计算水力学中的无网格方法

英文版

梁东方 张景新 王 华 陈日东 主编

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内容提要

相比传统的基于网格的数值方法，无网格法在处理流固耦合、变动的自由水面等问题中显示了优势，是计算水力学界的一个研究热点，但是在应用中也显示了一些局限性。本书通过对这种计算技术全面系统的总结，希望促进其在计算水力学领域得到合理的应用。

本书可供水利、土木、机械、力学等方向的科研与教学工作者学习参考。

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Preface

Most of the conventional numerical techniques, such as the finite difference method, finite volume method, boundary element method and finite element method, are based on computational grids. Each grid point has a fixed number of predefined neighbours, and the connectivity between neighbouring points is used to define mathematical operators like derivation and integration. Often, Eulerian formulation is adopted, where the grid is fixed. Such an approach encounters computational complexities in modelling problems involving moving boundaries and large convections. These complexities can be avoided with the Lagrangian approach, where the computational grid moves with the material. However, the mesh can become tangled if the deformation of the material is large.

Rather than relying on the pre-defined grid connectivity, the meshfree methods rely on discrete particles to remove and thus completely avoid the mesh tanglement. Therefore, they are often referred to as particle methods in the literature. However, care must be taken that these particles are actually interpolation points for solving the partial differential equations. Only a small minority of meshfree techniques employ physical particles. Meshfree methods have been increasingly applied in computational hydraulics. In particular, the meshfree algorithms have demonstrated advantages in handling fluid/solid interactions and free water surfaces. However, they have also shown some limitations when being applied to other problems. As a relatively new numerical method, the meshfree technique is one of the hottest research topics in computational mechanics. In computational hydraulics, it has attracted more and more research interest. Over time, various types of meshfree algorithms have been developed, with different complexities and characteristics.

We are pleased to assemble this book on the latest development of various meshfree methods in the field of computational hydraulics, after the successful organisation of the

International Symposium on Meshfree Methods in Shanghai Jiao Tong University in December 2015. The Symposium brings together many experienced researchers inside China and from abroad, and provides a platform to exchange ideas and forge collaborations in this rapidly developing field. By examining various meshfree techniques and the coupling algorithms between different meshfree and mesh-based methods, we want to highlight their differences and commonalities, together with their merits and shortcomings.

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Dongfang Liang, Shanghai Jiao Tong University
Jingxin Zhang, Shanghai Jiao Tong University
Hua Wang, Hohai University
Ridong Chen, Sichuan University

Contributors

Ante Munjiza
School of Engineering & Material Science, Queen Mary University of London, U. K

Abbas Khayyer
Department of Civil and Earth Resources Engineering, Kyoto University, Kyoto, Japan

Bing Ren
The State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, China

Chunning Ji
School of Civil Engineering, Tianjin University, China

Cheng Chen
The State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, China

Chuanhu Zhang
State Key Laboratory for Hydroscience and Engineering, Tsinghua University, Beijing, China

Chuanqi Liu
State Key Laboratory for Hydroscience and Engineering, Tsinghua University, Beijing China

Dong Xu
School of Civil Engineering, Tianjin University, China

Decheng Wan
State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China

Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

Dongfang Liang
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai Jiao Tong University, Shanghai, China

Eldad J. Avital
School of Engineering & Material Science, Queen Mary University of London, U. K

Efstathios Kaliviotis
Department of Mechanical Engineering and Materials Science and Engineering, Cyprus

Feng Jin
State Key Laboratory for Hydroscience and Engineering, Tsinghua University, Beijing, China

Guy T. Houlsby
Department of Engineering Science, University of Oxford, Oxford, UK

Gang Chen
State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

Haiou Che
School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China

Hitoshi Gotoh
Department of Civil and Earth Resources Engineering, Kyoto University, Kyoto, Japan

Hongjie Wen
The State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, China

John J. R. Williams
School of Civil Engineering, Tianjin University, China
School of Engineering & Material Science, Queen Mary University of London, U. K

Jiajie Liu
College of Water Resources & Civil Engineering, China Agricultural University, Beijing, China

Jingxin Zhang
MOE Key Laboratory of Hydrodynamics, Shanghai Jiao Tong University, Shanghai, China
School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China

Kai Wang
School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China

Kejun Yang
State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, China

Liuchao Qiu
College of Water Resources & Civil Engineering, China Agricultural University, Beijing, China

