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概率论
及其在投资、保险、工程中的应用

（英文版）

Probability:
The Science of Uncertainty
with Applications to Investments, Insurance, and Engineering

Michael A. Bean

THE BROOKS/COLE SERIES IN ADVANCED MATHEMATICS
Paul J. Sally, Jr., EDITOR

（加）Michael A. Bean 著
西 安 大 略 大 学

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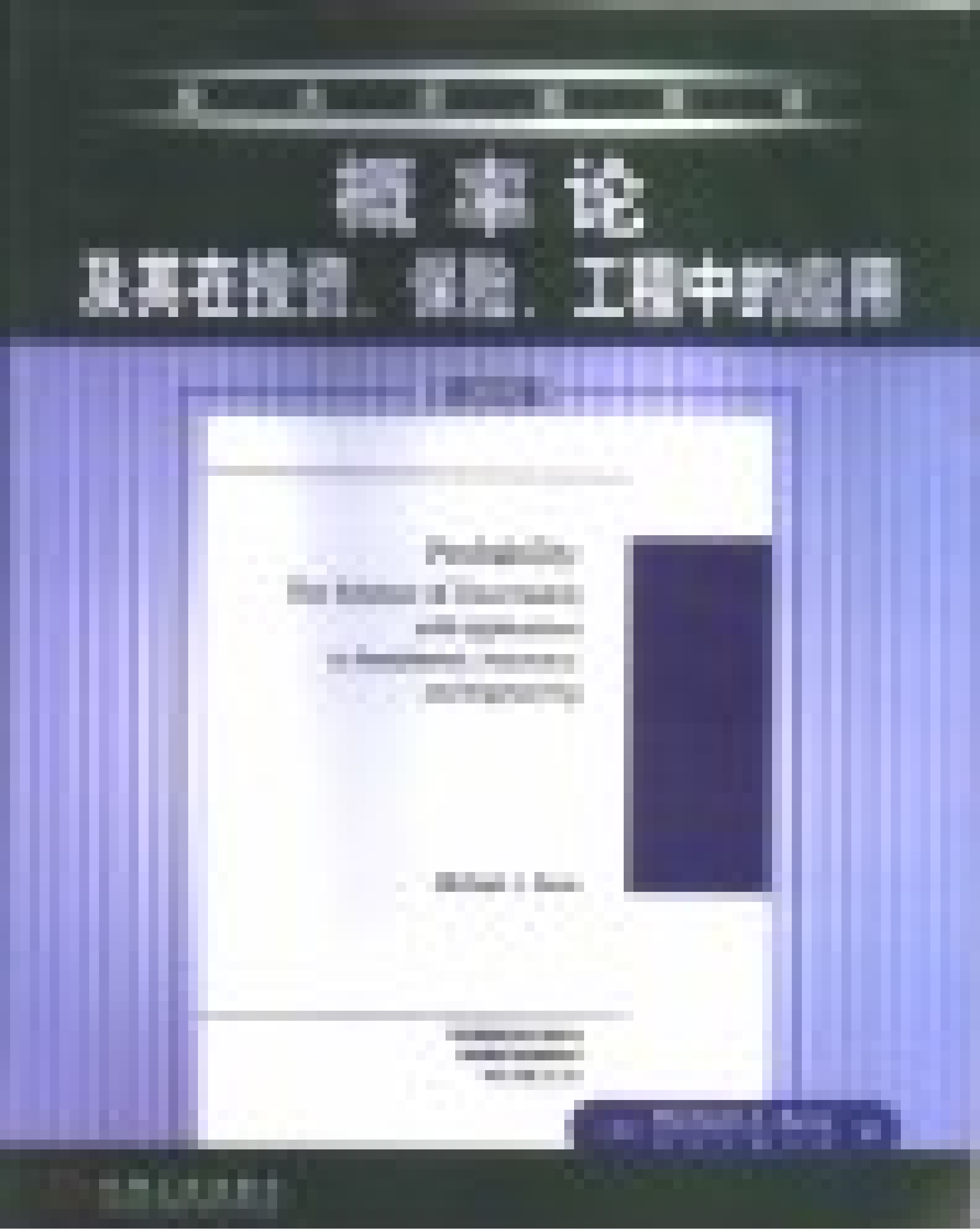
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Preface

