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HEAT KERNELS AND DIRAC OPERATORS

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Preface

This book, which began as a seminar in 1985 at MIT, contains complete proofs of the local index theorem for Dirac operators using the heat kernel approach, together with its generalizations to equivariant Dirac operators and families of Dirac operators, as well as background material on superconnections and equivariant differential forms.

Since the publication of the first edition, the subjects treated here have continued to find new applications. Equivariant cohomology plays an important role in the study of symplectic reduction, and Bismut superconnections and the local index theorem for families have had many applications, through the construction of higher analytic torsion forms and currents. (For a survey of some of these developments, we recommend reading Bismut's talk at the Berlin International Congress of Mathematicians, reference [33].)

Although this book lacks some of the usual attributes of a textbook (such as exercises), it has been widely used in advanced courses in differential geometry; for many of the topics discussed here, there are no other treatments available in monograph form. Because of the continuing demand from students for the book, we were very pleased when our editor Catriona Byrne at Springer Verlag proposed reissuing it in the series "Grundlehren Text Editions." The proofs in this book remain among the simplest available, and we have decided to retain them without any change in the new edition.

We have not attempted to give a definitive bibliography of this very large subject, but have only tried to draw attention to the articles that have influenced us.

We would like to take the opportunity to thank the other participants in the MIT seminar, especially Martin Andler and Varghese Mathai, for their spirited participation. Discussions with many other people have been important to us, among whom we would like to single out Jean-Michel Bismut, Dan Freed and Dan Quillen. Finally, we are pleased to be able to thank all of those people who read all or part of the book as it developed and who made many comments which were crucial in improving the book, both mathematically and stylistically, especially Jean-François Burnol, Michel Duflo, Sylvie Paycha, Christophe Soulé, and Shlomo Sternberg. We also thank the referee for suggestions which have improved the exposition.

To all of the following institutes and funds, we would like to express our gratitude: the Centre for Mathematical Analysis of the ANU, the ENS-Paris, the Harvard Society of Fellows, the IHES, MIT, and the Université de Paris-Sud. We also received some assistance from the CNRS, the NSF, and the Sloan Foundation.

We wish to thank readers of the first edition who were kind enough to send us corrections: any remaining errors are of course our own.

Paris and Chicago,
September, 2003

N. Berline,
E. Getzler,
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Contents

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Introduction

Dirac operators on Riemannian manifolds, which were introduced in the articles of Atiyah and Singer [13] and Lichnerowicz [81], are of fundamental importance in differential geometry: they occur in situations such as Hodge theory, gauge theory, and geometric quantization, to name just a few examples. Most first-order linear differential operators of geometric origin are Dirac operators.

After Atiyah and Singer's fundamental work on the index for general elliptic operators, methods based on the heat kernel were applied to prove the Atiyah-Singer Index Theorem in the special case of Dirac operators, by Patodi [90], Gilkey [65] and Atiyah-Bott-Patodi [8]. In recent years, new insights into the local index theorem of Patodi and Gilkey have emerged, which have simplified the proofs of their results, and permitted the extension of the local index theorem to other situations. Thus, we felt it worthwhile to write a book in which the Atiyah-Singer Index Theorem for Dirac operators on compact Riemannian manifolds and its more recent generalizations would receive elementary proofs. Many of the theorems which we discuss are due to J.-M. Bismut, although we have replaced his use of probability theory by classical asymptotic expansion methods.

Our book is based on a simple principle, which we learned from D. Quillen: Dirac operators are a quantization of the theory of connections, and the supertrace of the heat kernel of the square of a Dirac operator is the quantization of the Chern character of the corresponding connection. From this point of view, the index theorem for Dirac operators is a statement about the relationship between the heat kernel of the square of a Dirac operator and the Chern character of the associated connection. This relationship holds at the level of differential forms and not just in cohomology, and leads us to think of index theory and heat kernels as a quantization of Chern-Weil theory.

Following the approach suggested by Atiyah-Bott and McKean-Singer, and pursued by Patodi and Gilkey, the main technique used in the book is an explicit geometric construction of the kernel of the heat operator $e^{-t\mathsf{D}^2}$ associated to the square of a Dirac operator D. The importance of the heat kernel is that it interpolates between the identity operator, when $t = 0$, and the projection onto the kernel of the Dirac operator D, when $t = \infty$. However, we will study the heat kernel, and more particularly its restriction to the diagonal, in its own right, and not only as a tool in understanding the kernel of D.