Pengzhi Lin
State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, China

Qicheng Sun
State Key Laboratory for Hydroscience and Engineering, Tsinghua University, Beijing, China

Qingwei Ma
College of Shipbuilding Engineering, Harbin Engineering University, China
Schools of Engineering and Mathematical Science, City University, UK

Ridong Chen
State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, China

Stefano Utili
School of Civil Engineering and Geosciences (CEG), Newcastle University, Newcastle, UK

Shenglong Gu
School of Water Resources and Electric Power, Qinghai University, Xining, China

Songdong Shao
Sanjiangyuan Collaborative Innovation Center, Tsinghua University, Beijing, China
School of Water Resources and Electric Power, Qinghai University, Xining, China

Songgui Chen
Tianjin Research Institute of Water Transport Engineering, Tianjin, China

Tao Zhao
State Key Laboratory of Hydraulics and Mountain River Engineering, College of Water Resource and Hydropower, Sichuan University, Chengdu, China

Wei Jian
Maritime Institute, Nanyang Technological University, Nanyang Avenue, Singapore

Wenyang Duan
College of Shipbuilding Engineering, Harbin Engineering University, China

Xianpei Zheng
School of Water Resources and Electric Power, Qinghai University, Xining, China

Xin Liu
College of Harbor, Coastal and Offshore Engineering, Hehai University, Nanjing, China

Xing Zheng
College of Shipbuilding Engineering, Harbin Engineering University, China

Xuzhen He
Department of Engineering, University of Cambridge, UK

Youlin Zhang
State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

Zhenhong Hu
College of Shipbuilding Engineering, Harbin Engineering University, China

Zhenyuan Tang
State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China
Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

Contents

MLParticle-SJTU Solver and Its Applications in Free Surface Flows

Zhenyuan Tang, Youlin Zhang, Decheng Wan *, Gang Chen
State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

* E-mail: dcwan@sjtu.edu.cn

Abstract

Our in-house particle solver MLParticle-SJTU is introduced to solve numerically the complex free surface flows in this paper. The Improved Moving Particle Semi-implicit (IMPS) method is applied in the solver MLParticle-SJTU, which includes ① the non-singularity kernel function, ② the mixed source term for Pressure Poisson Equation (PPE), ③ the accuracy detection method for free surface particles in the basis of the asymmetric distribution of neighboring particles. To extend the solver into 3D numerical simulations, three kinds of acceleration techniques are employed in the context of MLParticle-SJTU solver, including ① multi-CPU parallel computation, ② GPU parallel, ③ spatially local refine technique. The acceleration performances of these techniques are also tested by free surface flows respectively. Afterwards, the MLParticle-SJTU solver is applied to the 3D liquid sloshing in tank with baffles to numerically investigate the effects of different kinds of baffle.

Keywords

improved Moving Particle Semi-implicit (MPS) method, MLParticle-SJTU solver, free surface flows, dam-break, sloshing, parallel computation, GPU, Overlapping Particle Technique (OPT), multi-resolution technique

1 Introduction

Moving Particle Semi-implicit (MPS) is a meshless particle method for incompressible fluid [1, 2]. Similar to Smoothed Particle Hydrodynamics (SPH) method, the fluid is represented by a set of arbitrarily distributed particles which carry the necessary properties

such as the mass, momentum in the MPS method. Thanks to its Lagrangian nature, the MPS method has great potentials in those flows characterized by the large free surface deformation. Nevertheless, it suffers from two main shortcomings in terms of the pressure fluctuation and high computational cost. In the development of the MPS method, numerous excellent improvements are made to enhance the performance of the MPS method and the acceleration techniques are developed to extend the application of MPS method into large-scale flows.

