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# The Kinetic Theory of Inert Dilute Plasmas

惰性稀薄等离子体动理论

(影印版)

〔墨〕加西亚-科林 (L. S. García-Colín) 〔墨〕达各杜格(L. Dagdug)



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by Leopoldo S. García-Colin and Leonardo Dagdug

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#### 序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花 齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的 作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教 材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科 学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍 是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了"中外物理学精品书系",试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,"中外物理学精品书系"力图完整呈现近现代世界和中国物理

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科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经 典物理丛书。

"中外物理学精品书系"另一个突出特点是,在把西方物理的精华要义"请进来"的同时,也将我国近现代物理的优秀成果"送出去"。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的"硬实力",也向世界主动传播我国科技文化领域不断创新的"软实力",对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,"中外物理学精品书系"还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是"只闻其声,未见其真"。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,"加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件"。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套"中外物理学精品书系"的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

"中外物理学精品书系"编委会 主任 中国科学院院士,北京大学教授

王恩哥

2010年5月于燕园

Leopoldo S. García-Colín Leonardo Dagdug

# The Kinetic Theory of Inert Dilute Plasmas



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#### Introduction

The contents of this book are the result of work performed in the past three years to provide some answers to questions raised by several colleagues working in astrophysics. Examining several transport processes in plasmas related to dissipative effects in phenomena such as cooling flows, propagation of sound waves, thermal conduction in the presence of magnetic fields, angular momentum transfer in accretion disks, among many, one finds a rather common pattern. Indeed when values for transport coefficients are required the overwhelming majority of authors refer to the classical results obtained by L. Spitzer and S. Braginski over forty years ago. Further, it is also often mentioned that under the prescribed working conditions the values of such coefficients are usually insufficient to provide agreement with observations.

The methodology followed by these authors is based upon Landau's pioneering idea that collisions in plasmas may be substantially accounted for when viewed as a diffusive process. Consequently the ensuing basic kinetic equation is the Fokker-Planck version of Boltzmann's equation as essentially proposed by Landau himself nearly 70 years ago. Curiously enough the magnificent work of the late R. Balescu in both Classical and Non-Classical transport in plasmas published in 1988 and also based on the Fokker-Planck equation is hardly known in the astrophysical audience. The previous work of Spitzer and Braginski is analyzed with much more rigorous vision in his two books on the subject.

With this background in hand the question that came to our minds is why, if true, the full Boltzmann's equation had never been used in dealing at least with the kinetic theory of dilute plasmas. In their well known and comprehensive treatment on the kinetic theory of non-uniform gases, Chapman and Cowling never developed the theory as they did with ordinary gases. A further attempt was made in 1960 by W. Marshall in three unpublished reports issued by the Harwell Atomic Energy Establishment in

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Harwell, England. And also, none of all the authors in this field with the sole exception of Balescu who did it partially, took the kinetic equation of their choice to provide the microscopic basis of linear irreversible thermodynamics therefore, providing, among many other results, a microscopic basis of magnetohydrodynamics.

This is the main objective of this book. Starting from the full Boltzmann equation for an inert dilute plasma and using the Hilbert-Chapman-Enskog method to solve the first two approximations in Knudsen's parameter we construct all the transport properties of the system within the framework of linear irreversible thermodynamics. This includes a systematic study of all possible cross effects which except for a few cases dealt with by Balescu, today to our knowledge, have never been mentioned in the literature. The equations of magnetohydrodynamics, including the rather surprising results here obtained for the viscomagnetic effects, for dilute plasmas may be then fully assessed. We expect that this material will thus be useful to graduate students and researchers involved in work with non-confined plasmas specially in astrophysical problems.

July 2008

L.S. García-Colín L. Dagdug

# Part I Vector Transport Processes



### Chapter 1

# Non-equilibrium Thermodynamics

The main objective of this book is to place the kinetic theory of a dilute plasma within the tenets of what is known as Classical (Linear) Irreversible Thermodynamics (CIT). Since this subject is quite often beyond the average knowledge of the younger generation of physicists and physical chemists we feel that it is useful to give a brief review of its basic concepts so that the reader appreciates better how and why we are seeking the results to be presented in the main text.

CIT, being a phenomenological theory is based essentially on four basic assumptions, namely,

- 1. The local equilibrium assumption (LEA)
- 2. The validity of the conservation equations
- 3. The linear constitutive equations and positive definiteness of the uncompensated heat (entropy production)
- 4. Onsagers' reciprocity theorem

In what follows we shall discuss as thoroughly as possible the basic ideas behind each assumption, leaving the reader to pursue more details in the standard texts on the subject [1]-[7]. Let us start with the LEA. Consider any arbitrary system which is not in thermodynamic equilibrium. For purely didactical reasons the reader may think of a fluid enclosed in a volume V.

