

Introduction to High Energy Physics

DONALD H. PERKINS

4th edition

Introduction to High Energy Physics, 4th Edition

This highly regarded text provides an up-to-date and comprehensive introduction to modern particle physics.

Extensively rewritten and updated, this fourth edition includes all the recent developments in elementary particle physics, as well as its connections with cosmology and astrophysics. As in previous editions, the balance between experiment and theory is continually emphasised. The stress is on the phenomenological approach and basic theoretical concepts rather than rigorous mathematical detail. Short descriptions are given of some of the key experiments in the field, and how they have influenced our thinking. Although most of the material is presented in the context of the Standard Model of quarks and leptons, the shortcomings of this model and new physics beyond its compass (such as supersymmetry, neutrino mass and oscillations, GUTs and superstrings) are also discussed. The text includes many problems and a detailed and annotated further reading list.

This is a text suitable for final-year physics undergraduates and graduate students studying experimental or theoretical particle physics.

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Contents

<i>Preface</i>	<i>page xi</i>
1 Quarks and leptons	1
1.1 Preamble	1
1.2 The Standard Model of particle physics	7
1.3 Particle classification: fermions and bosons	12
1.4 Particles and antiparticles	13
1.5 Free particle wave equations	16
1.6 Helicity states: helicity conservation	19
1.7 Lepton flavours	20
1.8 Quark flavours	22
1.9 The cosmic connection	26
Problems	33
2 Interactions and fields	35
2.1 Classical and quantum pictures of interactions	35
2.2 The Yukawa theory of quantum exchange	36
2.3 The boson propagator	37
2.4 Feynman diagrams	38
2.5 Electromagnetic interactions	40
2.6 Renormalisation and gauge invariance	42
2.7 Strong interactions	43
2.8 Weak and electroweak interactions	46
2.9 Gravitational interactions	51
2.10 The interaction cross-section	51
2.11 Decays and resonances	55
Problems	61

3	Invariance principles and conservation laws	63
3.1	Translation and rotation operators	63
3.2	The parity operation	65
3.3	Pion spin and parity	66
3.4	Parity of particles and antiparticles	69
3.5	Tests of parity conservation	72
3.6	Charge conjugation invariance	73
3.7	Charge conservation and gauge invariance	75
3.8	Baryon and lepton conservation	79
3.9	<i>CPT</i> invariance	81
3.10	<i>CP</i> violation and <i>T</i> violation	81
3.11	Neutron electric dipole moment	83
3.12	Isospin symmetry	87
3.13	Isospin in the two-nucleon and pion–nucleon systems	88
3.14	Isospin, strangeness and hypercharge	91
	Problems	93
4	Quarks in hadrons	95
4.1	Charm and beauty; the heavy quarkonium states	95
4.2	Comparison of quarkonium and positronium levels	102
4.3	The baryon decuplet	109
4.4	Quark spin and colour	114
4.5	The baryon octet	115
4.6	Quark–antiquark combinations: the light pseudoscalar mesons	118
4.7	The light vector mesons	121
4.8	Other tests of the quark model	123
4.9	Mass relations and hyperfine interactions	126
4.10	Electromagnetic mass differences and isospin symmetry	129
4.11	Magnetic moments of baryons	130
4.12	Mesons built of light and heavy quarks	132
4.13	The top quark	134
	Problems	139
5	Lepton and quark scattering	140
5.1	The process $e^+e^- \rightarrow \mu^+\mu^-$	140
5.2	e^+e^- annihilation to hadrons	144
5.3	Electron–muon scattering, $e^-\mu^+ \rightarrow e^-\mu^+$	147
5.4	Neutrino–electron scattering, $\nu_e e \rightarrow \nu_e e$	150
5.5	Elastic lepton–nucleon scattering	154
5.6	Deep inelastic scattering and partons	155
5.7	Deep inelastic scattering and quarks	159