The first disadvantage of the original MPS method is the large amplitude and high frequency oscillation of the pressure, which is highly similar to SPH method. However, amount of modifications are made to improve the accuracy and the stability of the MPS method in relation to the discretized models of differential operator and the judgment of surface particles, i. e. , the gradient model, the Laplacian model, the source term of Pressure Poisson Equation (PPE) and the free surface assessment. Firstly, to avoid the momentum non-conservation of the gradient model in the original MPS method, Khayyer and Gotoh [3] derived a fully anti-symmetric gradient model by introducing an imaginary point in the center of two neighboring particles. Similar to the conservative gradient formula in the SPH method [4, 5], Tanaka and Masunaga [6] directly gave an anti-symmetric and conservative gradient model in the MPS method. Tsuruta et al. [7] proposed a Dynamically Stabilized (DS) gradient model considering the instantaneous distribution of particles. Secondly, the source term of the PPE plays an important role in suppressing the pressure oscillation. Following their previous works on anti-symmetric gradient model, Khayyer and Gotoh [8] replaced the original source term of the PPE with the accurate time variation of Particle Number Density (PND) (denote higher order source term, HS). Tanaka and Masunaga [6] proposed a mixed source term for pressure Poisson equation by combining the Divergence-Free (DF) condition and the Particle Number Density Invariant (PNDI) condition. This mixed source term was further investigated by Lee et al. [9] and the range of the weight of the DF term in the mixed source term was given in the basis of a large number of numerical simulations. In addition, Tanaka and Masunaga [6] introduced a quasi-compressibility term in the PPE to stabilize the numerical simulations. Furthermore, Kondo and Koshizuka [10] derived a mixed source term consisted of one main part and two error-compensation parts. However, the corresponding coefficients for these three terms were not given in their article. Then, Khayyer and Gotoh [10] improved this source term and the corresponding coefficients of each term can be given directly and did not depend on special flow problems (denote multi-term source for PPE). Thirdly, the Laplacian model in the standard MPS method is derived according to a transient diffusion problem. Then, some modifications are carried out based on the divergence of gradient model. Zhang et al. [11] gave a Laplacian model combining the divergence model and gradient model, which was applied into heat transfer problems. Khayyer and Gotoh [12, 13] derived a higher order Laplacian model (HL) based on the divergence of the SPH gradient model and the original MPS kernel function, and they further extended it into 3D numerical simulations. Considering the corrected gradient model, Ikari et al. [14] investigated a corrected higher order Laplacian model. Fourthly, the misjudgment of free surface particles may lead to unphysical pressure fluctuation when solving the PPE. In the original MPS method, the target particle with small particle number density can be considered as surface particle by Koshizuka et al. [1]. Then, Tanaka and

Masunaga [6] adopted the number of neighboring particles to detect the free surface particle. Shibata et al. [15] developed a free surface detection method by using a virtual light source and virtual screen. Zhang and Wan [16] presented a surface judgment method based on the asymmetric distribution of neighboring particles, which was firstly proposed by Khayyer et al. [17]. Different from the surface assessment by Khayyer et al. [17], the weight function was considered in the summation function by Zhang and Wan [16]. Chen et al. [18] introduced the conceptual particles to avoid the free surface particle detection in the MPS method and developed No Surface Detection (NSD) - MPS method to suppress the pressure oscillation. Different from the above treatments in the MPS method, Chien et al. [19] proposed a pressure-convection MPS method in the basis of the pressure iteration algorithm following the idea of the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE). These improvements have been proved that they can enhance the accuracy and stability of the MPS method, which can extend the application of the MPS method in free surface flows.

The second disadvantage of the MPS method is its high computational cost since the pressure field is predicted by solving the pressure Poisson equation implicitly. This shortcoming may restrict its application into large-scale flows. To overcome this problem, the acceleration techniques in the concept of MPS method are quite significant. One kind of acceleration method is to increase the hardware sources, i. e. GPU and multi-CPU. Amount of works on GPU and multi-CPU parallel computations are carried out in the context of SPH method, and enormous 3D applications are also reported. The relative details can be found in Refs. [20 - 28]. However, the relative studies in the MPS is rarely reported since the pressure is predicted implicitly. Optimization algorithms should be developed carefully to achieve high efficiency for parallel computation in the MPS method. Hori et al. [29] investigated the GPU parallel technique and discussed its efficiency according to 2D free surface flows. Ovaysi and Piri [30] presented a multi-GPU acceleration based on MMPS (Modified Moving Particle Semi-implicit) method. For multi-CPU parallel computation, two represent parallel decomposition strategies are particle-based decomposition and domain-based decomposition in the MPS method. Iribe et al. [31] investigated particle decomposition by introducing the renumbering technique. Different from the decomposition strategy by Iribe et al. [31], Zhang et al. [32] developed a parallel improved MPS code in the basis of domain decomposition. Another kind of acceleration method is to perform local refine simulations with non-uniform particle distribution. In the SPH method, amounts of studies on the spatially local refine techniques have been carried out and the corresponding applications are also reported in the Refs. [33 - 39]. Recently, the relative work in the framework of MPS method have also carried out. Three major represent methods can be found in the Refs. [40 - 46], which includes the variable size technique [40], Overlapping Particle Technique (OPT) [44, 45] and multi-resolution technique [46]. In the OPT [44], the mass of the high-resolution particle system maybe non-conservative, which may restrict the further application of the OPT. Comparing the local refine techniques in the SPH method, much more work and improvements should be carried out in the MPS method.

