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高等学校数学双语教学推荐教材

线性代数

（英文版·第四版）

Linear Algebra (Fourth Edition)

A Modern Introduction

戴维·普尔（David Poole） 著

阳庆节 审

中国人民大学出版社

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内容简介

本书根据戴维·普尔的创新之作《线性代数：现代教程（第四版）》删减而成，详细介绍了线性代数的基本内容。

本书共有七章，内容包括：向量、线性方程组、矩阵、特征值与特征向量、正交性、向量空间以及距离与逼近。

本书以向量为切入点，为学生从计算数学过渡到理论数学做好铺垫。编写上结合了传统的叙述方法和现代以学生为中心的教学方式，强调几何理解，通过向量和向量几何帮助学生直观理解概念，提升数学的抽象思维能力。本书注重理论与应用的平衡，使理论、计算和应用各方面的内容均以灵活且完整的方式呈现。本书包含不同学科的大量应用，进一步说明线性代数是现实生活问题建模的有力工具。

本书语言流畅，通俗易懂，既可以作为高等院校线性代数课程的双语教材和教师参考书，也可以作为国际课程或国际培训机构所需要的线性代数教材。

作者简介

戴维 · 普尔是加拿大特伦特大学的数学教授，从1984年开始就在此任教。普尔博士获得过许多教学奖，包括特伦特大学杰出教学最高奖（Symons Award），三次杰出教学荣誉奖，安大略省教师协会奖（2002），加拿大数学会杰出教学奖（2009）等。普尔曾于2002—2007年担任特伦特大学主管教学的副校长。他的研究领域是离散数学、环论和数学教育。他于1976年在阿卡迪亚大学获得学士学位，并分别于1977年和1984年在麦克马斯特大学获得硕士和博士学位。戴维 · 普尔喜欢徒步旅行和烹饪，还是一个超级电影迷。

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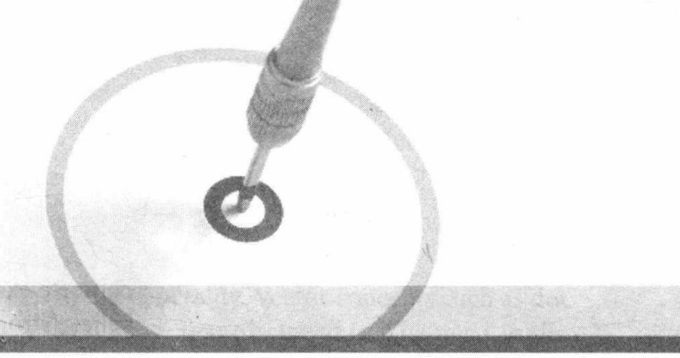
Dedicated to the memory of
Peter Hilton, who was an
exemplary mathematician,
educator, and citizen—a unit
vector in every sense.

Preface



The last thing one knows when writing a book is what to put first.

—Blaise Pascal
Pensées, 1670

The fourth edition of *Linear Algebra: A Modern Introduction* preserves the approach and features that users found to be strengths of the previous editions. However, I have streamlined the text somewhat, added numerous clarifications, and freshened up the exercises.

I want students to see linear algebra as an exciting subject and to appreciate its tremendous usefulness. At the same time, I want to help them master the basic concepts and techniques of linear algebra that they will need in other courses, both in mathematics and in other disciplines. I also want students to appreciate the interplay of theoretical, applied, and numerical mathematics that pervades the subject.

This book is designed for use in an introductory one- or two-semester course sequence in linear algebra. First and foremost, it is intended for students, and I have tried my best to write the book so that students not only will find it readable but also will *want* to read it. As in the first three editions, I have taken into account the reality that students taking introductory linear algebra are likely to come from a variety of disciplines. In addition to mathematics majors, there are apt to be majors from engineering, physics, chemistry, computer science, biology, environmental science, geography, economics, psychology, business, and education, as well as other students taking the course as an elective or to fulfill degree requirements. Accordingly, the book balances theory and applications, is written in a conversational style yet is fully rigorous, and combines a traditional presentation with concern for student-centered learning.