5.8	Experimental results on quark distributions in the nucleon	162
5.9	Sum rules	166
5.10	Summary	168
	Problems	168
6	Quark interactions and QCD	171
6.1	The colour quantum number	171
6.2	The QCD potential at short distances	172
6.3	The QCD potential at large distances: the string model	178
6.4	Gluon jets in e^+e^- annihilation	180
6.5	Running couplings in QED and QCD	181
6.6	Evolution of structure functions in deep inelastic scattering	186
6.7	Gluonium and the quark–gluon plasma	190
	Problems	192
7	Weak interactions	194
7.1	Classification	194
7.2	Lepton universality	195
7.3	Nuclear β -decay: Fermi theory	197
7.4	Inverse β -decay: neutrino interactions	201
7.5	Parity nonconservation in β -decay	202
7.6	Helicity of the neutrino	205
7.7	The $V - A$ interaction	206
7.8	Conservation of weak currents	209
7.9	The weak boson and Fermi couplings	210
7.10	Pion and muon decay	210
7.11	Neutral weak currents	213
7.12	Observation of W^\pm and Z^0 bosons in $p\bar{p}$ collisions	215
7.13	Z^0 production at e^+e^- colliders	220
7.14	Weak decays of quarks. The GIM model and the CKM matrix	221
7.15	Neutral K mesons	226
7.16	CP violation in the neutral kaon system	232
7.17	Cosmological CP violation	237
7.18	$D^0-\bar{D}^0$ and $B^0-\bar{B}^0$ mixing	238
	Problems	239
8	Electroweak interactions and the Standard Model	242
8.1	Introduction	242
8.2	Divergences in the weak interactions	243
8.3	Introduction of neutral currents	245
8.4	The Weinberg–Salam model	246

8.5	Intermediate boson masses	248
8.6	Electroweak couplings of leptons and quarks	249
8.7	Neutrino scattering via Z exchange	250
8.8	Asymmetries in the scattering of polarised electrons by deuterons	253
8.9	Observations on the Z resonance	255
8.10	Fits to the Standard Model and radiative corrections	260
8.11	W pair production	262
8.12	Spontaneous symmetry breaking and the Higgs mechanism	263
8.13	Higgs production and detection	271
	Problems	274
9	Physics beyond the Standard Model	276
9.1	Supersymmetry	277
9.2	Grand unified theories: the $SU(5)$ GUT	278
9.3	Unification energy and weak mixing angle	280
9.4	Supersymmetric $SU(5)$	282
9.5	Proton decay	282
9.6	Neutrino mass: Dirac and Majorana neutrinos	284
9.7	Neutrino oscillations	287
9.8	Magnetic monopoles	299
9.9	Superstrings	300
	Problems	301
10	Particle physics and cosmology	303
10.1	Hubble's law and the expanding universe	303
10.2	Friedmann equation	304
10.3	Cosmic microwave radiation: the hot Big Bang	307
10.4	Radiation and matter eras	311
10.5	Nucleosynthesis in the Big Bang	313
10.6	Baryon–antibaryon asymmetry	317
10.7	Dark matter	319
10.8	Inflation	326
10.9	Neutrino astronomy: SN 1987A	330
	Problems	336
11	Experimental methods	338
11.1	Accelerators	338
11.2	Colliding-beam accelerators	343
11.3	Accelerator complexes	346
11.4	Secondary particle separators	346
11.5	Interaction of charged particles and radiation with matter	349

11.6	Detectors of single charged particles	355
11.7	Shower detectors and calorimeters	368
	Problems	375
<i>Appendix A</i>	<i>Table of elementary particles</i>	377
<i>Appendix B</i>	<i>Milestones in particle physics</i>	379
<i>Appendix C</i>	<i>Clebsch–Gordan coefficients and d-functions</i>	386
<i>Appendix D</i>	<i>Spherical harmonics, d-functions and Clebsch–Gordan coefficients</i>	393
<i>Appendix E</i>	<i>Relativistic normalisation of cross-sections and decay rates</i>	396
	<i>Glossary</i>	398
	<i>Answers to problems</i>	408
	<i>Bibliography</i>	412
	<i>References</i>	418
	<i>Index</i>	421

Preface

The main object in writing this book has been to present the subject of elementary particle physics at a level suitable for advanced physics undergraduates or to serve as an introductory text for graduate students.

