

线性 代数 基础

XIAN XING
DAI SHU

〔美〕 J·索普 P·佩尔合著
钱辉镜 杨宗仁 等译

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译者的话

由于线性代数课程的基本概念较为抽象,其理论具有较强的逻辑性而使不少学生望而生畏,被认为是一门难度较大的课程.对于业余自学者来说,学习本门课程的困难是不言而喻的.

本书作者在美国著名的纽约布鲁克港大学任教,具有丰富的教学经验和卓越的教学艺术.他们写作本书的目的就是要使具备高中数学基础的学生能够自学线性代数基础内容.本书内容安排由浅入深,循序渐进,力求在学生打下坚实基础后,再引入较抽象和困难的概念,以便于学生理解和吸收.鉴于线性代数课程具有广泛的实用性,本书在每一章内都列有应用实例使学生开阔眼界.

参加本书翻译工作的有钱辉镜(第一章)、杨宗仁(第二章)、周燕(第三章、前言、习题答案)、孙美春(第四章)、沈桂芝(第五章)、冯泰(第六章、附录).

在本书出版过程中,北京邮电学院数学系王玉孝副教授对全书作了认真的审稿工作,对此我们表示衷心的感谢.

由于我们翻译水平有限,难免有错,敬请广大读者批评指正.

译者

1988年3月

前 言

本教科书是大学线性代数的初级教程，凡是掌握中学代数、几何与初等三角的学生均能理解本书的内容。

在本书的第一章里，首先学习用高斯消元法求解线性方程组，这是学习线性代数的一个自然起点，它可以把学生已经学过的中学代数知识平稳地过渡到线性代数。从线性方程组的解，我们自然地引出 R^n 中向量的概念以及向量空间 R^n 的结构。在学过 R^n 中的代数运算以后，我们就着手学习 R^2 中的向量几何，然后是 R^n 的向量几何问题。我们用向量方程来表示 R^n 中的直线；随后，作为一种求两个向量之间夹角的方法引进内积的概念，并用内积的概念来表示 R^2 中的直线、 R^3 中的平面和 R^n 中的超平面。在 R^3 中作为一种求与两个已知向量垂直的向量的方法，我们引入了叉积的概念。我们还阐述了叉积的性质，并利用叉积来求平行六面体和四面体的体积，证明毕达哥拉斯定理对于 R^3 中四面体的推广。第一章最后以“应用”结束：介绍求解 R^2 中的线性规划问题的顶点法。

第二章介绍由矩阵所确定的线性变换的概念。我们从平面到自身的线性变换着手讨论，用大量浅显的例子着重说明它们的几何意义。然后将讨论转移到较为普遍的情况：从 R^n 到 R^m 的线性变换。我们还要讨论线性变换和矩阵的代数运算，利用矩阵和线性变换之间的对应关系引出矩阵乘法，矩阵乘法是对应于线性变换的乘积的矩阵运算。接着，再讨论逆变换和逆矩阵。最后，以对于马尔科夫过程的应用结束。

第三章讨论行列式。我们先以通常的公式定义2阶矩阵，然后考察行变换对2阶行列式的作用，再用这些性质定义一个 n 阶矩阵的行列式， n 为任意数。这样做可使学生很快便学到一种较优的计算行列式的算法，而不必在一开始就去研究那些广泛的理论。随后我们再证明，还可以用子式的展开式或是用排列来求行列式的值。我们还要推导可逆矩阵求逆的代数余子式公式，并用此公式来证明克莱姆法则。对各定理的证明仅限于3阶行列式，以便使证明易于被读者领会。最后，我们要证明乘积的行列式是行列式的乘积，同时还要讨论在线性变换下，如何以行列式来度量面积和体积的伸缩。

第四章讨论 R^n 中的子空间、生成集以及线性相关性。我们要讨论向量在有序基下的坐标、正交规范基和格兰姆-施密特正交化法。最后，以讨论最小二乘近似法作为本章的结束。

在第五章里我们要研究 n 维空间到自身的线性变换的特性，讨论特征向量和特征值、方阵的对角化和等距变换。在这一章里，我们要充分讨论三维空间内的旋转变换。最后，讨论对角化方法如何应用于二次曲线的研究。