Lastly, we attempt to express all of our constructions in such a way that they generalize easily to the equivariant setting, in which a compact Lie group G acts on the manifold and leaves the Dirac operator invariant.

We will consider the most general type of Dirac operators, associated to a Clifford module over a manifold, to avoid restricting ourselves to manifolds with spin structures. We will also work within Quillen's theory of superconnections, since this is conceptually simple, and is needed for the formulation of the local family index theorem of Bismut in Chapters 9 and 10.

We will now give a rapid account of some of the main results discussed in our book. Dirac operators on a compact Riemannian manifold M are closely related to the Clifford algebra bundle. The Clifford algebra $C_x(M)$ at the point $x \in M$ is the associative complex algebra generated by cotangent vectors $\alpha \in T_x^*M$ with relations

$$\alpha_1 \cdot \alpha_2 + \alpha_2 \cdot \alpha_1 = -2(\alpha_1, \alpha_2),$$

where (α_1, α_2) is the Riemannian metric on T_x^*M. If e_i is an orthonormal basis of T_xM with dual basis e^i, then this amounts to saying that $C_x(M)$ is generated by elements c^i subject to the relations

$$(c^i)^2 = -1, \text{ and } c^i c^j + c^j c^i = 0 \text{ for } i \neq j.$$

The Clifford algebra $C_x(M)$ is a deformation of the exterior algebra ΛT_x^*M, and there is a canonical bijection $\sigma_x : C_x(M) \to \Lambda T_x^*M$, the symbol map, defined by the formula

$$\sigma_x(c^{i_1} \dots c^{i_j}) = e^{i_1} \wedge \dots \wedge e^{i_j}.$$

The inverse of this map is denoted by $\mathbf{c}_x : \Lambda T_x^*M \to C_x(M)$.

Let $\mathscr{E}$ be a complex $\mathbb{Z}_2$-graded bundle on M, that is, $\mathscr{E} = \mathscr{E}^+ \oplus \mathscr{E}^-$. We say that $\mathscr{E}$ is a bundle of Clifford modules, or just a Clifford module, if there is a bundle map $c : T^*M \to \operatorname{End}(\mathscr{E})$ such that

1. $c(\alpha_1)c(\alpha_2) + c(\alpha_2)c(\alpha_1) = -2(\alpha_1, \alpha_2)$, and
2. $c(\alpha)$ swaps the bundles $\mathscr{E}^+$ and $\mathscr{E}^-$.

That is, $\mathscr{E}_x$ is a $\mathbb{Z}_2$-graded module for the algebra $C_x(M)$. If M is even-dimensional, the Clifford algebra $C_x(M)$ is simple, and we obtain the decomposition

$$\operatorname{End}(\mathscr{E}) \cong C(M) \otimes \operatorname{End}_{C(M)}(\mathscr{E}).$$

From now on, most of our considerations only apply to even-dimensional oriented manifolds. If M is a spin manifold, that is, a Riemannian manifold satisfying a certain topological condition, there is a Clifford module $\mathscr{S}$, known as the spinor bundle, such that $\operatorname{End}(\mathscr{S}) \cong C(M)$. On such a manifold, any Clifford module may be written as a twisted spinor bundle $\mathscr{W} \otimes \mathscr{S}$, with $\mathscr{W} = \operatorname{Hom}_{C(M)}(\mathscr{S}, \mathscr{E})$. Let $\Gamma_M \in \Gamma(M, \mathbb{C}(M))$ be the chirality operator in $C(M)$, given by the formula

$$\Gamma_M = i^{\dim(M)/2} c^1 \dots c^n,$$

so that $\Gamma_M^2 = 1$.

If $\mathscr{E}$ is a vector bundle on M, let $\Gamma(M,\mathscr{E})$ be the space of smooth sections of $\mathscr{E}$, and let $\mathscr{A}(M,\mathscr{E}) = \Gamma(M,\Lambda T^*M \otimes \mathscr{E})$ be the space of differential forms on M with values in $\mathscr{E}$. We make the obvious, but crucial, remark that if $\mathscr{E}$ is a Clifford module, then by the symbol map, the space of sections

$$\Gamma(M,\mathrm{End}(\mathscr{E})) \cong \Gamma(M, C(M) \otimes \mathrm{End}_{C(M)}(\mathscr{E}))$$

is isomorphic to the space of bundle-valued differential forms

$$\mathscr{A}(M,\mathrm{End}_{C(M)}(\mathscr{E})) \cong \Gamma(M,\Lambda T^*M \otimes \mathrm{End}_{C(M)}(\mathscr{E})).$$