There is no such thing as a universally best learning style. In any class, there will be some students who work well independently and others who work best in groups; some who prefer lecture-based learning and others who thrive in a workshop setting, doing explorations; some who enjoy algebraic manipulations, some who are adept at numerical calculations (with and without a computer), and some who exhibit strong geometric intuition. In this edition, I continue to present material in a variety of ways—*algebraically, geometrically, numerically*, and *verbally*—so that all types of learners can find a path to follow. I have also attempted to present the theoretical, computational, and applied topics in a flexible yet integrated way. In doing so, it is my hope that all students will be exposed to the many sides of linear algebra.

For more on the recommendations of the Linear Algebra Curriculum Study Group, see *The College Mathematics Journal* **24** (1993), 41–46.

This book is compatible with the recommendations of the Linear Algebra Curriculum Study Group. From a pedagogical point of view, there is no doubt that for most students concrete examples should precede abstraction. I have taken this approach here. I also believe strongly that linear algebra is essentially about vectors and that students need to see vectors first (in a concrete setting) in order to gain some geometric insight. Moreover, introducing vectors early allows students to see how systems of linear equations arise naturally from geometric problems. Matrices then arise equally naturally as coefficient matrices of linear systems and as agents of change (linear transformations). This sets the stage for eigenvectors and orthogonal projections, both of which are best understood geometrically. The dart that appears on the cover of this book symbolizes a vector and reflects my conviction that geometric understanding should precede computational techniques.

I have tried to limit the number of theorems in the text. For the most part, results labeled as theorems either will be used later in the text or summarize preceding work. Interesting results that are not central to the book have been included as exercises or explorations. For example, the cross product of vectors is discussed only in explorations (in Chapters 1 and 4). Unlike most linear algebra textbooks, this book has no chapter on determinants. The essential results are all in Section 4.2, with other interesting material contained in an exploration. The book is, however, comprehensive for an introductory text. Wherever possible, I have included elementary and accessible proofs of theorems in order to avoid having to say, "The proof of this result is beyond the scope of this text." The result is, I hope, a work that is self-contained.

I have not been stingy with the applications: There are many more in the book than can be covered in a single course. However, it is important that students see the impressive range of problems to which linear algebra can be applied. I have included some modern material on finite linear algebra and coding theory that is not normally found in an introductory linear algebra text. There are also several impressive real-world applications of linear algebra and one item of historical, if not practical, interest; these applications are presented as self-contained "vignettes."

I hope that instructors will enjoy teaching from this book. More important, I hope that students using the book will come away with an appreciation of the beauty, power, and tremendous utility of linear algebra and that they will have fun along the way.

Features

Clear Writing Style

The text is written is a simple, direct, conversational style. As much as possible, I have used "mathematical English" rather than relying excessively on mathematical notation. However, all proofs that are given are fully rigorous. Concrete examples almost always precede theorems, which are then followed by further examples and applications. This flow—from specific to general and back again—is consistent throughout the book.

Key Concepts Introduced Early

Many students encounter difficulty in linear algebra when the course moves from the computational (solving systems of linear equations, manipulating vectors and matrices) to the theoretical (spanning sets, linear independence, subspaces, basis, and dimension). This book introduces all of the key concepts of linear algebra early, in a concrete setting, before revisiting them in full generality. Vector concepts such as dot product, length, orthogonality, and projection are first discussed in Chapter 1 in the concrete setting of $\mathbb{R}^2$ and $\mathbb{R}^3$ before the more general notions of inner product, norm, and orthogonal projection appear in Chapters 5 and 7. Similarly, spanning sets and linear independence are given a concrete treatment in Chapter 2 prior to their generalization to vector spaces in Chapter 6. The fundamental concepts of subspace, basis, and dimension appear first in Chapter 3 when the row, column, and null spaces of a matrix are introduced; it is not until Chapter 6 that these ideas are given a general treatment. In Chapter 4, eigenvalues and eigenvectors are introduced and explored for 2×2 matrices before their $n \times n$ counterparts appear. By the beginning of Chapter 4, all of the key concepts of linear algebra have been introduced, with concrete, computational examples to support them. When these ideas appear in full generality later in the book, students have had time to get used to them and, hence, are not so intimidated by them.