本书的最后一章论述一般向量空间。我们要讨论有限维向量空间的线性变换、线性相关性、基和维数的概念。我们以前几章的内容为例，详尽阐明这些抽象概念。和其它各章一样，本章以

对于线性差分方程的应用结束。

在写作本书的过程中，我们始终遵循下述原则：

1. 教科书的内容安排应该由浅入深，较简单的内容安排在前面，待学生对本学科的具体面貌有所了解，打下坚实基础后，再介绍较抽象和困难的概念。根据这一精神，我们在前三章里从几何和计算的角度阐述线性代数问题；到第四章再介绍较难学的线性相关性和基本的概念；而一般向量空间这样抽象的概念则安排在第六章。

2. 好的教学方法应能使使学生处于一个平稳的、始终建立在他们现有知识水平基础上的学习过程中。我们将这一原则贯穿全书。本书在一开始就安排学习线性方程组，就是出于应用并扩大学生已掌握的中学代数知识的考虑。

3. 引入新概念要慎重，只有在其必要性对学生来说是显而易见的情况下，才应该提出新概念。特别要避免漫无目标地介绍许多定义。例如，我们引入内积的概念，是为了度量两个向量之间的夹角；介绍叉积的概念，是为了求一个垂直于两个已知向量的向量；讲述矩阵乘法，是为了找到一个对应于线性变换乘积的矩阵运算。

4. 举例比之抽象的讨论更易于被学生接受。为此，在证明一个定理之前，我们经常先讨论一个精心选择的例子，用例子来阐明该定理，从而使证明过程的意义更加明确。

5. 对于大学低年级水平的学生来说，通过做计算来学习，其效果最佳。尽管他们能够理解抽象概念，但是不能缺少经常的计算技巧训练，以进一步理解和证实这些抽象概念。我们在本书的每一节里都安排了许多计算实例和练习，学生决不致只就理论来思考问题。

6. 学生们是不大注意去学习严格的数学证明的。虽然这些证明令数学家们满意，对数学程度较高的、认真的学生来说是基本的知识，但是对于初学的学生来说，却往往是不适应的。为此，我们采取以下措施来克服这个难题：

(i) 尽量用恰当、通俗的语言准确地叙述每一条定理。

(ii) 细致、严格地证明各项定理。无论这些证明是相当浅显的，还是对于加深学生对定理的理解有重大意义的，一概都不例外。

(iii) 如果只对于低维情况下给出的证明，但却包含了一般情况下的全部基本概念，我们就毫不犹豫地采用这个证明。通过这个证明过程，学生理解的透彻性将会大大提高。

(iv) 有时，我们把特别困难的证明完全略去，代之以一个精心设计的讨论，使学生明白为什么我们所阐述的结论是有道理的。

7. 从几何角度来学习线性代数是容易理解的，其效果比从代数角度学习为好。我们利用一切机会提醒学生去注意线性代数的几何内容。所以，在定义平面的线性变换以后，我们立即以一节的内容详细地讨论各种几何实例：伸展、压缩、旋转和反射。我们准备了200多张图，以便在提出每个概念时，用这些图来说明此概念的几何意义。事实上，光是这些图和详细的图注，就能使学生对本门学科的内容有一个很深的印象。

8. 既然安排了“应用”这一节，就必须严肃地对待它。每一章的最后一节都是“应用”，每一章的“应用”都有一定深度。我们避免牵强地谈论将线性代数应用于天下万物的可能性，宁可只

采用少量有意义的“应用”，而把其它的应用留给专业教程。

在每一章的结尾都有复习题，以帮助学生加强其薄弱环节。

我们希望通过自己的努力奉献出一本学生们能看得懂的书。显然，线性代数这门学科，大学一、二年级学生，甚至高中生都可以学习。对这些读者来说，在学习过程中勤于提出疑问，无疑是增加了一种合适的学习手段，这正是我们一直在追求的目标。

J. A. 索普

P. G. 孔佩尔

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Chapter One

I

'Linnet Ridgeway!'

'That's *her!*' said Mr. Burnaby, the landlord of the Three Crowns.