Thus, a section $k \in \Gamma(M,\mathrm{End}(\mathscr{E}))$ corresponds to a differential form $\sigma(k)$ with values in $\mathrm{End}_{C(M)}(\mathscr{E})$. If M is a spin manifold and $\mathscr{E} = \mathscr{W} \otimes \mathscr{S}$ is a twisted spinor bundle, $\sigma(k)$ is a differential form with values in $\mathrm{End}(\mathscr{W})$.

A Clifford connection on a Clifford module $\mathscr{E}$ is a connection $\nabla^{\mathscr{E}}$ on $\mathscr{E}$ satisfying the formula

$$\left[\nabla^{\mathscr{E}}_X, c(\alpha)\right] = c\left(\nabla_X \alpha\right),$$

where α is a one-form on M, X is a vector field, and $\nabla_X \alpha$ is the Levi-Civita derivative of α.

The Dirac operator D associated to the Clifford connection $\nabla^{\mathscr{E}}$ is the composition of arrows

$$\Gamma(M,\mathscr{E}) \xrightarrow{\nabla^{\mathscr{E}}} \Gamma(M,T^*M \otimes \mathscr{E}) \xrightarrow{c} \Gamma(M,\mathscr{E}).$$

With respect to a local frame e^i of T^*M, D may be written

$$\mathsf{D} = \sum_i c^i \nabla^{\mathscr{E}}_{e_i}.$$

A number of classical first-order elliptic differential operators are Dirac operators associated to a Clifford connection. Let us list three examples:

1. The exterior bundle ΛT^*M is a Clifford bundle with Clifford action by the one-fom $\alpha \in \Gamma(M,T^*M)$ defined by the formula

$$c(\alpha) = \varepsilon(\alpha) - \varepsilon(\alpha)^*;$$

here $\varepsilon(\alpha) : \Gamma(M,\Lambda^\bullet T^*M) \to \Gamma(M,\Lambda^{\bullet+1} T^*M)$ is the operation of exterior multiplication by α. The Levi-Civita connection on ΛT^*M is a Clifford connection. The associated Dirac operator is $d + d^*$, where d is the exterior differential operator. The kernel of this operator is just the space of harmonic forms on M, which by Hodge's Theorem is isomorphic to the de Rham cohomology $H^\bullet(M)$ of M.
2. If M is a complex manifold and $\mathscr{W}$ is a Hermitian bundle over M, the bundle $\Lambda(T^{0,1}M)^* \otimes \mathscr{W}$ is a Clifford module, with $\alpha = \alpha_{1,0} + \alpha_{0,1} \in \mathscr{A}^{1,0}(M) \oplus \mathscr{A}^{0,1}(M)$ acting by

$$c(\alpha) = \sqrt{2}\big(\varepsilon(\alpha_{0,1}) - \varepsilon(\overline{\alpha_{1,0}})^*\big).$$

The Levi-Civita connection on ΛT^*M preserves $\Lambda(T^{0,1}M)^*$ and defines a Clifford connection if M is Kähler, and the Dirac operator associated to this connection is $\sqrt{2}(\bar\partial + \bar\partial^*)$. If $\mathscr{W}$ is a holomorphic vector bundle with its canonical connection, the tensor product of this connection with the Levi-Civita connection on $\Lambda(T^{0,1}M)^*$ is a Clifford connection with associated Dirac operator $\sqrt{2}(\bar\partial + \bar\partial^*)$. The kernel of this operator is the space of harmonic forms on M lying in $\mathscr{A}^{0,\bullet}(M,\mathscr{W})$, which by Dolbeault's theorem is isomorphic to the sheaf cohomology $H^\bullet(M,\mathscr{W})$.

3. If M is a spin manifold, its spinor bundle $\mathscr{S}$ is a Clifford module, the Levi-Civita connection is a Clifford connection, and the associated Dirac operator is known simply as **the** Dirac operator. Its kernel is the space of harmonic spinors.

Thus, we see from these examples that the kernel of a Dirac operator often has a topological, or at least geometric, significance.