Besides the improvements and acceleration techniques in the MPS method, coupling the MPS method with the other methods such as the Finite Element Method (FEM), or the Boundary Element Method (BEM) is also an alternative approach to extend its applications.

Sueyoshi et al. [47] carried out the simulation of wave-body interaction problems by combining MPS method and the BEM. In details, only the zone near the free surface was distributed by particles and the fluid field in the other zone was calculated by the BEM. Lee et al. [48] analyzed the fluid-shell interaction problems by combining the MPS method and the FEM, where the former and the latter were employed for fluid flow and structural deformation respectively. Following the explicit algorithm in the SPH method, Shakibaeinia and Jin [49] developed a Weakly Compressible model in the concept of the MPS method (WC - MPS) and further proposed an inflow-outflow boundary condition based upon particle recycling strategy. On the basis of the WC - MPS, they proposed a two-phase model [50, 51].

Thanks to the improvements and the acceleration techniques, the application of the MPS method could be extended into numerous free surface flows and the large-scale numerical simulations in the framework of the MPS can also be performed. Dam breaking is always considered as the benchmark to validate the particle code by MPS researchers [9, 50, 52]. Liquid sloshing is a highly nonlinear problem, and it always include the largely free surface deformation such as overturning of free surface and splashing. In our previous work, the liquid sloshing flows in rectangular and membrane tanks were carried out numerically [53 - 56], where the sloshing in a tank under multi-degree of freedom motions was also simulated [57]. Alam et al. [58] discussed the effect of the surface tension when a triangle wedge entering water. Shibata et al. [59] applied the MPS method into a 3D lifeboat falling into water freely. Sueyoshi et al. [60] applied the MPS method into the wave and floating body interaction. Shibata and Koshizuka [61] investigated the behavior of water shipping. Furthermore, Shibata et al. [62] studied the shipping water behavior of a advancing ship in the wave condition. In these cases, the numerical tank was simplified to reduce the computational cost in the 3D MPS method. In addition, Shibata et al. [63] calculated the interaction between ship and wave by using MPS method. Zhang et al. [32] applied the parallel improved MPS method into green water incidents. Chikazawa et al. [64] extended the MPS method into the elastic and visco-plastic structures and Fluid-Structure Interactions (FSI) problem, where the rotation model was given similar to the treatment of traditional gradient model and divergence model.

This article is organized as follows. Firstly, the Improved MPS (IMPS) method adopted by the meshless particle solver MLParticle-SJTU is introduced, and the following improvements are included: (a) the non-singularity kernel function; (b) the mixed source term for pressure Poisson equation; (c) the accuracy surface detection method based on the non-symmetric distribution of neighboring particles. Then, the acceleration techniques are also adopted in the MLParticle-SJTU solver, i. e. GPU, multi-CPU, overlapping particle and multi-resolution techniques. Finally, the solver is applied into liquid sloshing flows.

2 MPS Numerical Method

In the IMPS method, the governing equation for the incompressible fluid can be read as following:

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot \boldsymbol{V} = 0, \tag{1}$$

$$\frac{D\boldsymbol{V}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{V} + \boldsymbol{g}, \tag{2}$$

where ρ and ν are the density and the kinematic viscosity of fluid, $\boldsymbol{V}$, P, $\boldsymbol{g}$ are the velocity vector, the pressure, and the gravitational acceleration vector, t indicates the time. Moreover, D/Dt is the material derivative.

The differential terms in the Eqs. (1) and (2) can be numerically calculated by the following equations in the improved MPS method [1, 6, 65].