He nudged his companion.

The two men stared with round bucolic eyes and slightly open mouths.

A big scarlet Rolls-Royce¹ had just stopped in front of the local post office.

A girl jumped out, a girl without a hat and wearing a frock that looked like

golden hair and a... a... a...
a lovely... girl... as with... in Mal...
under-Wo...

With a quick imper... step she passed into the post office.

'That's her!' said Mr. Burnaby again. And he went on in

a low awed voice: 'Millions she's got...Going to spend thousands on the place. Swimming-pools there's going to be, and Italian gardens and a ballroom and half of the house pulled down and rebuilt...'

'She'll bring money into the town,' said his friend. He was a lean, seedy-looking man. His tone was envious and grudging.

Mr. Burnaby agreed.

'Yes, it's a great thing for Malton-under-Wode. A great thing it is.'

Mr. Burnaby was complacent about it.

'Wake us all up proper,'³ he added.

'Bit of difference from Sir George,'⁴ said the other.

'Ah, it was the 'orses did for him,' said Mr. Burnaby indulgently. 'Never 'ad no luck.'⁵

'What did he get for the place?'

'A cool sixty thousand,⁶ so I've heard.'

The lean man whistled.

Mr. Burnaby went on triumphantly: 'And they say she'll have spent ~~another~~ ~~sixty~~ thousand before she's finished!'

'Wicked!' said the lean man. 'Where'd she get all that money from?'

'America, so I've heard. Her mother was the only daughter of one of those millionaire blokes.⁷ Quite like the pictures, isn't it?'

The girl came out of the post office and climbed into the car.

As she drove off, the lean man followed her with his eyes. He muttered:

'It seems all wrong to me—her looking like that. Money and looks—it's too much! If a girl's as rich as that she's no right to be a good-looker as well. And she is a good-looker...

Got everything, that girl has. Doesn't seem fair...'

2

Extract from the Social column of the *Daily Blague*.⁸

Among those supping at Chez Ma Tante⁹ I noticed beautiful Linnet Ridgeway. She was with the Hon. Joanna Southwood, Lord Windlesham and Mr. Toby Bryce. Miss Ridgeway, as everyone knows, is the daughter of Melhuish Ridgeway who married Anna Hartz. She inherits from her grandfather, Leopold Hartz, an immense fortune. The lovely Linnet is the sensation of the moment¹⁰ and it is rumoured that an engagement may be announced shortly. Certainly Lord Windlesham seemed very epris!!¹¹

3

The Hon. Joanna Southwood said:

'Darling, I think it's going to be all perfectly *marvellous!*'
She was sitting in Linnet Ridgeway's bedroom at Wode Hall.

From the window the eye passed over the gardens to open country with blue shadows of woodlands.

'It's rather perfect, isn't it?' said Linnet.

She leaned her arms on the window sill. Her face was eager, alive, dynamic. Beside her, Joanna Southwood seemed, somehow, a little dim—a tall thin young woman of twenty-seven, with a long clever face and freakishly plucked eyebrows.

'And you've done so much in the time! Did you have lots of architects and things?'¹²

'Three.'

'What are architects like? I don't think I've ever seen any.'

'They were all right. I found them rather unpractical sometimes.'

'Darling, you soon put *that* right! You are the *most* practical creature!'

Joanna picked up a string of pearls from the dressing-table.

'I suppose these are real, aren't they, Linnet?'

'Of course.'

'I know it's "of course" to you, my sweet, but it wouldn't be to most people. Heavily cultured or even Woolworth!¹³
Darling, they really are *incredible*, so exquisitely matched.

They must be worth the *most* fabulous sums!

'Rather vulgar, you think?'

'No, not at all—just pure beauty. What *are* they worth?'

'About fifty thousand.'

'What a lovely lot of money! Aren't you afraid of having them stolen?'

'No, I always wear them—and anyway they're insured.'

'Let me wear them till dinner-time, will you, darling? It would give me such a thrill.'

Linnet laughed.

'Of course, if you like.'

'You know, Linnet, I really do envy you! You've simply got *everything*. Here you are at twenty, your own mistress, with any amount of money, looks, superb⁹ health.¹⁴ You've even got *brains*! When are you twenty-one?'

