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Finite Size Effects in Correlated Electron Models: Exact Results

关联电子模型中的有限尺度效应——精确结果

（影印版）

〔乌克兰〕兹维亚金（A. A. Zvyagin）著

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序　言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础，同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天，物理学不仅囿于基础科学和技术应用研究的范畴，而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到，改革开放三十多年来，随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展，我国物理学取得了跨越式的进步，做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下，近年来物理学相关书籍也呈现百花齐放的良好态势，在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看，尽管国内各出版社相继推出了一些质量很高的物理教材和图书，但系统总结物理学各门类知识和发展，深入浅出地介绍其与现代科学技术之间的渊源，并针对不同层次的读者提供有价值的教材和研究参考，仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展，特别是展现近年来中国物理学者的研究水平和成果，北京大学出版社在国家出版基金的支持下推出了《中外物理学精品书系》，试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家，确保了整套丛书的权威性和前瞻性。

这套书系内容丰富，涵盖面广，可读性强，其中既有对我国传统物理学发展的梳理和总结，也有对正在蓬勃发展的物理学前沿的全面展示；既引进和介绍了世界物理学研究的发展动态，也面向国际主流领域传播中国物理的优秀专著。可以说，《中外物理学精品书系》力图完整呈现近现代世界和中国物理

科学发展的全貌，是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

《中外物理学精品书系》另一个突出特点是，在把西方物理的精华要义“请进来”的同时，也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻，引进和翻译世界物理的经典著作和前沿动态，可以满足当前国内物理教学和科研工作的迫切需求。另一方面，改革开放几十年来，我国的物理学研究取得了长足发展，一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域，使世界对中国物理学的过去和现状有更多的深入了解，不仅充分展示出中国物理学研究和积累的“硬实力”，也向世界主动传播我国科技文化领域不断创新的“软实力”，对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是，《中外物理学精品书系》还对中国近现代物理学科的经典著作进行了全面收录。20 世纪以来，中国物理界诞生了很多经典作品，但当时大都分散出版，如今很多代表性的作品已经淹没在浩瀚的图书海洋中，读者们对这些论著也都是“只闻其声，未见其真”。该书系的编者们在这方面下了很大工夫，对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值，不仅可以很好地保护和传承我国物理学的经典文献，充分发挥其应有的传世育人的作用，更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统，真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在 2006 年中国科学技术大会上指出，“加强基础研究是提升国家创新能力、积累智力资本的重要途径，是我国跻身世界科技强国的必要条件”。中国的发展在于创新，而基础研究正是一切创新的根本和源泉。我相信，这套《中外物理学精品书系》的出版，不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣，也将进一步推动其他相关基础科学更好更快地发展，为我国今后的科技创新和社会进步做出应有的贡献。

《中外物理学精品书系》编委会　主任

中国科学院院士，北京大学教授

王恩哥

2010 年 5 月于燕园

FINITE SIZE EFFECTS IN CORRELATED ELECTRON MODELS

EXACT RESULTS

Andrei A. Zvyagin
National Academy of Sciences, Ukraine

ICP

Imperial College Press

To My Family

Preface

Low-dimensional correlated electron systems are a fascinating topic in condensed matter physics which started in the 1930s, developed in the 1960s and received great attention from physicists during the last decade. Research into low-dimensional magnetism occupied an important part of this field, because it originated from electron correlations due to Coulomb interaction. In the last twenty years the problems of low-dimensional correlated electron systems has turned from a narrow, special topic of mathematical physics into one of the central problems of condensed matter physics. This became true due to the great progress in the miniaturization of technology, and many great findings in the composition of new materials like metal-oxides, carbon nanotubes, organic compounds, optical lattices for ultra-cold quantum gases, *etc.*, in most of which the scale of research is micro- and nano-physics. New mesoscopic and nano-devices are based on quantum dots, wires, rings in which the low-dimensionality and quantum nature are basic features. This, in turn, resulted in the creation and development of powerful theoretical and mathematical approaches, like Bethe's ansatz, bosonization and conformal field theory. These approaches are not only important to the pure quantum many-particle theory, but also happen to be extremely useful in a number of areas related to this theory. The reason for such an application is that certain important phenomena, like high-T_c superconductivity, physics of magnetic impurities, *etc.*, cannot seem to be explained in the framework of weak couplings, *i.e.*, perturbative theory, or mean-field-like approach. One-dimensional exact solutions provide a complete and unambiguous picture of correlated electron systems and play a role in the basis to further applications of perturbative methods.