Since the first edition of this book was produced over 25 years ago, and the third edition over 10 years ago, there have been many revolutionary developments in the subject, and this has necessitated a complete rewriting of the text in order to reflect these changes in direction and emphasis. In comparison with the third edition, the main changes have been in the removal of much of the material on hadron–hadron interactions as well as most of the mathematical appendices, and the inclusion of much more detail on the experimental verification of the Standard Model of particle physics, with emphasis on the basic quark and lepton interactions. Although much of the material is presented from the viewpoint of the Standard Model, one extra chapter has been devoted to physics outside of the Standard Model and another to the role of particle physics in cosmology and astrophysics.

Many – indeed most – texts on this subject place particular emphasis on the power and beauty of the theoretical description of high energy processes. However, progress in this field has in fact depended crucially on the close interplay of theory and experiment. Theoretical predictions have challenged the ingenuity of experimentalists to confirm or refute them, and equally there have been long periods when unexpected experimental discoveries have challenged our theoretical description of high energy phenomena. In this text, I have tried to emphasise some of the experimental aspects of the subject and have given brief descriptions of some of the key experiments. Some knowledge of elementary quantum mechanics has been assumed, but generally I have tried to present the material from a phenomenological and empirical viewpoint, with a minimum of theoretical formalism. A short chapter on experimental methods and techniques has been included, placed at the end of the book so as not to interrupt the flow of the main material.

Although the intention is that the different chapters should be read in sequence, I have tried to make each one reasonably self-contained, at the price of occasional repetition. For a shorter course, sections or even whole chapters could be left out without too much loss to the remaining material. For example, Chapters 9, 10 and 11 and possibly much of Chapter 3 could be omitted on a first pass through the text.

References to original papers are not comprehensive but have been cited where I thought this was necessary. At the end of the book I have included short bibliographies for further reading, relating to the chapter material in general as well as to specific topics. Sets of problems, mostly numerical, are included at the ends of chapters.

No textbook can cover this entire subject, even at a superficial level. I have tried to compensate for this shortcoming, and to put the subject matter on a historical footing, by including as Appendix B a chronological list of the most important advances in the subject over the last 100 years. This is accompanied by a short summary of the significance and importance of these developments.

For those readers who wish to delve into the theoretical aspects of the subject at a deeper level, I suggest the following texts, in ascending order of difficulty:

Gottfried, K., and V. F. Weisskopf, *Concepts of Particle Physics* (Oxford: Oxford University Press 1984)

Halzen, F., and A. D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics* (New York: Wiley 1984)

Close, F. E., *Introduction to Quarks and Partons* (London: Academic 1979)

Griffiths, D., *Introduction to Elementary Particles* (New York: Wiley 1987)

Aitchison, I. J., and A. J. Hey, *Gauge Theories in Particle Physics* (Bristol: Adam Hilger 1982)

For a comprehensive text on the key experimental developments in particle physics, including many original papers, I recommend

Cahn, R., and G. Goldhaber, *The Experimental Foundations of Particle Physics* (Cambridge: Cambridge University Press 1991)

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Donald H. Perkins

1

Quarks and leptons

1.1 Preamble

The subject of elementary particle physics may be said to have begun with the discovery of the electron 100 years ago. In the following 50 years, one new particle after another was discovered, mostly as a result of experiments with cosmic rays, the only source of very high energy particles then available. However, the subject really blossomed after 1950, following the discovery of new elementary particles in cosmic rays; this stimulated the development of high energy accelerators, providing intense and controlled beams of known energy that were finally to reveal the quark substructure of matter and put the subject on a sound quantitative basis.

1.1.1 Why high energies?

Particle physics deals with the study of the elementary constituents of matter. The word ‘elementary’ is used in the sense that such particles have no known structure, i.e. they are pointlike. How pointlike is pointlike? This depends on the spatial resolution of the ‘probe’ used to investigate possible structure. The resolution is Δr if two points in an object can just be resolved as separate when they are a distance Δr apart. Assuming the probing beam itself consists of pointlike particles, the resolution is limited by the de Broglie wavelength of these particles, which is $\lambda = h/p$ where p is the beam momentum and h is Planck’s constant. Thus beams of high momentum have short wavelengths and can have high resolution. In an optical microscope, the resolution is given by