Let us now partition this volume in small cells such that the number of particles in each cell with coordinates  $\vec{r}$ ,  $\vec{r} + d\vec{r}$  at time t contains enough particles to be considered as a continuum but small compared with the total number of particles in the system, say N. The LEA asserts that within each cell a thermodynamic equilibrium state prevails. For instance, if  $n(\vec{r},t)$  is the particle density in the cell characterized by its position  $\vec{r}$  at time t and  $T(\vec{r},t)$  the temperature inside the cell, any other thermodynamic quantity, for instance the entropy  $s(\vec{r},t)$  will be related to  $n(\vec{r},t)$  and  $T(\vec{r},t)$  as

$$s(\vec{r}, t) = s[n(\vec{r}, t), T(\vec{r}, t)] \tag{1.1}$$

precisely by the same relationship that holds for these variables in the equilibrium state. The local equation of state for an ideal gas would read

$$p(\vec{r},t) = n(\vec{r},t)k_B T(\vec{r},t) \tag{1.2}$$

 $k_B$  being Boltzmann's constant. And so on.

These equations bring us in a natural way to the second assumption. Think of a monatomic fluid for the moment in the absence of sources and sinks. If we chose to describe the states of this fluid by the "natural" variables, the local particles density  $n(\vec{r},t)$ , the local hydrodynamic velocity  $\vec{u}(\vec{r},t)$  (or  $m\vec{u}(\vec{r},t)$  its momentum) and the local energy density  $e(\vec{r},t)$  these variables will satisfy clearly, conservation equations. Use of this fact and Eq. (1.1) with  $e(\vec{r},t)$  instead of  $T(\vec{r},t)$  plus the standard techniques of ordinary calculus lead us in a straightforward fashion to an equation describing the evolution of the local entropy  $s(\vec{r},t)$ . In fact if  $\rho(\vec{r},t)=mn(\vec{r},t)$ , m being the mass of the particles, one finds that,

$$\frac{\partial(\rho s)}{\partial t} + \operatorname{div} \vec{J_s} = \sigma \tag{1.3}$$

which is a balance type equation for  $\rho s$ .  $\vec{J_s}$ , the entropy flux, gives the amount of entropy flowing through the boundaries to the system and  $\sigma$ , the uncompensated heat or entropy production, measures the entropy generated inside the system due to the dissipative processes. Its existence goes back to Clausius who indeed identified it with the uncompensated heat which should arise from dissipation. Its analytical expression was first identified by T. de Donder in chemical reactions and later brought into its present form by Meixner. Indeed, in the derivation of Eq. (1.3) one finds that

$$\sigma = \sum \overleftrightarrow{J}_i \odot \overleftrightarrow{X}_i = -\frac{\overrightarrow{J}_q}{T} \cdot \operatorname{grad} T - \frac{1}{T} \overset{\circ}{\tau} : (\operatorname{grad} \vec{u})^s - \frac{1}{T} \tau \operatorname{div} \vec{u}$$
 (1.4)

where  $\overleftrightarrow{J}_i$  and  $\overleftrightarrow{X}_i$  denote the fluxes and their corresponding forces respectively, and  $\odot$  the contraction of tensors of equal rank. The second equality illustrates its nature for an ordinary monatomic fluid.  $\overrightarrow{J}_q$  is the heat flow vector and the momentum flow  $\overleftrightarrow{\tau}$  is split into its symmetric traceless part  $\overset{\circ}{\tau}$  and its trace  $\tau$ . Eq. (1.4) clearly fulfills Clausius' predictions.

These results bring us to the third assumption. The conservation equations for a monatomic fluid are the set of five differential equations for the state variables  $\rho$ ,  $\vec{u}$  and e but contain fourteen unknowns, these variables plus the three components of  $\vec{J_q}$  plus the six independent components of the stress tensor  $\overrightarrow{\tau}$  assumed to be symmetric. We thus need nine additional equations to express  $\vec{J_q}$  and  $\overrightarrow{\tau}$  in terms of the independent variables. Notice that  $T(\vec{r},t)$  may be introduced through the LEA since  $e(\vec{r},t)=e(n(\vec{r},t),T(\vec{r},t))$ . These additional equations known in the literature as the "constitutive equations" are completely foreign to thermodynamics. They may be extracted from experiment or from a microscopic theory. If we now assume (assumption 3) that the relationship between fluxes and forces is linear so that in general,

$$\overleftrightarrow{J}_{i} = \sum L_{ik} \overleftrightarrow{X}_{k}, \tag{1.5}$$

we may obtain a complete set for the time evolution equations of the local state variables. For a monatomic fluid, Eq. (1.5) reduces to

$$\vec{J_q} = -\kappa \operatorname{grad} T \quad \text{Fourier} \tag{1.6}$$

$$\begin{array}{l} \stackrel{\circ}{\leftarrow} \stackrel{s}{\tau} = -\eta (\operatorname{grad} \ \vec{u})^s \\ \tau = -\zeta \operatorname{div} \ \vec{u} \end{array} \right\} \quad \text{Naviere-Newton}$$

As it is shown in any standard text on the subject, when Eqs. (1.6) are substituted into the conservation equations one gets a set of non-linear, second order in space, first order in time differential equations for n,  $\vec{u}$  and T known as the Navier-Stokes-Fourier equations of hydrodynamics. These equations require the knowledge of a local equation of state (c.f. Eq. (1.1)), of the transport coefficients  $\kappa$ , the thermal conductivity,  $\eta$ , the shear viscosity, and  $\zeta$ , the bulk viscosity in addition to well defined boundary and initial conditions to seek for a solution.

In spite of its centennial age these equations still pose immense problems to mathematical physicists and hydrodynamicists in finding stable solutions [8].