The aim of the book is to present an introduction to recent achievements in theoretical studies of exactly integrable low-dimensional models of strongly correlated electrons and spin models. The central topic of this book is finite size effects in lattice exactly solvable spin and electron models. However, this book is not a review. A great number of papers were published since the 1920s till now on exact solutions in one-dimensional quantum models and it would be completely hopeless to discuss and even mention them all. (I would like to use this opportunity to sincerely apologise to those authors whose important contributions are not mentioned here.) Unfortunately, it is impossible to write about the many very important aspects of the Bethe's ansatz, the main subject of this book, like its purely mathematical developments, exact solutions of field theory models, continuous models of interacting electrons, models of systems with more than one internal degree of freedom (*e.g.*, orbital moments of electrons), systems with lower symmetries, models with long-range interactions, multi-chain quantum models, magnetic and hybridization impurities in three-dimensional metals, *etc.*, though they are related to the topic of the book. Some effects, like the behaviour of elementary excitations in quantum correlated electron and spin chains are also not presented here. I can only refer the interested reader to some excellent monographs, review articles, collections of reprints, like [Baxter (1982); Gaudin (1983); Andrei, Furuya and Lowenstein (1983); Tsvelick and Wiegmann (1983); Schlottmann (1989); Izyumov and Skryabin (1990); Jimbo (1990); Korepin, Bogoliubov and Izergin (1993); Schlottmann and Sacramento (1993); Korepin and Eßler (1994); Ha (1996); Schlottmann (1997); Takahashi (1999)].

The structure of this book is as follows. After a short introduction to statistical mechanics and thermodynamics, I remind the reader of some important facts about thermodynamics of quantum spins and free electrons in crystals in Chapter 1. Very important Mermin–Wagner and Hohenberg theorems are also presented in this chapter, to explain to the reader the importance of exact studies in low-dimensional quantum systems. In Chapter 2 several exact results of one-dimensional theory of quantum spins are presented. Those theories are relatively simple, but their knowledge permits us to understand the deeper nature of homogeneous quantum spin chains. Chapter 3 is devoted to the description of the main aim of this book — the Bethe's ansatz in its most known form, the co-ordinate Bethe ansatz. The development of this method for models of correlated electrons, the nested Bethe ansatz, is presented in Chapter 4. Chapter 5 explains features of the elegant algebraic Bethe ansatz, or the quantum inverse scattering method to the reader. Hence, the reader experienced in Bethe ansatz is, probably, aware of those studies. The main results of the book are presented in Chapters 6, 7 and 8. Chapter 6 describes the difference in thermodynamic behaviours of bulk particles with those, situated at edges (boundaries) of open chains. Similar effects of isolated impurities are presented in Chapter 7. The reader can see how great the difference in behaviours of host electrons (spins), and "surface" or impure ones is. Very often effects for homogeneous hosts and "distinguished" sites are qualitatively different, and they have to be taken into account when one interprets data of experiments in low-dimensional electron systems. I present results for various hosts and impurities, from the simplest ones, to the more complicated. Chapter 8 gives the description for thermodynamic behaviour of finite concentration of impurities in correlated electron and quantum spin hosts. To the best of my knowledge, Bethe ansatz solvable models are the *only* example, where it is possible to obtain exact thermodynamic characteristics for correlated electron and spin systems with *ensembles* of impurities, *e.g.*, to investigate the interplay between correlation effects and disorder exactly. The important method of modern Bethe ansatz thermodynamics, the quantum transfer matrix approach, is also presented in that chapter. In Chapter 9 other finite size effects are described. For example, recent experiments drew attention to studies of quantum topological effects, like persistent currents in quantum rings with or without embedded quantum dots. Another aspect of similar finite size effects is the possibility to extract from them the information about the asymptotic behaviour of correlation functions, using the conformal field theory approach. This is why, in Chapter 9 a short introduction to the scaling theory and conformal field theory is given. Finally, in Chapter 10 I discuss which methods can be used beyond exact ones. Here some short descriptions of scaling theory of quantum phase transitions and bosonization are given. However, all these theories cannot be presented in detail, and I refer the reader to several excellent books and review articles like [Ma (1976); Sólyom (1979); Cardy (1996); Di Franchesco, Mathieu and Sénéshal (1997); Sondhi, Girvin, Carini and Shahar (1997); Gogolin, Nersesyan and Tsvelik (1998); van Delft and Schoeller (1998); Nagaosa (1998); Sachdev (1999); Kadanoff (2000)]. I hope that those readers, who are familiar with exact solutions, will find some new interesting facts about finite size effects in one-dimensional quantum spin and electron systems, while the book can serve as an introduction for beginners to introduce them to the beautiful world of exact solutions.