$$\Delta r \simeq \lambda / \sin \theta$$

where θ is the angular aperture of the light beam used to view the structure of an object. The object scatters light into the eyepiece, and the larger the angle of scatter θ and the smaller the wavelength λ of the incident beam the better is the resolution. For example an ultraviolet microscope has better resolution and reveals

more detail than one using visible light. Substituting the de Broglie relation, the resolution becomes

$$\Delta r \simeq \frac{\lambda}{\sin \theta} = \frac{h}{p \sin \theta} \simeq \frac{h}{q}$$

so that Δr is inversely proportional to the momentum q transferred to the photons, or other particles in an incident beam, when these are scattered by the target.[†] Thus a value of momentum transfer such that $qc = 10 \text{ GeV} = 10^{10} \text{ eV}$ – easily attainable with present accelerator beams – gives a spatial resolution $hc/(qc) \sim 10^{-16} \text{ m}$, about 10 times smaller than the known radius of the charge and mass distribution of a proton (see Table 1.1 for the values of the units employed).

In the early decades of the twentieth century, particle-beam energies from accelerators reached only a few MeV (10^6 eV), and their resolution was so poor that protons and neutrons could themselves be regarded as elementary and pointlike. At the present day, with a resolution thousands of times better, the fundamental pointlike constituents of matter appear to be quarks and leptons, which are the main subject of this text. Of course, it is possible that they in turn may have an inner structure, but there is no present evidence for this, and whether they do will be for future experiments to decide.

The second reason for high energies in experimental particle physics is simply that many of the elementary particles are extremely massive and the energy mc^2 required to create them is correspondingly large. The heaviest elementary particle detected so far, the ‘top’ quark (which has to be created as a pair with its antiparticle) has $mc^2 \simeq 175 \text{ GeV}$, nearly 200 times the mass–energy of a proton.

At this point it should be mentioned that the total energy in accelerator beams required to create such massive particles in sufficient intensities is quite substantial. For example, an energy per particle of 1 TeV (10^{12} eV) in beams consisting of bunches of 10^{13} accelerated particles every second will correspond to a total kinetic energy in each bunch of 1.6 megajoules, equal to the energy of 30 000 light bulbs, or of a 15 tonne truck travelling at 30 mph.

1.1.2 Units in high energy physics

The basic units in physics are length, mass and time and the SI system expresses these in metres, kilograms and seconds. Such units are not very appropriate in high energy physics, where typical lengths are 10^{-15} m and typical masses are 10^{-27} kg .

Table 1.1 summarises the units commonly used in high energy physics. The unit of length is the *femtometre* or *fermi*, where $1 \text{ fm} = 10^{-15} \text{ m}$; for example, the root mean square radius of the charge distribution of a proton is 0.8 fm. The

[†] To be exact, in an elastic collision with a massive target, the momentum transfer will be $q = 2p \sin(\theta/2)$, if θ is the angle of deflection.

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV = 10^9 eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/ c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s $^{-1}$	2.998×10^8 m s $^{-1}$
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

(b)

natural units, $\hbar = c = 1$		
mass, Mc^2/c^2	1 GeV	
length, $\hbar c/(Mc^2)$	1 GeV ⁻¹ = 0.1975 fm	
time, $\hbar c/(Mc^3)$	1 GeV ⁻¹ = 6.59 × 10 ⁻²⁵ s	
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$		
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$	
Relations between energy units		
1 MeV = 10 ⁶ eV	1 GeV = 10 ³ MeV	1 TeV = 10 ³ GeV

commonly used unit of energy is the GeV, convenient because it is typical of the mass–energy mc^2 of strongly interacting particles. For example, a proton has $M_p c^2 = 0.938$ GeV.

In calculations, the quantities $\hbar = h/(2\pi)$ and c occur frequently, sometimes to high powers, and it is advantageous to use units in which we set $\hbar = c = 1$. Having chosen these two units, we are still at liberty to specify one more unit, e.g. the unit of energy, and the common choice, as indicated above, is the GeV. With $c = 1$ this is also the mass unit. As shown in the table, the unit of length will then be 1 GeV $^{-1}$ = 0.197 fermi, while the corresponding unit of time is 1 GeV $^{-1}$ = 6.59×10^{-25} s.

