

# 并行计算导论

(英文版·第2版)

ANANTH GRAMA · ANSHUL GUPTA  
GEORGE KARYPIS · VIPIN KUMAR

Introduction to  
**Parallel Computing**

Second Edition

ADDISON  
WESLEY

Ananth Grama  
Anshul Gupta  
(美) George Karypis 著  
Vipin Kumar



机械工业出版社  
China Machine Press

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Parallel Computing



经典原版书库

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## 出版者的话

文艺复兴以降，源远流长的科学精神和逐步形成的学术规范，使西方国家在自然科学的各个领域取得了垄断性的优势；也正是这样的传统，使美国在信息技术发展的六十多年间名家辈出、独领风骚。在商业化的进程中，美国的产业界与教育界越来越紧密地结合，计算机学科中的许多泰山北斗同时身处科研和教学的最前线，由此而产生的经典科学著作，不仅擘划了研究的范畴，还揭橥了学术的源变，既遵循学术规范，又自有学者个性，其价值并不会因年月的流逝而减退。

近年，在全球信息化大潮的推动下，我国的计算机产业发展迅猛，对专业人才的需求日益迫切。这对计算机教育界和出版界都既是机遇，也是挑战；而专业教材的建设在教育战略上显得举足轻重。在我国信息技术发展时间较短、从业人员较少的现状下，美国等发达国家在其计算机科学发展的几十年间积淀的经典教材仍有许多值得借鉴之处。因此，引进一批国外优秀计算机教材将对我国计算机教育事业的发展起积极的推动作用，也是与世界接轨、建设真正的世界一流大学的必由之路。

机械工业出版社华章图文信息有限公司较早意识到“出版要为教育服务”。自1998年开始，华章公司就将工作重点放在了遴选、移译国外优秀教材上。经过几年的不懈努力，我们与Prentice Hall, Addison-Wesley, McGraw-Hill, Morgan Kaufmann等世界著名出版公司建立了良好的合作关系，从它们现有的数百种教材中甄选出Tanenbaum, Stroustrup, Kernighan, Jim Gray等大师名家的一批经典作品，以“计算机科学丛书”为总称出版，供读者学习、研究及收藏。大理石纹理的封面，也正体现了这套丛书的品位和格调。

“计算机科学丛书”的出版工作得到了国内外学者的鼎力襄助，国内的专家不仅提供了中肯的选题指导，还不辞劳苦地担任了翻译和审校的工作；而原书的作者也相当关注其作品在中国的传播，有的还专诚为其书的中译本作序。迄今，“计算机科学丛书”已经出版了近百个品种，这些书籍在读者中树立了良好的口碑，并被许多高校采用为正式教材和参考书籍，为进一步推广与发展打下了坚实的基础。

随着学科建设的初步完善和教材改革的逐渐深化，教育界对国外计算机教材的需求和应用都步入一个新的阶段。为此，华章公司将加大引进教材的力度，在“华章教育”的总规划之下出版三个系列的计算机教材：除“计算机科学丛书”之外，对影印版的教材，则单独开辟出“经典原版书库”；同时，引进全美通行的教学辅导书“Schaum's Outlines”系列组成“全美经典学习指导系列”。为了保证这三套丛书的权威性，同时也为了更好地为学校和老师服务，华章公司聘请了中国科学院、北京大学、清华大学、国防科技大学、复旦大学、上海交通大学、南京大学、浙江大学、中国科技大学、哈尔滨工业大学、西安交通大学、中国人民大学、北京航空航天大学、北京邮电大学、中山大学、解放军理工大学、郑州大学、湖北工学院、中国国



家信息安全测评认证中心等国内重点大学和科研机构在计算机的各个领域的著名学者组成“专家指导委员会”，为我们提供选题意见和出版监督。

这三套丛书是响应教育部提出的使用外版教材的号召，为国内高校的计算机及相关专业的教学度身订造的。其中许多教材均已为M. I. T., Stanford, U.C. Berkeley, C. M. U. 等世界名牌大学所采用。不仅涵盖了程序设计、数据结构、操作系统、计算机体系结构、数据库、编译原理、软件工程、图形学、通信与网络、离散数学等国内大学计算机专业普遍开设的核心课程，而且各具特色——有的出自语言设计者之手、有的历经三十年而不衰、有的已被全世界的几百所高校采用。在这些圆熟通博的名师大作的指引之下，读者必将在计算机科学的宫殿中由登堂而入室。

权威的作者、经典的教材、一流的译者、严格的审校、精细的编辑，这些因素使我们的图书有了质量的保证，但我们的目标是尽善尽美，而反馈的意见正是我们达到这一终极目标的重要帮助。教材的出版只是我们的后续服务的起点。华章公司欢迎老师和读者对我们的工作提出建议或给予指正，我们的联系方法如下：

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# Preface

Since the 1994 release of the text “Introduction to Parallel Computing: Design and Analysis of Algorithms” by the same authors, the field of parallel computing has undergone significant changes. Whereas tightly coupled scalable message-passing platforms were the norm a decade ago, a significant portion of the current generation of platforms consists of inexpensive clusters of workstations, and multiprocessor workstations and servers. Programming models for these platforms have also evolved over this time. Whereas most machines a decade back relied on custom APIs for messaging and loop-based parallelism, current models standardize these APIs across platforms. Message passing libraries such as PVM and MPI, thread libraries such as POSIX threads, and directive based models such as OpenMP are widely accepted as standards, and have been ported to a variety of platforms.