My understanding of quantum low-dimensional magnetic systems and correlated metallic systems has grown over many years. It is a great pleasure for me to thank the many friends and colleagues who contributed to it. First of all I would like to thank my father, who introduced me to the physics of low-dimensional systems, and V. M. Tsukernik, whose deep knowledge of quantum physics of magnetism supported me so much. I especially want to thank my long-term co-authors Pedro Schlottmann, Andreas Klümper, Henrik Johannesson, and Holger Frahm for stimulating interaction, helpful suggestions and essential support. I have benefited from and appreciate interesting discussions with I. Affleck, B. L. Altshuler, N. Andrei, R. Z. Bariev, B. Douçot, U. Eckern, F. H. L. Eßler, P. Fulde, T. Giamarchi, L. I. Glazman, F. D. M. Haldane, A. G. Izergin, Y. S. Kivshar, V. E. Korepin, I. V. Krive, V. E. Kravtsov, H.-J. Mikeska, P. Nozières, A. A. Ovchinnikov, L. A. Pastur, N. M. Plakida, A. S. Rozhavsky, E. Runge, F. Steglich, P. Thalmeier, A. M. Tsvelik, P. B. Wiegmann, Yu Lu, and J. Zittartz. Finally, I am very grateful for all the valuable support as well as suggestions from my wife, Natasha.

A. A. Zvyagin

Contents

Chapter 1

Introduction

In this chapter we shall remind the reader of some basic ideas of thermodynamics and statistical physics of interacting electron and spin systems. We shall show how thermal fluctuations destroy long-range order in low-dimensional quantum interacting systems at any nonzero temperature if only short-range interactions are present.

1.1 Why is the Topic of the Book Worthwhile Studying?

Low-dimensional electron systems (insulating magnets and conductors) have been an active topic of scientific research long before their experimental realization in organic conductors, polymers, Peierls insulators and nanoscale and mesoscopic systems, *e.g.*, quantum wires and edge states of the fractional quantum Hall effect devices. There are several principal differences between one space dimension and higher dimensions, most of which can be traced back to the reduced phase space in one dimension. Key properties distinguishing one-dimensional systems used to be connected with thermal fluctuations destroying long-range order at any nonzero temperature if only short-range interactions are present and quantum fluctuations tending to suppress a broken continuous symmetry, the *spin-charge separation* (the charge and spin content of wave functions of electrons move with different speeds), the breakdown of the *Fermi liquid* description, *i.e.*, absence of Fermi liquid quasiparticle pole in the Green's function (it becomes a *marginal Fermi liquid* or *Tomonaga–Luttinger liquid* with collective excitations due to global *conformal symmetry*), and the localization of electrons with even a small amount of disorder.