Throughout this text we shall be dealing with interactions between charges – which can be the familiar electric charge of electromagnetic interactions, the strong charge of the strong interaction or the weak charge of the weak interaction. In the SI system the unit electric charge, e , is measured in coulombs and the fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

Here ϵ_0 is the permittivity of free space, while its permeability is defined as μ_0 ,

such that $\epsilon_0\mu_0 = 1/c^2$. For interactions in general, such units are not useful and we can define e in Heaviside–Lorentz units, which require $\epsilon_0 = \mu_0 = \hbar = c = 1$, so that

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

with similar definitions that relate charges and coupling constants analogous to α in the other interactions.

1.1.3 Relativistic transformations

In most of the processes to be considered in high energy physics, the individual particles have relativistic or near relativistic velocities, $v \sim c$. This means that the result of a measurement, e.g. the lifetime of an unstable particle, will depend on the reference frame in which it is made. It follows that one requirement of any theory of elementary particles is that it should obey a fundamental symmetry, namely invariance under a relativistic transformation, so that the equations will have the same form in all reference frames. This can be achieved by formulating the equations in terms of 4-vectors, which we now discuss briefly, together with the notation employed in this text.

The relativistic relation between total energy E , the vector 3-momentum \mathbf{p} (with Cartesian components p_x, p_y, p_z) and the rest mass m for a free particle is

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4$$

or, in units with $c = 1$

$$E^2 = \mathbf{p}^2 + m^2$$

The components p_x, p_y, p_z, E can be written as components of an energy–momentum 4-vector p_μ , where $\mu = 1, 2, 3, 4$. In the Minkowski convention used in this text, the three momentum (or space) components are taken to be real and the energy (or time) component to be imaginary, as follows:

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

so that

$$p^2 = \sum_{\mu} p_{\mu}^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2 = \mathbf{p}^2 - E^2 = -m^2 \quad (1.1)$$

Thus p^2 is a relativistic invariant. Its value is $-m^2$, where m is the rest mass, and clearly has the same value in all reference frames. If E, \mathbf{p} refer to the values measured in the lab frame Σ then those in another frame, say Σ' , moving along

the x -axis with velocity βc are found from the Lorentz transformation, given in matrix form by

$$p'_\mu = \sum_{\nu=1}^4 \alpha_{\mu\nu} p_\nu$$

where

$$\alpha_{\mu\nu} = \begin{vmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$

and $\gamma = 1/\sqrt{1 - \beta^2}$. Thus

$$\begin{aligned} p'_1 &= \gamma p_1 + i\beta\gamma p_4 \\ p'_2 &= p_2 \\ p'_3 &= p_3 \\ p'_4 &= -i\beta\gamma p_1 + \gamma p_4 \end{aligned}$$

In terms of energy and momentum

$$\begin{aligned} p'_x &= \gamma(p_x - \beta E) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - \beta p_x) \end{aligned}$$

with, of course, $\mathbf{p}'^2 - E'^2 = -m^2$. The above transformations apply equally to the space-time coordinates, making the replacements $p_1 \rightarrow x_1 (= x)$, $p_2 \rightarrow x_2 (= y)$, $p_3 \rightarrow x_3 (= z)$ and $p_4 \rightarrow x_4 (= it)$.

The 4-momentum squared in (1.1) is an example of a Lorentz scalar, i.e. the invariant scalar product of two 4-vectors, $\sum p_\mu p_\mu$. Another example is the phase of a plane wave, which determines whether it is at a crest or a trough and which must be the same for all observers. With \mathbf{k} and ω as the propagation vector and the angular frequency, and in units $\hbar = c = 1$,

$$\text{phase} = \mathbf{k} \cdot \mathbf{x} - \omega t = \mathbf{p} \cdot \mathbf{x} - Et = \sum p_\mu x_\mu$$

The Minkowski notation used here for 4-vectors defines the *metric*, namely the square of the 4-vector momentum $p = (\mathbf{p}, iE)$ so that

$$\text{metric} = (4\text{-momentum})^2 = (3\text{-momentum})^2 - (\text{energy})^2$$

In analogy with the space-time components, the components $p_{x,y,z}$ of 3-momentum are said to be *spacelike* and the energy component E , *timelike*. Thus,