Consider a matter system originally invariant under a global $U(1)$ group of gauge transformations. We "gauge" this symmetry, i.e., enlarge it to a local $U(1)$ gauge invariance. This means that an independent $U(1)$ gauge group shall be associated with each space-time point. To do this it is necessary to introduce a vector gauge field, to which the matter field current becomes coupled. The coupling constant is the electric charge, the generator of $U(1)$. The original global symmetry can be gauged only if it is an exact symmetry.

We shall use a generalized gauge principle formulated by Yang and Mills[10], which applies to a multicomponent matter field. Instead of $U(1)$, the gauge group is now a larger group of transformations that mix the different components of the matter field. There will now be more than one gauge field—the Yang-Mills fields. Their number is equal to the number of generators of the gauge group. The relevant group for the weak, electromagnetic and strong interactions is $SU(2) \times U(1) \times SU(3)$. To define this group, we must first describe the matter fields.

A well-known characteristic of the weak interactions is that they violate parity conservation to a maximal degree[11] by virtue of the V-A coupling[12]. That is, only left-handed components of the leptons are coupled in the charge-changing sector; the right-handed components play a rather passive role—to provide mass. Similarly, hadronic weak interactions can be accounted for by assuming that quarks have the same kind of weak couplings. Thus, to the weak interactions, the elementary entities are states of definite chirality[e], which have

[d] H. Weyl, *Ann. d. Physik*, **59**, 101 (1919), first introduced the term "gauge transformation" in an interesting but unsuccessful attempt to unify electromagnetism with gravity in a geometric theory, by extending the non-integrability of the direction of a vector in curved space-time to a non-integrability of its length (gauge) in an extended space called "gauge space".

[e] Chirality is defined as the eigenvalue of γ_5, with $\gamma_5 = 1$ corresponding to right-handedness, and $\gamma_5 = -1$ to left-handedness.

[10] C. N. Yang and R. L. Mills, *Phys. Rev.* **96**, 191 (1954).

[11] T. D. Lee and C. N. Yang, *Phys. Rev.* **104**, 254 (1956); C .S. Wu *et al.*, *Phys. Rev.*, **105**, 1413 (1957).

[12] R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958).

zero mass. (An eigenstate of finite mass is a superposition of left and right-handed states with equal weight). Glashow[13] first proposed a unified gauge theory of electroweak interactions based on a gauge group $SU(2) \times U(1)$, which mixes different massless chiral states. However, the fact that physical particles have finite masses seems to violate this symmetry. The seeming impasse was overcome by Weinberg[14] and Salam[15] by appealing to the notion of "spontaneous symmetry breaking". In the now-standard Weinberg-Salam model, "Higgs fields" are introduced to implement this idea, though they may be phenomenological parameters to be replaced by something more basic in a future theory. It is fair to say that at present we have no deep understanding of where masses come from.

The symmetries to be gauged refer to transformations among massless quarks and leptons of definite chirality. They come in at least six "flavors" (the sixth one being not yet experimentally confirmed). The lepton flavors are (e, ν), (μ, ν'), (τ, ν''), where the ν's denote massless left-handed neutrinos. The quark flavors bear a one-to-one correspondence to the above: (u, d), (s, c), (t, b). The parentheses group the particles into three families, which are indistinguishable copies as far as the weak interactions are concerned[f]. In addition, each quark flavor comes in three (and only three) "colors", while leptons have no color. Thus, the elementary particles are

$$\text{quarks:}\quad q_{fn}\begin{cases}(f = 1, \ldots, 6) & \text{(flavor index)}\\ (n = 1, 2, 3) & \text{(color index)}\end{cases}$$
$$\text{leptons:}\quad l_f \quad (f = 1, \ldots, 6)\quad \text{(flavor index)}$$

It is understood that, for example, q_{fn} denotes collectively $(q_R)_{fn}$ and $(q_L)_{fn}$, the right and left-handed components respectively, each regarded as an independent particle.

We list the quarks and leptons more explicitly in Table 1.2, and postulate the following internal symmetries:

(a) **Color $SU(3)$:** With respect to the color index, the three quarks of each flavor form a triplet representation of a "color group" $SU(3)$. The leptons are color singlets[g].

(b) **Weak isospin $SU(2)$:** In each family, the left-handed components of the upper and lower particles (e.g., ν_L and e_L) form a doublet representation of a "weak isospin group" $SU(2)$. All right-handed particles are $SU(2)$ singlets.

(c) **Weak hypercharge $U(1)$:** There is a $U(1)$ symmetry, called "weak hypercharge", associated with simultaneous phase changes of each particle. The relative phases are fixed by definite "weak hypercharge" assignments.

The gauge group is then $SU(2) \times U(1) \times SU(3)$, a direct product of the three mutually commuting groups defined above. Gauging this group necessitates the

[f] Rabi's question on p. 3 can be generalized, but remains unanswered.
[g] This means that the theory is invariant under the group in question, and that the particles transform under the group according to the representations specified.

[13] S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961).
[14] S. Weinberg, *Phys. Rev. Lett.* **19** 1264 (1967).
[15] A. Salam, in *Elementary Particle Theory*, ed. N. Svarthholm (Almquist and Wiksell, Stockholm, 1968).