The idea to write this book first came to me in the spring of 1995, shortly after I joined the faculty of the University of Michigan. At that time, a major review of the entire undergraduate curriculum was underway, the purpose of which was to ensure that undergraduate education remain relevant in the face of a rapidly changing world. About the same time, the Society of Actuaries, which oversees the education of actuaries in North America through the administration of its professional examinations, embarked on a major redesign of its own curriculum to keep abreast of the extraordinary changes taking place in the financial services industry. This time also saw the emergence of financial engineering as a new profession and the rise of programs in financial engineering and financial mathematics around the world. My goal in writing this book was to update the undergraduate probability curriculum to reflect these changes and to incorporate many of the new and interesting applications of probability arising in the fields of engineering, insurance, and investments.

Key Features of This Text

This book has several features that distinguish it from other probability texts currently on the market:

- Key concepts are introduced through detailed motivating examples.
- Random variables and probability distributions are introduced early in the text.
- The text has a large number of detailed worked-out examples and problems with an emphasis on applications from engineering, insurance, and investments.
- There is a wide range of exercises of varying difficulty, many of which are suitable for student projects or group work.
- The text includes topics not covered or not emphasized in other probability texts, such as the geometric expected value, normal power approximations, mixtures, and portfolio selection models.
- The text is written in a clear, concise, expository style, with extensive graphical illustrations throughout, making it well suited for individual study or self-learning.

How to Use This Book

This book can be used in a variety of probability courses with a variety of teaching styles. There is considerably more material in this book than would normally be covered in a one-semester course. Hence, an instructor will have to be selective in what is covered.

What I consider to be core material for an undergraduate probability course is contained in Chapters 3, 4, 5, and 6. An instructor teaching probability should plan on covering most of the material in these chapters, although discussions of some specialized topics such as the Pareto distribution and the beta distribution can be omitted without loss of continuity.

The material in Chapter 7 and Chapter 8 is also important and should be covered to some extent. Instructors teaching engineering students will probably want to discuss the techniques for determining the distribution of a transformed random variable and the distributions of sums and products (§7.1 and §8.1) quite thoroughly. Instructors teaching other types of students may wish to focus on the law of large numbers (§8.4) instead. The sections labeled as being "optional" may be omitted without loss of continuity.

Chapter 2 is unique in that it uses four extended examples to motivate many of the key concepts covered in the rest of the book. I have found that by discussing these examples at the beginning of the course (i.e., before covering Chapters 3 through 8), students are able to make important conceptual discoveries early on and end up learning a great deal of probability theory in a relatively short period of time. Instructors familiar with the discovery method of learning should be quite comfortable using Chapter 2 in this way. Instructors accustomed to teaching in a more traditional way can begin the course at Chapter 3 (after a brief survey of Chapter 1) and use Chapter 2 selectively or omit it entirely.

The material in Chapters 9 and 10 is supplementary and would not normally be covered in a one-semester course in probability. However, this material is good for student projects.

Chapter summaries are provided in the first four chapters to recap the main ideas and help the reader acquire perspective on the subject. Chapters 5 and 6 are written in a summary style throughout and hence do not require separate summary sections. Chapters 7 through 10 are designed to be covered selectively and do not contain summary sections.

An instructor's manual with solutions to all of the exercises in the book is available with a bound-in CD. This manual contains a wealth of material including detailed descriptions of the *Mathematica* commands for constructing the graphs in this book. It is freely available to instructors who adopt this book as a text for their course. For details on how to obtain a copy, contact your Brooks/Cole representative or visit the Brooks/Cole Web site at www.brookscole.com.