With respect to applications, fluid dynamics, structural mechanics, and signal processing formed dominant applications a decade back. These applications continue to challenge the current generation of parallel platforms. However, a variety of new applications have also become important. These include data-intensive applications such as transaction processing and information retrieval, data mining and analysis, and multimedia services. Applications in emerging areas of computational biology and nanotechnology pose tremendous challenges for algorithms and systems development. Changes in architectures, programming models, and applications are also being accompanied by changes in how parallel platforms are made available to the users in the form of grid-based services.

This evolution has a profound impact on the process of design, analysis, and implementation of parallel algorithms. Whereas the emphasis of parallel algorithm design a decade back was on precise mapping of tasks to specific topologies such as meshes and hypercubes, current emphasis is on programmability and portability, both from points of view of algorithm design and implementation. To this effect, where possible, this book employs an architecture independent view of the underlying platforms and designs algorithms for an abstract model. With respect to programming models, Message Passing Interface (MPI), POSIX threads, and OpenMP have been selected. The evolving application mix for parallel computing is also reflected in various examples in the book.

This book forms the basis for a single concentrated course on parallel computing or a two-part sequence. Some suggestions for such a two-part sequence are:

1. Introduction to Parallel Computing: Chapters 1–6. This course would provide the basics of algorithm design and parallel programming.



## XX Preface

2. Design and Analysis of Parallel Algorithms: Chapters 2 and 3 followed by Chapters 8–12. This course would provide an in-depth coverage of design and analysis of various parallel algorithms.

The material in this book has been tested in Parallel Algorithms and Parallel Computing courses at the University of Minnesota and Purdue University. These courses are taken primarily by graduate students and senior-level undergraduate students in Computer Science. In addition, related courses in Scientific Computation, for which this material has also been tested, are taken by graduate students in science and engineering, who are interested in solving computationally intensive problems.

Most chapters of the book include (i) examples and illustrations; (ii) problems that supplement the text and test students' understanding of the material; and (iii) bibliographic remarks to aid researchers and students interested in learning more about related and advanced topics. The comprehensive subject index helps the reader locate terms they might be interested in. The page number on which a term is defined is highlighted in boldface in the index. Furthermore, the term itself appears in bold italics where it is defined. The sections that deal with relatively complex material are preceded by a '\*'. An instructors' manual containing slides of the figures and solutions to selected problems is also available from the publisher (<http://www.booksites.net/kumar>).

As with our previous book, we view this book as a continually evolving resource. We thank all the readers who have kindly shared critiques, opinions, problems, code, and other information relating to our first book. It is our sincere hope that we can continue this interaction centered around this new book. We encourage readers to address communication relating to this book to [book-vk@cs.umn.edu](mailto:book-vk@cs.umn.edu). All relevant reader input will be added to the information archived at the site <http://www.cs.umn.edu/~parbook> with due credit to (and permission of) the sender(s). An on-line errata of the book will also be maintained at the site. We believe that in a highly dynamic field such as ours, a lot is to be gained from a healthy exchange of ideas and material in this manner.

# Acknowledgments

We would like to begin by acknowledging our spouses, Joanna, Rinku, Krista, and Renu to whom this book is dedicated. Without their sacrifices this project would not have been seen completion. We also thank our parents, and family members, Akash, Avi, Chethan, Eleni, Larry, Mary-Jo, Naina, Petros, Samir, Subhasish, Varun, Vibhav, and Vipasha for their affectionate support and encouragement throughout this project.

Our respective institutions, Computer Sciences and Computing Research Institute (CRI) at Purdue University, Department of Computer Science & Engineering, the Army High Performance Computing Research Center (AHPCRC), and the Digital Technology Center (DTC) at the University of Minnesota, and the IBM T. J. Watson Research Center at Yorktown Heights, provided computing resources and active and nurturing environments for the completion of this project.

This project evolved from our first book. We would therefore like to acknowledge all of the people who helped us with both editions. Many people contributed to this project in different ways. We would like to thank Ahmed Sameh for his constant encouragement and support, and Dan Challou, Michael Heath, Dinesh Mehta, Tom Nurkkala, Paul Saylor, and Shang-Hua Teng for the valuable input they provided to the various versions of the book. We thank the students of the introduction to parallel computing classes at the University of Minnesota and Purdue university for identifying and working through the errors in the early drafts of the book. In particular, we acknowledge the patience and help of Jim Diehl and Rasi Eskicioglu, who worked through several early drafts of the manuscript to identify numerous errors. Ramesh Agarwal, David Bailey, Rupak Biswas, Jim Bottum, Thomas Downar, Rudolf Eigenmann, Sonia Fahmy, Greg Frederickson, John Gunnels, Fred Gustavson, Susanne Hambrusch, Bruce Hendrickson, Christoph Hoffmann, Kai Hwang, Ioannis Ioannidis, Chandrika Kamath, David Keyes, Mehmet Koyuturk, Piyush Mehrotra, Zhiyuan Li, Jens Palsberg, Voicu Popescu, Alex Pothen, Viktor Prasanna, Sanjay Ranka, Naren Ramakrishnan, Elisha Sacks, Vineet Singh, Sartaj Sahni, Vivek Sarin, Wojciech Szpankowski, Srikanth Thirumalai, Jan Vitek, and David Yau have been great technical resources. It was a pleasure working with the cooperative and helpful staff at Pearson Education. In particular, we would like to thank Keith Mansfield and Mary Lince for their professional handling of the project.

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