

ANALYSIS OF HEAT AND MASS TRANSFER

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INTERNATIONAL STUDENT EDITION

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Nomenclature

CONDUCTION AND CONVECTION

a	acceleration	v	specific volume
b	wall thickness; fin thickness	v_i	velocity components
c	specific heat	v^*	shearing-stress velocity
c_p	specific heat at constant pressure	x, y, z	coordinates
d	diameter	x_i	coordinates
d_h	hydraulic diameter	λ	wavelength
e	energy per unit mass	A	area
f	friction coefficient for flow through a tube; dimensionless stream function	C	constant; circumference
f_d	drag coefficient of a cylinder	E	voltage; energy
f_s	friction coefficient of a plane plate	F	force
f_v	velocity slip coefficient	H	thermal field, enthalpy (in Sect. 11-3)
g	acceleration of gravity; body force per unit mass	J	turbulence intensity; electric current; integral
h	film heat-transfer coefficient	L	length
i	enthalpy	L_e	entrance length
i_{fg}	heat of evaporation	M	molecular weight
j	degrees of freedom	N_A	Avogadro's number
k	thermal conductivity	Q	heat flow per unit time
k	Boltzmann constant	R	resistance; gas constant; radius
l	length, mixing length	R_c	thermal-conduction resistance
m	mass	R_t	thermal-convection resistance
\dot{m}	mass rate of flow	T	absolute temperature
n	normal; number of molecules	U	internal energy; overall transfer coefficient
p	pressure; stress	V	velocity
q	heat flow per unit time and area (specific rate of heat flow)	\mathcal{V}	volume
r	radius; recovery factor	W	weight; work
s	thickness; distance; molecular speed ratio	α	angle; thermal diffusivity; parameter for properties
t	temperature	β	angle; parameter for stream velocity; expansion coefficient
t_b	bulk temperature	γ	parameter for wall temperature; ratio of specific heats
u, U	internal energy	δ	boundary-layer thickness; phase lag
u, v, w	velocity components		

δ	boundary-layer momentum thickness
δ_t	thermal boundary-layer thickness
δ^*	boundary-layer displacement thickness
ϵ	porosity; energy flux
ϵ_H	eddy diffusivity for heat
ϵ_m	eddy diffusivity for momentum
ζ	ratio of thermal and flow boundary-layer thickness
η	dimensionless wall distance; fin effectiveness
ϑ	temperature difference; parameter describing temperature
λ	form parameter (for velocity profile); eigenvalue; mean-free-path length
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density; radius; electric resistivity
σ	surface tension
τ	shearing stress; time
τ_0	period; relaxation time
φ	potential
Φ	heat dissipation; heat source; fin effectiveness
ψ	stream function

DIMENSIONLESS PARAMETERS

Bi	Biot number
c_f	friction factor
Ec	Eckert number
f	friction factor for tube
Fo	Fourier number
Fr	Froude number
Gr	Grashof number
Kn	Knudsen number
Le	Lewis number
Le_t	turbulent Lewis number
Ma	Mach number

Nu	Nusselt number
Pe	Peclet number
Pr	Prandtl number
Pr_t	turbulent Prandtl number
Ra	Rayleigh number
Re	Reynolds number
Sc	Schmidt number
Sc_t	turbulent Schmidt number
Sh	Sherwood number
St	Stanton number

INDICES

b	values on the border between laminar sublayer and turbulent boundary layer
cr	critical values for transition to turbulent flow
d	based on diameter d
dy	dynamic values (dynamic pressure, etc.)
e	exit
f	fluid
i	inlet; based on enthalpy
l	liquid
m	mean value
p	partial
r	recovery
s	values in the stream outside the boundary layer
st	static values (static pressure, etc.)
t	turbulent
v	vapor
w	values on the wall surface
x	based on length x
B	bulk
M	arithmetic mean value
0	values at a reference point
∞	values at great distance from a body
$-$	mean value
$'$	fluctuating value; dimensionless

SUPERSCRIPTS

- ° total values (total pressure, . . .)
 * reference

ADDITIONALLY FOR MASS TRANSFER

- c mass concentration
 h_M mass-transfer coefficient
 i enthalpy
 j specific mass flux
 n mole density
 r relative humidity
 s specific humidity
 w mass ratio
 C_p specific heat per unit volume
 D mass diffusion coefficient
 M molecular weight

- N number of moles
 \mathcal{R} universal gas constant
 α thermal diffusion ratio
 δ_D diffusion boundary-layer thickness
 ϵ_M eddy diffusivity for mass
 φ parameter describing mass fraction
 ψ mass generation rate
 ω mole fraction

INDICES

- a air
 m component m
 s saturation
 v vapor

RADIATION

- c wave velocity
 e emissive power
 h Planck's quantum
 i intensity of emitted flux
 j coefficient of emission
 k Boltzmann's gas constant
 n refractive index
 s path length
 u radiation density
 A area
 B radiosity, flux leaving a surface per unit area
 E energy
 F shape factor, angle factor
 H irradiance, flux arriving at a surface per unit area
 K intensity of radiant flux
 L length
 L_e equivalent beam length
 L_g geometric beam length
 S path length