$$\langle \nabla P\rangle_i = \frac{D}{n^0}\sum_{j\neq i}\frac{P_j + P_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)W(|\boldsymbol{r}_j - \boldsymbol{r}_i|), \tag{3}$$

$$\langle \nabla\cdot \boldsymbol{V}\rangle_i = \frac{D}{n^0}\sum_{j\neq i}\frac{(\boldsymbol{V}_j - \boldsymbol{V}_i)\cdot(\boldsymbol{r}_j - \boldsymbol{r}_i)}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}W(|\boldsymbol{r}_j - \boldsymbol{r}_i|), \tag{4}$$

$$\langle \nabla^2\phi\rangle_i = \frac{2D}{n^0\lambda_i}\sum_{j\neq i}(\phi_j - \phi_i)W(|\boldsymbol{r}_j - \boldsymbol{r}_i|), \tag{5}$$

$$\lambda_i = \frac{\sum_{j\neq i}W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)\cdot|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum_{j\neq i}W(|\boldsymbol{r}_j - \boldsymbol{r}_i|)}, \tag{6}$$

where D is the space dimension, ϕ is a physical quantity. $\boldsymbol{W}(|\boldsymbol{r}_j - \boldsymbol{r}_i|)$ is the kernel function, and the non-singularity kernel function is adopted in the IMPS method as Eq. (7) [53].

$$W(r) = \begin{cases}\dfrac{r_e}{0.85r + 0.15r_e} - 1, & 0 \leqslant r < r_e,\\ 0, & r_e \leqslant r,\end{cases} \tag{7}$$

where r_e is the influent radius for the candidate particle.

In the IMPS method, the pressure field is obtained by solving the following PPE [6, 9].

$$\langle \nabla^2 P^{k+1}\rangle_i = (1-\gamma)\frac{\rho}{\Delta t}\nabla\cdot \boldsymbol{V}_i^* - \gamma\frac{\rho}{\Delta t^2}\frac{\langle n^k\rangle_i - n^0}{n^0}, \tag{8}$$

where the superscripts k and $k+1$ are the physical quantities in the k th and $k+1$ th time step, γ is a bending parameter.

Before solving the pressure Poisson equation, the free surface particles should be judged firstly. In the present work, the detection method of surface particles is by combining the small PND and the asymmetric distribution of neighboring particles [16]. Particularly, the target particle i satisfies the Eq. (10) or Eq. (11) will be set as surface particle.

$$\boldsymbol{F}_i = \frac{D}{n^0} \sum_{j \neq i} \frac{1}{| \boldsymbol{r}_i - \boldsymbol{r}_j |} (\boldsymbol{r}_i - \boldsymbol{r}_j) W(r_{ij}), \tag{9}$$

$$n_i^* < 0.8 n^0, \tag{10}$$

$$0.8 n^0 \leqslant n_i^* < 0.97 n^0 \text{ and } | \boldsymbol{F}_i | > \alpha. \tag{11}$$

3 Multi-CPU Parallel Computation for 3D Dam Breaking

3D dam breaking with obstacle: For large-scale numerical simulations, multi-CPU technique is an alternative tool to accelerate the computation in the framework of MPS method. In this section, a 3D dam breaking with an obstacle is carried out by using multi-CPU parallel computation to validate the parallel efficiency of the meshless particle solver MLParticle-SJTU. For parallel computation, there are two key problems. The first one is the distribution of the computational load. In the particle method such as the SPH and the MPS, particle decomposition and domain decomposition are two major strategies to distribute the computational load into different cores. Here, only the domain decomposition strategy is adopted in the MLParticle-SJTU solver. In addition, the initial uniform distribution of computational load may become unbalanced in the following numerical simulation. In the MLParticle-SJTU solver, dynamic load balance is also employed to achieve high parallel efficiency. The details about the domain decomposition and the dynamic load balance can be found in our previous work [32, 66].

Here, the 3D dam breaking with an obstacle is performed by MLParticle-SJTU solver [66]. The numerical tank for the dam breaking is portrayed in Fig. 1 and the detailed dimensional size of the obstacle is also given in Fig. 2. The water column with initial height 0.55 m is locked in the right part of the tank. Much more details about the experiment can be seen in Kleefsman et al. [67]. In this case, the number of particles are nearly 1,010,000. The case is carried out on Tianhe-1A supercomputer which is located at the National Supercomputing Center in Tianjin, China, equipped with 14,336 Intel Xeon X5670 processors.

For parallel computation, speed-up is usually employed to analyze the parallel efficiency. The speed-up can be defined as following:

$$S_n = \frac{T_1}{T_n}, \tag{12}$$

where T_1 and T_n are the computational CPU time with only one and n processor(s) respectively. For the parallel computation, the computational load are distributed into n processors. During the calculation in each time step, processors have to exchange the necessary information with their neighboring processor(s) for the complete neighbor particles of the target particle. Different from the explicit algorithm in the weakly compressible SPH method, the pressure is obtained by solving the pressure Poisson equation implicitly in the MPS method and the processors have to exchange information