Acknowledgments

Writing a textbook of this magnitude is a major undertaking which requires the assistance of many people. I would like to begin by thanking my publisher, Gary W. Ostedt, for agreeing to take on this project and by thanking Carol Benedict, Kelsey McGee, Karin Sandberg, Dan Thiem, and the rest of the Brooks/Cole team for their part in making this book a reality. Thanks also go to Robin Gold at Forbes Mill Press for keeping production on track under a tight schedule and for being patient with me when other responsibilities demanded my attention.

I would also like to thank the reviewers for their valuable comments and suggestions, many of which have been incorporated into the final manuscript. These reviewers include Phillip Beckwith of Michigan Technological University, John Holcomb of Cleveland State University, Paul Holmes of Clemson University, Ian McKeague of Florida State University, and Harry Panjer of the University of Waterloo, former president of the Canadian Institute of Actuaries.

Special thanks also go to my colleague Jack Goldberg for his helpful advice on the publication process (from a textbook author's perspective) and to John Birge for supporting this project in its early stages. I am also grateful to the National Science Foundation and the Center for Research on Learning and Teaching at the University of Michigan for their support of the curriculum development initiatives that ultimately led to my writing this book. Finally, I would like to thank my parents for instilling in me an appreciation of the importance of education and for supporting me in all my endeavors.

Michael Bean

About the Author

Michael A. Bean, Ph.D., FSA, FCIA, has held teaching and research appointments at universities throughout the United States and Canada, including the University of Michigan at Ann Arbor, the University of Toronto, the University of California at Berkeley, the University of Waterloo, and the University of Western Ontario.

He has also worked as a practicing investment actuary for a major international financial services company based in Toronto, and has written numerous articles in the mathematical sciences that have been published in some of the leading journals of the field.

He has a Ph.D. from the University of Waterloo, a Licentiate from Trinity College of Music in London, England, and is a fully qualified Fellow of the Society of Actuaries and a Fellow of the Canadian Institute of Actuaries.

Contents

1 Introduction

Uncertainty is very much a part of the world in which we live. Indeed, one often hears the well-known cliché that the only certainties in life are death and taxes. However, even these supposed certainties are far from being completely certain, as any actuary or accountant can attest; for although one's eventual death and the requirement that one pay taxes may be facts of life, the timing of one's death and the amount of taxes one must pay are far from certain and are generally beyond one's control.

Uncertainty can make life interesting. Indeed, the world would likely be a very dull place if everything were perfectly predictable. However, uncertainty can also cause grief and suffering. For example, the sudden and premature death of a family breadwinner can cause great financial distress for surviving family members with limited means of support. The age-old fascination of humans with predicting the future, as evidenced by the ever-present popularity of astrology and fortune-telling, and the development of institutions such as insurance to make the effects of an uncertain future less severe are no doubt due in large part to a recognition of the malevolent role that uncertainty can play in one's life.

This book presents the scientific approach to uncertainty, known as probability, which has been developed over the past 350 years and is generally accepted in the scientific community. There are undoubtedly many other approaches, such as mysticism and astrology, which some people use to understand uncertainty. However, these approaches lie beyond the realm of science and will not be considered in this book.

In this introductory chapter, we consider what the nature and scope of probability is and how it arises in engineering and the sciences. We also consider how the notion of a probability should be defined and how it can be interpreted. We then discuss how probability models are constructed in practice. We end this introductory chapter with an outline of the topics covered in the rest of the book.

1.1 What Is Probability?

Probability is the branch of science concerned with the study of mathematical techniques for making quantitative inferences about uncertainty. The key words in this definition are *quantitative* and *inferences*. Indeed, as we will soon see, probability provides a mechanism for making quantitative statements about uncertainty and, more important, allows one to draw quantitative conclusions from such statements using the rules of logic.