During recent years the interest in the strongly correlated electron and quantum spin systems has grown considerably. Usually low-lying electron-hole excitations of three-dimensional metals are successfully described within the phenomenological Landau's Fermi liquid theory. A Fermi liquid is the Fermi sphere and a gas of weakly interacting between each other *quasiparticles* defined *via* poles in one-particle Green's functions. Quasiparticles continuously evolve from free fermions when the interaction is adiabatically switched on. This is why, they have the same sets of quantum numbers and statistics as noninteracting electrons. In one space dimension the residue of the Fermi liquid quasiparticle pole vanishes and it is replaced by incoherent collective excitations, which follow from the global conformal symmetry. These excitations involve non-universal power-law singularities, which, in turn, determine the asymptotic behaviour of low-energy correlation functions. Although the Fermi surface is still properly defined, the discontinuity (jump) of the momentum distribution at the Fermi surface disappears, due to the above mentioned singularities. Systems displaying such breakdown of the Fermi liquid picture and exotic low-energy spectral properties are known as Tomonaga–Luttinger liquids.

On the other hand, in a number of recent experiments on the low-temperature behaviour of the rare-earth compounds and alloys, which are essentially three-dimensional, the *non Fermi liquid* character of the behaviour of the specific heat, magnetic susceptibility and (magneto) resistivity has also been observed during last couple of decades. It was pointed out recently that these features of the *non Fermi liquid* characteristics can be explained using the concept of the disordered behaviour of magnetic impurities in such systems. Superconductivity and antiferromagnetism in low dimensions has regained interest with the discovery of high-T_c superconductors and new heavy fermion superconductors. Very anisotropic magnetic and transport properties of the former arise primarily from the CuO planes there. Many of normal state properties of the two-dimensional high-T_c superconductors are very different from normal metals and cannot be reconciled with a standard Fermi liquid theory. A marginal Fermi liquid picture, similar to the one of one-dimensional electron systems, has been proposed to explain some of these features. Models of stripe-like effectively one-dimensional structures were proposed to explain some essential properties of high-T_c cuprates and heavy fermion Kondo lattices. The one-dimensional Kondo lattice model, realization of which is often considered as realistic model for heavy fermion materials, is still poorly understood. One can say that the finite concentration of magnetic impurities and random distribution of their characteristics (Kondo temperatures) will give rise to frustration, spin gap, non Fermi liquid critical behaviour and possible additional magnetic phase transitions (similar to metamagnetic ones). Possible implications of one-dimensional strongly correlated electron systems other than high-T_c superconductors could be new metal-oxides with ladder structure, and the edge states of the fractional quantum Hall effect, heavy fermion systems, *etc.* Ladder spin or correlated electron structures are nontrivial examples of quantum systems with the properties of both one- and two-dimensional models.

A substantial level of understanding of one-dimensional quantum correlated electron and/or spin systems has been reached over the past years. The exact solution with the help of the *Bethe's ansatz* of numerous models together with field-theoretical studies have provided deep insight into the ground state of systems, the complete classification of states, thermodynamic properties, and an asymptotic behaviour of correlation functions. Within the Bethe ansatz method the eigenfunctions and eigenvalues of the stationary Schrödinger equation are parametrized by a set of parameters known as *rapidities.* A system with internal degrees of freedom (such as a spin) requires a sequence of additional, nested Bethe ansatz for the wave function. In fact, each internal degree of freedom gives rise to one set of rapidities. Independently of the symmetry of the wave function and spin, eigenstates are occupied according to the Fermi-Dirac statistics, because hard-core particles are considered. Usually, in the ground state (and at low temperatures) each internal degree of freedom contributes with one Fermi (Dirac) sea. Fermi velocities of these Fermi seas are, generally speaking, different from each other, giving rise to the effect of charge and spin separation.

Over the past decade finite size effects in one-dimensional systems have been of considerable interest. The finite size of a system (*e.g.*, an electron conductor or a magnetic chain) manifests itself in several ways. First, impurities are important in low-dimensional quantum systems, since their contribution to extensive quantities (*e.g.*, the energy, charge and magnetic susceptibilities, resistance and specific heat) can become relatively large and observable. Boundaries and edges of open chains behave as some special sort of impurities with many similarities and differences in their behaviours. Second, the finite length of a mesoscopic (nanoscale) quantum chain (wire) or ring gives rise to quantum topological effects, *i.e.*, to *persistent currents* with oscillation periods given by interferences of the Aharonov–Bohm and Aharonov–Casher type. Finally, finite size corrections to the energy of a one-dimensional system determine critical exponents of the asymptotic dependence at large distances and long times of correlation functions via conformal field theory.