The heat kernel $\langle x \mid e^{-t\mathsf{D}^2} \mid y\rangle \in \operatorname{Hom}(\mathscr{E}_y, \mathscr{E}_x)$ of the square of the Dirac operator D is the kernel of the heat semigroup $e^{-t\mathsf{D}^2}$, that is,

$$(e^{-t\mathsf{D}^2}s)(x) = \int_M \langle x \mid e^{-t\mathsf{D}^2} \mid y\rangle s(y)\,|dy| \quad \text{for all } s \in \Gamma(M,\mathscr{E}),$$

where $|dy|$ is the Riemannian measure on M. The following properties of the heat kernel are proved in Chapter 2:

1. it is smooth;
2. the fact that smooth kernels are trace-class, from which we see that the kernel of D is finite-dimensional;
3. the uniform convergence of $\langle x \mid e^{-t\mathsf{D}^2} \mid y\rangle$ to the kernel of the projection onto $\ker(\mathsf{D})$ as $t \to \infty$;
4. the existence of an asymptotic expansion for $\langle x \mid e^{-t\mathsf{D}^2} \mid y\rangle$ at small t, (where $\dim(M) = n = 2\ell$),

$$\langle x \mid e^{-t\Delta^2} \mid y\rangle \sim (4\pi t)^{-\ell} e^{-d(x,y)^2/4t} \sum_{i=0}^{\infty} t^i f_i(x,y),$$

where f_i is a sequence of smooth kernels for the bundle $\mathscr{E}$ given by local functions of the curvature of $\nabla^{\mathscr{E}}$ and the Riemannian curvature of M, and $d(x,y)$ is the geodesic distance between x and y.

Note that the restriction to the diagonal $x \mapsto \langle x \mid e^{-t\mathsf{D}^2} \mid x\rangle$ is a section of $\operatorname{End}(\mathscr{E})$. The central object of our study will be the behaviour at small time of the differential form

$$\sigma(\langle x \mid e^{-t\mathsf{D}^2} \mid x\rangle) \in \mathscr{A}(M, \operatorname{End}_{C(M)}(\mathscr{E})),$$

obtained by taking the image under the symbol map σ of $\langle x \mid e^{-t\mathsf{D}^2} \mid x\rangle$.

Let us describe the differential forms which enter in the study of the asymptotic expansion of $\langle x \mid e^{-t\mathsf{D}^2} \mid x \rangle$. If $\mathfrak{g}$ is a unimodular Lie algebra, let $j_{\mathfrak{g}}(X)$ be the analytic function on $\mathfrak{g}$ defined by the formula

$$j_{\mathfrak{g}}(X) = \det\left(\frac{\sinh(\operatorname{ad}X/2)}{\operatorname{ad}X/2}\right);$$

it is the Jacobian of the exponential map $\exp : \mathfrak{g} \to G$. We define $j_{\mathfrak{g}}^{1/2}$ in a neighbourhood of $0 \in \mathfrak{g}$ to be the square-root of $j_{\mathfrak{g}}$ such that $j_{\mathfrak{g}}^{1/2}(0) = 1$.

Let $R \in \mathscr{A}^2(M, \mathfrak{so}(TM))$ be the Riemannian curvature of M. Choose a local orthonormal frame e_i of TM, and consider the matrix R with two-form coefficients,

$$\mathsf{R}_{ij} = (Re_i, e_j) \in \mathscr{A}^2(M).$$

Then $\left(\frac{\sinh(\mathsf{R}/2)}{\mathsf{R}/2}\right)$ is a matrix with even degree differential form coefficients, and since the determinant is invariant under conjugation by invertible matrices,

$$j(\mathsf{R}) = \det\left(\frac{\sinh(\mathsf{R}/2)}{\mathsf{R}/2}\right)$$

is an element of $\mathscr{A}(M)$ independent of the frame of TM used in its definition. Note that the zero-form component of $j(\mathsf{R})$ equals 1. Thus, we can define the $\hat{A}$-genus $\hat{A}(M)$ of the manifold M by

$$\hat{A}(M) = j(\mathsf{R})^{-1/2} = \det{}^{1/2}\left(\frac{R/2}{\sinh(R/2)}\right) \in \mathscr{A}(M);$$

it is a closed differential form whose cohomology class is independent of the metric on M. It is a fascinating puzzle that the function $j_{\mathfrak{g}}^{-1/2}$ occurs in a basic formula of representation theory for Lie groups, the Kirillov character formula, while its cousin, the $\hat{A}$-genus, plays a similar role in a basic formula of differential geometry, the index theorem for Dirac operators. Understanding the relationship between these two objects is one of the aims of this book.