Most historians consider the work of Fermat (1601–1665) and Pascal (1623–1662) on games of chance to be the first significant contribution to the study of probability; however, many of Fermat's and Pascal's ideas can be traced to earlier works of Cardan, Kepler, and Galileo. There is also some evidence that the Romans, many centuries before, used mortality tables[1] to predict human lifespans. Since Fermat and Pascal's time, nearly every great mathematician has made some contribution to probability. Among the more famous contributors are the Bernoullis, Laplace, DeMoivre, Poisson, DeMorgan, Venn, Bayes, Markov, and Kolmogorov. A complete and readable account of the history of the subject from the early 17th to the mid-19th century is given in the classic book by Todhunter listed at the end of this chapter. Subsequent developments up to the early 20th century are discussed in the scholarly book of the famous economist John Maynard Keynes, which is also listed at the end of the chapter.

While many scholars have studied probability purely for its intellectual and philosophical appeal, a good deal of the motivation for the subject has come, and continues to come, from practical problems outside of mathematics. Indeed, the development of probability since Fermat's time has been heavily influenced by investigations in gaming, demography, insurance, genetics, and quantum physics, to name just a few. Moreover, the subject itself has had profound implications on everything from economics to engineering and, it could be argued, has played a significant role in the history of the world over the last 200 years. To give a simple example, consider marine insurance, whose issuance can be justified by the well-known law of averages: The availability of marine insurance enabled commercial shipping to develop on a large scale (because it freed maritime shippers from the worry of financial ruin due to a catastrophe at sea), which in turn contributed to the economic and political ascendancy of Britain in the 19th century and to international commerce as we know it.[2]

Today, probability is used in a wide range of fields including engineering, finance, medicine, meteorology, and management. We will encounter numerous applications of probability to these and other fields throughout this book.

How Is Uncertainty Quantified?

If we agree that probability, from a scientific perspective, is the study of mathematical techniques for making quantitative inferences about uncertainty, then for the subject to have any meaningful content, we must have some precise way of quantifying uncertainty and making inferences about that quantification. That there is considerable controversy over how to precisely formulate such a quantification of uncertainty is an understatement, to say the least. Indeed, some philosophers have gone so far as to argue that the very notion of uncertainty cannot be precisely quantified since to do so would, in effect, make uncertainty certain.

One approach to quantifying uncertainty is to use the concept of *relative frequency*. To describe this concept, consider an experiment with several possible outcomes which

[1] A mortality table lists the number of deaths each year for a hypothetical group of individuals assumed to be born at the same time.

[2] We will have more to say about the connection between insurance and probability in §1.4.

can be repeated a large number of times.[3] The **relative frequency** of a particular outcome of such an experiment in a sequence of repetitions of the experiment is the fraction of the total number of repetitions of the experiment that result in the desired outcome. For example, in the sequence of coin tosses resulting in H, T, T, T, H (where H signifies heads and T signifies tails), the relative frequency of heads is $2/5$, whereas in the sequence of tosses resulting in T, H, T, H, H, the relative frequency of heads is $3/5$. Experience suggests that as the number of repetitions of the experiment increases, the relative frequencies associated with a particular outcome converge to a common value. For example, the relative frequency of heads approaches $1/2$ as the number of coin tosses increases, provided that the coin is not biased.[4] This common value to which the relative frequencies converge is called the **probability** of the desired outcome.

This approach to quantifying uncertainty, while intuitively appealing, has some major drawbacks, the most serious of which is the reliance on the ambiguous notion of a *limiting* relative frequency. The early probabilists overcame these logical difficulties by restricting their attention to experiments in which the number of outcomes is *finite* and by assuming that all outcomes of such experiments are *equally likely*, (i.e., have the same probability). While the assumption of equal likelihood of outcomes is admittedly idealized,[5] in the context of the games of chance with which the early probabilists were concerned, it is not unrealistic *provided that one correctly identifies the outcomes of the experiment.* The key to applying the classical principle of equal likelihood correctly is to identify the outcomes in such a way that all information on the underlying experiment is captured and no information is suppressed.