- T absolute temperature
 V volume
 α absorptance, absorptivity
 β angle measured from the surface normal
 γ azimuth angle
 ϵ emittance, emissivity
 κ coefficient of absorption
 λ wavelength
 Λ mean molecular path length
 Λ_r mean photon path length
 ν frequency
 ρ density, reflectance, reflectivity
 σ scattering coefficient, Stefan-Boltzmann constant
 τ transmittance, transmissivity
 Φ radiative energy flux, energy source
 φ radiative energy flux per unit area
 ω solid angle

DEFINITIONS OF RADIATION PARAMETERS

	On surfaces	
	directional	Hemispherical
Total flux	K	B, H
Emitted flux	i	e
	In space	
Total flux	K	
Emitted flux	j	
	Dimensionless parameters	
On surfaces	ϵ α ρ τ	
In space	κ σ	

Preface

The present book has evolved from a series of lectures to graduate students. The text is therefore envisioned as one which will enable the student to learn and understand the fundamental laws in depth and detail, to study and appreciate the power of analytical methods, and to recognize their limitations, after he has gained some familiarity with the phenomena of heat-transfer processes from such texts as Eckert, "Introduction to Heat and Mass Transfer" or Eckert and Drake, "Heat and Mass Transfer."

We have a strong conviction that the student should not be overburdened with numerous details, which may prevent him from developing a clear understanding of the fundamental principles required for application to specific situations as they occur in technological developments. Therefore no attempt is made to discuss all or even most of the newer developments. Instead, a few approaches which have broad applications are treated in sufficient detail for the student to follow the developments and derivations with minimum reference to other sources.

The literature of heat transfer has grown to such an extent that a clear separation between textbooks, like the present one, and reference books is desirable. In the latter category there are now available the annual *Advances in Heat Transfer*, edited by T. F. Irvine and J. P. Hartnett and the yearly reviews of heat-transfer literature in the *International Journal of Heat and Mass Transfer*. In preparation is the "Handbook of Heat Transfer," edited by W. M. Rohsenow and J. P. Hartnett. It is hoped that these volumes and others of a similar nature will provide a factual coverage not only of useful developments from the past but also of new developments as they occur.

Analysis in the field of heat transfer, as in many other subjects, has undergone radical changes in methodology in recent years because of the greater availability and use of large computers. The changes brought about by increasingly sophisticated computer technology are reflected in the field of education. The present text therefore emphasizes first the development and exposition of the basic conservation equations; then it treats the applications of the resulting equations to special situations, through simplification by similarity transformations, for example; and finally it discusses results. The necessary numerical computations are not treated, on the assumption that the graduate student of today is familiar with the programming techniques required to produce the results from the equations describing physical phenomena by the use of the digital or analog computer.

Modern computers make it possible to obtain numerical solutions for the complete system of conservation equations for quite complex situations, e.g., the determination of timewise unsteady three-dimensional flow and temperature fields. Extensive industrial applications have already been made of such calculations. Many groups, both industrial and academic, have developed computer programs of interest, and it is hoped that such programs will become generally available through computer libraries.

On the other hand, to gain an insight into the heat-transfer and flow processes involved and to develop an understanding of the trends in the phenomena, one must study simplified situations with a limited number of boundary conditions. This choice is quite analogous to the situation in experimental work, where basic understanding is sought by means of the selection and exploration of simple experiments with a restricted number of well-defined influencing parameters. It is hoped that the phenomena and examples discussed in this text are those best suited to improving and broadening the understanding of the processes of heat transfer, although we realize that such a selection will always be somewhat subjective.

Dean Drake again contributed Part I, Heat Conduction, Sec. 11-3,

Exchange Processes in Rarefied Gases, and Appendix B, Thermophysical Properties, and thus essentially helped round out the coverage in the text. Physical dimensions, including the thermophysical properties of matter, are given in *Système International* (SI) units, as it is expected that they will become more and more common in the United States, which is apparently destined to be the only country in the world using a nonmetric system in the near future. There seems to be a consensus today that a change to SI units must be made, and opinion differs only as to when the change will occur. Perhaps this book will help speed that day. The reader may find Appendix A helpful in acquainting himself with the SI system.

It is hoped that this book will be useful as a text in graduate courses, as well as in self-study by practicing engineers who need to solve problems in heat transfer or who want to look at the field of heat transfer from a more basic point of view and thus obtain a deeper understanding of the more advanced methods of analysis.

We wish to express here our thanks to our wives, Josefine (Eckert) and Jane (Drake), who patiently accepted the fact that considerable time had to be devoted to the preparation of the book and kept the distracting everyday chores and disturbances at a distance. We also thank our secretaries, Miss Ruth Fenton and Miss Marsha Harman, who typed the manuscript with their usual care and helped with the organization of the material. Thanks are expressed here to the many graduate students of the senior author for their assistance in carrying out the necessary computations and help in proofreading of the textual material. Professors Wendell DeMarcus and J. E. Funk graciously provided an invaluable review of Part I, for which we here note our gratitude. We are indebted to L. B. Davis, Jr. for the recalculation of the data in Appendix B into the *Système International d'Unités*. The help provided by McGraw-Hill Book Company through careful editing and in preparing the final drawings should also be acknowledged.

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