

研究生前沿教材书系

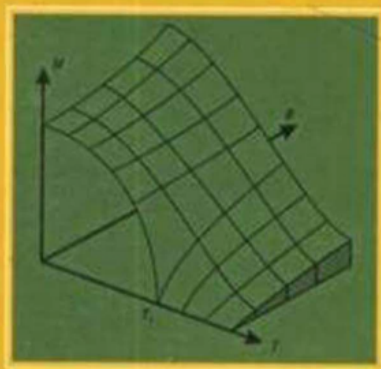
Topics in Statistical Mechanics

统计力学论题

(英文影印版)

Brian Cowan

(Royal Holloway, University of London, UK)




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内 容 简 介

伦敦地区的几所大学,在硕士研究生的最后一年,都要联合起来,通过网络教育的方式,给硕士生讲授几门统一的高级课程,本书就是其中的教程之一。本门教程在成书之前,作者已经系统地讲授了十多年,成书过程中又组织学生、同行和由出版社委派的专家一道,对书稿提出许多建议,然后再修改而成现在这个样子。

全书用一种统一的观点处理热力学和统计物理论题。第一、第二章分别讲述统计力学的方法论和理想体系的实际计算。其中差不多有一半内容属于本科期间已有的基础知识,但采用更高的、完全用统一的观点,看待热力学和统计力学。第三章非理想气体,重点讲述维里展开、配分函数、节流和状态方程。第四章相变,介绍相图、对称性、序参量、临界指数、标度理论、一级相变、二级相变、伊辛模型、朗道理论、铁电体、二元混合物、量子相变、平均场理论等等。这是全书的重点。第五章讲述涨落和动力学行为,重点是涨落的关联特性、布朗运动、朗之万方程和线性响应理论。各章末尾都安排一定数量的习题,习题解答可通过 <http://www.worldscibooks.com/physics/p365.htm/> 取得。

书后还有 4 个附录,便于读者应用时查取。

序 言

David Goodstein……

路德维希·玻尔兹曼 1906 年自杀身亡,他毕生的大部分时光都在从事统计力学的研究工作。继续这项工作的保罗·厄仑菲斯同样于 1933 年自杀身亡。现在轮到我们来研究统计力学了。

也许小心谨慎地接近这一学科才是明智的。

写于 *States of Matter*, 1975, 纽约 Dover

统计力学与物理学的任何其他分支学科相比,更易受到方法论和表述问题的困扰。哲学家对概率的含义争论不休,尤其是对单一“事件”应用概率时尤甚。数学家则用回避物理解释的方法躲过了这一问题,他们只是简单地把概率作为由一系列规则所限定的“测量”。不过以这种方式脱离实际就不太适用于物理学了。对物理学家而言,概率和统计方法一直使他们非常苦恼。统计方法是造成玻尔兹曼自杀的因素之一,保罗·厄仑菲斯的自杀也可能出于这个原因。即使到了今天,量子力学中猜不透的谜也在于核心部位的概率在作祟。

统计力学中,数学家的操作方法是同 E. T. Jaynts True 的信息论方法相似的,这种方法经过几代杰出教育家的拼搏,已近完美,不过我承认这种方法并没有特别吸引人之处。当然这种方法或许只是得到结果的一种权宜之计,但是就我内心深处而言,还只是模糊的理解而已。真正理解是物理学家殚思竭虑的目标。相比之下, T. L. Hill 等学者的系综形式不但非常清晰而且物理含义明确。有人或许发现系综太形式化了,我却认为它十分迷人。

第二个结构问题是统计力学与热力学这门较老的学科的关系

问题。它们应当各自独立地发展呢还是用一种统一的办法来处理更好呢？朗道坚决支持后一种看法，我本人几乎被他的论据所征服。在大学本科阶段，这一哲学观点得益于 Reif 的重要阐释（他在伯克利物理丛书的《统计物理》一书中同样有此类解释）。这种观点很可能对当今职业物理学家中很多人的知识结构产生过重大的影响。这也必定为研究朗道和栗弗席茨的书奠定了理想的基础。但不管怎么说，经典热力学的重要性是毋庸置疑的。最重要的特征是模型与其结果无关。热力学的抽象形式多少有些令人生畏，的确，它的深层逻辑体系确实有些模糊不清。从 Zemansky 多次再版的热力学教科书中可以看到，热力学的逻辑体系逐步地转为 Carathéodory 的观点，那段时间里，经典热力学的逻辑结构已经相当清晰并经过 Callen 的条理化处理，但对大学本科水平的学生来说，要深入理解这种逻辑结构恐怕是不恰当的。

撇开深刻的哲学问题，接下来的另一个突出的问题，是要重新解决必然性的问题。在大多数英国大学里，本科生的课程压力越来越大，简直没有可能再插入完备的经典热力学和统计力学的课程。的确，伦敦的 Royal Holloway 大学就是这种情况。因此，把对四年级本科生的导介性材料同硕士生的学位课程（科学硕士或物理学硕士）合并在一起，就有利于把许多更加高级的素材纳入到学位课程的计划之中。

伦敦大学，King's, Queen Mary, Royal Holloway, 以及 University Colleges 一起，对科学硕士四年级课程组织联合教学，讲座地点就设在伦敦市中心。这当中的课程门类很多，许多课程都属于边缘学科。这样一来，就有可能更多地包括那些从传统的本科四年级学士学位课程中被排挤出来的素材。