Emphasis on Vectors and Geometry

In keeping with the philosophy that linear algebra is primarily about vectors, this book stresses geometric intuition. Accordingly, the first chapter is about vectors, and it develops many concepts that will appear repeatedly throughout the text. Concepts such as orthogonality, projection, and linear combination are all found in Chapter 1, as is a comprehensive treatment of lines and planes in $\mathbb{R}^3$ that provides essential insight into the solution of systems of linear equations. This emphasis on vectors, geometry, and visualization is found throughout the text. Linear transformations are introduced as matrix transformations in Chapter 3, with many geometric examples, before general linear transformations are covered in Chapter 6. In Chapter 4, eigenvalues are introduced with "eigenpictures" as a visual aid. The proof of Perron's Theorem is given first heuristically and then formally, in both cases using a geometric argument. The geometry of linear dynamical systems reinforces and summarizes the material on eigenvalues and eigenvectors. In Chapter 5, orthogonal projections, orthogonal complements of subspaces, and the Gram-Schmidt Process are all presented in the concrete setting of $\mathbb{R}^3$ before being generalized to $\mathbb{R}^n$ and, in Chapter 7, to inner product spaces. The nature of the singular value decomposition is also explained informally in Chapter 7 via a geometric argument. Of the more than 300 figures in the text, over 200 are devoted to fostering a geometric understanding of linear algebra.

Examples and Exercises

There are over 400 examples in this book, most worked in greater detail than is customary in an introductory linear algebra textbook. This level of detail is in keeping with the philosophy that students should want (and be able) to read a textbook. Accordingly, it is not intended that all of these examples be covered in class; many can be assigned for individual or group study, possibly as part of a project. Most examples have at least one counterpart exercise so that students can try out the skills covered in the example before exploring generalizations.

There are over 2000 exercises, more than in most textbooks at a similar level. Answers to most of the computational odd-numbered exercises can be found in the back of the book. Instructors will find an abundance of exercises from which to select homework assignments. The exercises in each section are graduated, progressing from the routine to the challenging. Exercises range from those intended for hand computation to those requiring the use of a calculator or computer algebra system, and from theoretical and numerical exercises to conceptual exercises. Many of the examples and exercises use actual data compiled from real-world situations. For example, there are problems on modeling the growth of caribou and seal populations, radiocarbon dating of the Stonehenge monument, and predicting major league baseball players' salaries. Working such problems reinforces the fact that linear algebra is a valuable tool for modeling real-life problems.

Additional exercises appear in the form of a review after each chapter. In each set, there are 10 true/false questions designed to test conceptual understanding, followed by 19 computational and theoretical exercises that summarize the main concepts and techniques of that chapter.

Margin Icons

The margins of the book contain several icons whose purpose is to alert the reader in various ways. Calculus is not a prerequisite for this book, but linear algebra has many interesting and important applications to calculus. The $\frac{dy}{dx}$ icon denotes an example or exercise that requires calculus. (This material can be omitted if not everyone in the class has had at least one semester of calculus. Alternatively, this material can be assigned as projects.) The $a + bi$ icon denotes an example or exercise involving complex numbers. The CAS icon indicates that a computer algebra system (such as Maple, Mathematica, or MATLAB) or a calculator with matrix capabilities (such as almost any graphing calculator) is required—or at least very useful—for solving the example or exercise.

In an effort to help students learn how to read and use this textbook most effectively, I have noted various places where the reader is advised to pause. These may be places where a calculation is needed, part of a proof must be supplied, a claim should be verified, or some extra thought is required. The → icon appears in the margin at such places; the message is "Slow down. Get out your pencil. Think about this."

Acknowledgments

The reviewers of the previous edition of this text contributed valuable and often insightful comments about the book. I am grateful for the time each of them took to do this. Their judgement and helpful suggestions have contributed greatly to the development and success of this book, and I would like to thank them personally:
Jamey Bass, City College of San Francisco; Olga Brezhneva, Miami University; Karen Clark, The College of New Jersey; Marek Elzanowski, Portland State University; Christopher Francisco, Oklahoma State University; Brian Jue, California State University, Stanislaus; Alexander Kheyfits, Bronx Community College/CUNY; Henry Krieger, Harvey Mudd College; Rosanna Pearlstein, Michigan State University; William Sullivan, Portland State University; Matthias Weber, Indiana University.

I am indebted to a great many people who have, over the years, influenced my views about linear algebra and the teaching of mathematics in general. First, I would like to thank collectively the participants in the education and special linear algebra sessions at meetings of the Mathematical Association of America and the Canadian

Mathematical Society. I have also learned much from participation in the Canadian Mathematics Education Study Group and the Canadian Mathematics Education Forum.