Note that our normalization of the $\hat{A}$-genus, and of other characteristic classes, is not the same as that preferred by topologists, who multiply the $2k$-degree component by $(-2\pi i)^k$, so that it lies in $H^{2k}(M, \mathbb{Q})$. We prefer to leave out these powers of $-2\pi i$, since it is in this form that they will arise in the proof of the local index theorem.

Let $\mathscr{E}$ be a Clifford module on M, with Clifford connection $\mathscr{E}$ and curvature $F^{\mathscr{E}}$. The twisting curvature $F^{\mathscr{E}/S}$ of $\mathscr{E}$ is defined by the formula

$$F^{\mathscr{E}/S} = F^{\mathscr{E}} - R^{\mathscr{E}} \in \mathscr{A}^2(M, \operatorname{End}_{C(M)}(\mathscr{E})),$$

where

$$R^{\mathscr{E}}(e_i, e_j) = \tfrac{1}{2}\sum_{k<l}(R(e_i, e_j)e_k, e_l)c^k c^l.$$

If M is a spin manifold with spinor bundle $\mathscr{S}$ and $\mathscr{E} = \mathscr{W} \otimes \mathscr{S}$, $F^{\mathscr{E}/S}$ is the curvature of the bundle $\mathscr{W}$.

For $a \in \Gamma(M, C(M)) \cong \Gamma(M, \Lambda T^*M)$, we denote the k-form component of $\sigma(a)$ by $\sigma_k(a)$.

The first four chapters lead up to the proof of the following theorem, which calculates the leading order term, in a certain sense, of the heat kernel of a Dirac operator restricted to the diagonal.

Theorem A. *Consider the asymptotic expansion of $\langle x \mid e^{-t\mathsf{D}^2} \mid x \rangle$ at small times t,*

$$\langle x \mid e^{-t\mathsf{D}^2} \mid x \rangle \sim (4\pi t)^{-\ell} \sum_{i=0}^{\infty} t^i k_i(x)$$

with coefficients $k_i \in \Gamma(M, C(M) \otimes \mathrm{End}_{C(M)}(\mathscr{E}))$. Then

1. *$\sigma_j(k_i) = 0$ for $j > 2i$*
2. *Let $\sigma(k) = \sum_{i=0}^{\ell} \sigma_{2i}(k_i) \in \mathscr{A}(M, \mathrm{End}_{C(M)}(\mathscr{E}))$. Then*

$$\sigma(k) = \det{}^{1/2}\Big(\frac{R/2}{\sinh(R/2)}\Big) \exp(-F^{\mathscr{E}/S}).$$

In Chapter 4, we give a proof of Theorem A which relies on an approximation of D^2 by a harmonic oscillator, which is easily derived from Lichnerowicz's formula for the square of the Dirac operator, and properties of the normal coordinate system.

Since the zero-form piece of the $\hat{A}$-genus equals 1, we recover Weyl's formula

$$\lim_{t \to 0} (4\pi t)^{\ell} \langle x \mid e^{-t\mathsf{D}^2} \mid x \rangle = \mathrm{rk}(\mathscr{E});$$

in this sense, Theorem A is a refinement of Weyl's formula for the square of a Dirac operator.

Define the index of D to be the integer

$$\mathrm{ind}(\mathsf{D}) = \dim(\ker(\mathsf{D}^+)) - \dim(\ker(\mathsf{D}^-)),$$

where $\mathsf{D}^{\pm}$ is the restriction of D to $\Gamma(M, \mathscr{E}^{\pm})$. For example, $\mathrm{ind}(d + d^*)$ is the Euler characteristic

$$\mathrm{Eul}(M) = \sum_{i=0}^{n} (-1)^i \dim(H^i(M))$$

of the manifold M, while $\mathrm{ind}(\bar{\partial} + \bar{\partial}^*)$ is the Euler characteristic of the sheaf of holomorphic sections

$$\mathrm{Eul}(M, \mathscr{W}) = \sum_{i=0}^{\ell} (-1)^i \dim(H^i(M, \mathscr{W})).$$

These indexes are particular cases of the Atiyah-Singer Index Theorem, and are given by well-known formulas, respectively the Gauss-Bonnet-Chern theorem and the Riemann-Roch-Hirzebruch theorem; we will show in Section 4.1 how these formulas follow from Theorem A.

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