'Next June, I shall have a grand coming-of-age party¹⁵ in London.'

'And then are you going to marry Charles Windlesham? All the dreadful little gossip writers are getting so excited about it.¹⁶ And he really is frightfully devoted.'

Linnet shrugged her shoulders.

'I don't know. I don't really want to marry anyone yet.'

'Darling, how right you are! It's never quite the same afterwards, is it?'

The telephone shrilled and Linnet went to it.

'Yes? Yes?'

The butler's voice answered her:

'Miss de Bellefort is on the line. Shall I put her through?'

'Bellefort? Oh, of course, yes, put her through.'

A click and a voice, an eager, soft, slightly breathless voice:

'Hullo, is that Miss Ridgeway? *Linnet!*'

'*Jackie darling!* I haven't heard anything of you for ages and ages!'

'I know. It's awful. Linnet, I want to see you terribly.'

'Darling, can't you come down here? My new toy. I'd love to show it to you.'

'That's just what I want to do.'

'Well, jump into a train or a car.'

'Right, I will. A frightfully dilapidated two-seater.¹⁷ I bought it for fifteen pounds, and some days it goes beautifully. But it has moods.¹⁸ If I haven't arrived by tea-time you'll know it's had a mood. So long, my sweet.'

Linnet replaced the receiver. She crossed back to Joanna.

'That's my oldest friend, Jacqueline de Bellefort. We were together at a convent in Paris. She's had the most terrible bad luck. Her father was a French Count, her mother was American—a Southerner. The father went off with some woman, and her mother lost all her money in the Wall Street crash.¹⁹ Jackie was left absolutely broke. I don't know how she's managed to get along the last two years.'

Joanna was polishing her deep-blood-coloured nails with

her friend's nail pad. She leant back with her head on one side scrutinising the effect.

'Darling,' she drawled, 'won't that be rather *tiresome*? If any misfortunes happen to my friends I always drop them *at once*! It sounds heartless, but it saves such a lot of trouble later! They always want to borrow money off you, or else they start a dressmaking business and you have to get the most terrible clothes from them. Or they paint lampshades, or do batik scarves.'

'So, if I lost all my money, you'd drop me to-morrow?'

'Yes, darling, I would. You can't say I'm not honest about it! I only like successful people. And you'll find that's true of nearly everybody—only most people won't admit it. They just say that really they can't put up with Mary or Emily or Pamela any more! "Her troubles have made her so *bitter* and peculiar, poor dear!'

'How beastly you are, Joanna!'

'I'm only on the make,²⁰ like everyone else.'

'I'm not on the make!'

'For obvious reasons! You don't have to be sordid when good-looking, middle-aged American trustees pay you over a vast allowance every quarter.'

'And you're wrong about Jacqueline,' said Linnet. 'She's not a sponge. I've wanted to help her, but she won't let me. She's as proud as the devil.'

'What's she in such a hurry to see you for? I'll bet she

wants something! You just wait and see.'

'She sounded excited about something,' admitted Linnet. 'Jackie always did get frightfully worked up over things.²¹ She once stuck a penknife into someone!'

'Darling, how thrilling!'

'A boy who was teasing a dog. Jackie tried to get him to stop. He wouldn't. She pulled him and shook him, but he was much stronger than she was, and at last she whipped out a penknife and plunged it right into him. There was the *most* awful row!'

'I should think so. It sounds most uncomfortable!'

Linnet's maid entered the room. With a murmured word of apology, she took down a dress from the wardrobe and went out of the room with it.

'What's the matter with Marie?' asked Joanna. 'She's been crying.'

'Poor thing! You know I told you she wanted to marry a man who has a job in Egypt. She didn't know much about him, so I thought I'd better make sure he was all right. It turned out that he had a wife already—and three children.'

'What a lot of enemies you must make, Linnet.'

'Enemies?' Linnet looked surprised.

Joanna nodded and helped herself to a cigarette.

'Enemies, my sweet. You're so devastatingly efficient.²² And you're so frightfully good at doing the right thing.'²³

Linnet laughed.