本书就是一门适用于校际教学的统计力学课程，我在过去的十多年时间里，一直给硕士四年级学生讲授这门课。这种校际教学有其自身的挑战性。读完三年硕士课程之后，他们已经达到一个相似的标准和水平，都能理解第四年校际教学的课程了，尽管各所大学的侧重点和偏好会有所不同。热物理领域尤其属于这样的情形，其

原因正如前面几段文字所阐明的那样。正因为这一原因,这本教程中提供给学生的学习素材包括了很多“基本”的内容,当然表述的起点更高了。第一、第二章的一半左右内容,属于这种“基本”素材。

这本教程原先是一门“无纸教程”,当然学生可通过网站得到讲稿要点和其他学习资料。资料中包括讲课日程表、练习题等等,全部学生都可以在各自学校里,甚至在其他任何很远的地方下载这些资料。当我收到来自伦敦之外,甚至世界各地的学生要求索取习题解答(常常已到了截止期)并回答讲课要点上的疑问时,我简直处于四面楚歌的应急状态之中,当校园网的服务中断时更是如此,有时我发现有些 email 甚至气愤地责问,你们究竟把资料搁到哪里了?所以当帝国学院出版社(Imperial College Press)找到我,想考虑为这门教程出一本书时,我欣然答应了。这本书是集体成果,是来自学生、同事和帝国学院出版社委派专家的努力、听取建议、反复修改的结晶。

电磁学单位制是使学生在学习物理课时产生许多困难的原因。采用 SI 单位制之后,这类困难估计可以消除了,但阻力仍然存在。Kittel 的《固体物理》一书,有不少式子既有用高斯制的,也有用 SI 制的,似乎两种单位制混用也行。我几乎划一地全用 SI 制,尽管我对因此而引起 $B-H$ 的争论并不满意。不过读者会注意到,我用 M 表示总磁矩而不是单位体积的磁矩,我也承认将复动力学磁化率写成 $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$,的确不同于更常见的复共轭形式。

我应当对许多人表示谢意,我的妻子 Louise,我的女儿 Abigail,她们对于沉醉于学术工作的丈夫和父亲已习以为常了,因为我经常不在她们身边,如果不是事实至少在精神上如此。我的老师 Michael Richards 和 Bill Mullin 鼓励我献身于物理学这一分支的工作。他们千方百计地为我钻研朗道和栗弗席茨的著作提供帮助。我也被他们著作的明晰和深刻所折服。在我早期的教学生涯中,我从 Roland Dobbs 和 Mike Hoare 那里学了很多,再后来, Bob Jones 在回应我的模糊不清的提问时,一而再、再而三地阐释了他的百科全书

式的统计力学知识。他耐心细致的方法常常有助于通过难题的讨论而建立起深厚的师生感情。此外,John Saunders 始终是我的忘年之交,他持续不断地以各种方式鼓励我。除此之外,我还要对许多学生表示感谢,他们使我在教学上不断长进。他们的观察、质疑,甚至是反对,都有助于我在论证中消除混淆不清,把自己的观点解释得更透彻。不管怎么说,上面这些感谢并不推卸我在教学上的责任。差错、疏忽和过失,责任完全由我来承担。

我以本书作为对父亲 Stanley Cowan 的纪念,他于 1997 年 2 月故世。他是一位天才的工程师,是一个特别有耐心的人。他忍受着疾病的痛苦,尽量克制而不让亲人感受到他的痛苦。他坚信在求解问题中数学的威力,他使我也遵奉了这一信条。他鼓励我多问为什么,虽然我们之间有时也会有激烈的争论,但他使我始终保持着一一种开放的心态。

Brian Cowan
2004 年 12 月

Brian Cowan

物理学教授, 伦敦大学皇家 Holloway 学院物理系系主任。毕业于英国 Sussex 大学物理系, 曾先后就职于诺丁汉 (Nottingham) 大学和巴黎 (Paris) 大学, 致力于核磁共振 (NMR) 的理论和实验研究, 著有 *Nuclear Magnetic Resonance and Relaxation* (Cambridge University Press, 1997) 等著作。

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THE METHODOLOGY OF STATISTICAL MECHANICS

This chapter provides an overview of statistical mechanics and thermodynamics. Although the discussion is reasonably self-contained, it is assumed that this is not the reader's first exposure to these subjects. More than many other branches of physics, these topics are treated in a variety of different ways in the textbooks in common usage. The aim of the chapter is thus to establish the perspective of the book, the standpoint from which the topics in the following chapters will be viewed. The textbook by Bowley¹ is recommended to the novice as a clearly-argued introduction to the subject; indeed all readers will find many of the examples of this book treated by that book in a complementary fashion.

1.1. Terminology and Methodology

1.1.1. *Approaches to the subject*

Thermodynamics is the study of the relationship between *macroscopic* properties of systems such as temperature, volume, pressure, magnetisation, compressibility, etc. *Statistical Mechanics* is concerned with understanding how the various macroscopic properties arise as a consequence of the *microscopic* nature of the system. In essence it makes macroscopic deductions from microscopic models.

The power of thermodynamics, as formulated in the traditional manner (e.g. Zemansky²) is that its deductions are quite general; they do not rely, for their validity, on the microscopic nature of the system. Einstein