To illustrate the difference between a correct and an incorrect application of the principle of equal likelihood, consider the experiment in which two unbiased coins are tossed. If one identifies the possible outcomes as being head–head, head–tail, tail–head and tail–tail, then one correctly assigns a probability of 1/4 to each of these outcomes and one correctly deduces that the probability of getting exactly one head is 1/2. However, if one fails to distinguish between the coins and identifies the possible outcomes as two heads, one head–one tail, and two tails, then one incorrectly assigns a probability of 1/3 to each of the outcomes head–head, tail–tail, and one incorrectly deduces that the probability of getting exactly one head is 1/3.[6]

The great achievement of the classical probabilists was to initiate a *logical* approach to the study of uncertainty, which to a great extent is still with us today. By avoiding the difficulty inherent in considering probabilities to be limiting relative frequencies and instead assuming that all experimental outcomes are equally likely, they were able to focus their energies on developing a logical system for deducing the probabilities of particular *groups* of observations that were often too difficult to determine accurately by successive repetition of an experiment. This system of logical deduction also enabled them to avoid being misled by potentially faulty intuition.

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[3] The 'experiment' to which we refer here could be a scientific experiment or some other procedure, such as tossing a coin, which can be repeated and which has several possible outcomes.

[4] A coin is **biased** if it has a tendency to land on one side over the other. A coin is **unbiased** or **fair** if it has no such tendency.

[5] It fails, for example, when considering a coin that is biased.

[6] Interestingly enough, the mathematician D'Alembert is alleged to have believed this incorrect line of reasoning at one point in his career. We will have more to say about this particular example in Chapter 3.

TABLE 1.1 Results from 100 Repetitions of a "Ten Coin Toss"

Number of Heads	Frequency
0	0
1	1
2	3
3	12
4	21
5	24
6	22
7	13
8	4
9	0
10	0
	100

To give a simple illustration of a situation in which the deductive approach succeeds where intuition might fail, consider the probability of getting exactly five heads in ten tosses of a coin. One might think that if the coin is as likely to land heads as it is to land tails, then this probability should be 1/2 since, according to the relative frequency interpretation of probability, an unbiased coin that is tossed a large number of times will land heads approximately half the time. However, if you thought this, you would be wrong! In fact, under the assumption that the probability of getting heads in a single toss is 50%, one can show deductively (using the methods to be developed in Chapter 3) that the probability of getting exactly five heads in ten tosses of the coin is $63/256 = 0.24609375$. Interestingly enough, the value $63/256$ is in accord with the relative frequency interpretation of probability, as one can confirm by repeatedly tossing a fair coin ten times and computing the corresponding relative frequencies. The results of 100 such repetitions are given in Table 1.1.

The deductive approach to probability taken by the classical probabilists is an example of the *axiomatic approach*. The **axiomatic approach** in mathematics is a deductive technique in which the topic of interest is described by a collection of axioms in the language of sets, and all inferences about the topic are made using only these assumptions and the rules of set theory and formal logic. Mathematicians struggled for many years to find an axiomatic formulation for probability that would encompass all types of experiments, not just the ones considered by the classical probabilists. Finally, in the 1930s, the Russian mathematician A. N. Kolmogorov gave an axiomatic description for the theory of probability that permitted virtually every experiment to be considered. A greatly simplified version of this axiomatization will be discussed in Chapter 3.

In most applications, an intuitive understanding of probability based on relative frequencies is generally sufficient. However, it is nice to know that the subject rests on a firm foundation and that the conclusions we reach have some basis in logic!

Before moving on to the next section, it is instructive to make one more remark about the meaning of probability statements. While one often makes probability statements