I especially want to thank Ed Barbeau, Bill Higginson, Richard Hoshino, John Grant McLoughlin, Eric Muller, Morris Orzech, Bill Ralph, Pat Rogers, Peter Taylor, and Walter Whiteley, whose advice and inspiration contributed greatly to the philosophy and style of this book. My gratitude as well to Robert Rogers, who developed the student and instructor solutions, as well as the excellent study guide content. Special thanks go to Jim Stewart for his ongoing support and advice. Joe Rotman and his lovely book *A First Course in Abstract Algebra* inspired the etymological notes in this book, and I relied heavily on Steven Schwartzman's *The Words of Mathematics* when compiling these notes. I thank Art Benjamin for introducing me to the Codabar system and Joe Grcar for clarifying aspects of the history of Gaussian elimination. My colleagues Marcus Pivato and Reem Yassawi provided useful information about dynamical systems. As always, I am grateful to my students for asking good questions and providing me with the feedback necessary to becoming a better teacher.

I sincerely thank all of the people who have been involved in the production of this book. Jitendra Kumar and the team at MPS Limited did an amazing job producing the fourth edition. I thank Christine Sabooni for doing a thorough copyedit. Most of all, it has been a delight to work with the entire editorial, marketing, and production teams at Cengage Learning: Richard Stratton, Molly Taylor, Laura Wheel, Cynthia Ashton, Danielle Hallock, Andrew Coppola, Alison Eigel Zade, and Janay Pryor. They offered sound advice about changes and additions, provided assistance when I needed it, but let me write the book I wanted to write. I am fortunate to have worked with them, as well as the staffs on the first through third editions.

As always, I thank my family for their love, support, and understanding. Without them, this book would not have been possible.

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To the Student

"Where shall I begin, please your Majesty?" he asked.
"Begin at the beginning," the King said, gravely, "and go on till you come to the end: then stop."

—Lewis Carroll
Alice's Adventures in Wonderland, 1865

Linear algebra is an exciting subject. It is full of interesting results, applications to other disciplines, and connections to other areas of mathematics. The *Student Solutions Manual and Study Guide* contains detailed advice on how best to use this book; following are some general suggestions.

Linear algebra has several sides: There are *computational techniques, concepts,* and *applications.* One of the goals of this book is to help you master all of these facets of the subject and to see the interplay among them. Consequently, it is important that you read and understand each section of the text before you attempt the exercises in that section. If you read only examples that are related to exercises that have been assigned as homework, you will miss much. Make sure you understand the definitions of terms and the meaning of theorems. Don't worry if you have to read something more than once before you understand it. Have a pencil and calculator with you as you read. Stop to work out examples for yourself or to fill in missing calculations. The ⟶ icon in the margin indicates a place where you should pause and think over what you have read so far.

Answers to most odd-numbered computational exercises are in the back of the book. Resist the temptation to look up an answer before you have completed a question. And remember that even if your answer differs from the one in the back, you may still be right; there is more than one correct way to express some of the solutions. For example, a value of $1/\sqrt{2}$ can also be expressed as $\sqrt{2}/2$ and the set of all scalar multiples of the vector $\begin{bmatrix} 3 \\ 1/2 \end{bmatrix}$ is the same as the set of all scalar multiples of $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$.

As you encounter new concepts, try to relate them to examples that you know. Write out proofs and solutions to exercises in a logical, connected way, using complete sentences. Read back what you have written to see whether it makes sense. Better yet, if you can, have a friend in the class read what you have written. If it doesn't make sense to another person, chances are that it doesn't make sense, period.

You will find that a calculator with matrix capabilities or a computer algebra system is useful. These tools can help you to check your own hand calculations and are indispensable for some problems involving tedious computations. Technology also enables you to explore aspects of linear algebra on your own. You can play "what if?" games: What if I change one of the entries in this vector? What if this matrix is of a different size? Can I force the solution to be what I would like it to be by changing something? To signal places in the text or exercises where the use of technology is recommended, I have placed the icon CAS in the margin. The companion website that accompanies this book contains computer code working out selected exercises from the book using Maple, Mathematica, and MATLAB, as well as *Technology Bytes*, an appendix providing much additional advice about the use of technology in linear algebra.

You are about to embark on a journey through linear algebra. Think of this book as your travel guide. Are you ready? Let's go!

目录

第5章 正交性 257

第6章 向量空间 306

第7章 距离与逼近 398

Contents