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王宽诚教育基金会

学术讲座汇编

主编 钱伟长

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主编：钱伟长

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王宽诚教育基金会敬赠

年　月　日

谨以此书纪念本会创建人、故董事会主席王宽诚先生

DEDICATED TO THE MEMORY OF MR K.C. WONG, FOUNDER OF THE FOUNDATION AND THE LATE CHAIRMAN OF THE BOARD OF DIRECTORS

K.C. WONG EDUCATION FOUNDATION

王宽诚先生

K. C. WONG (1907–1986)

王宽诚教育基金会简介

王宽诚先生(1907～1986)为香港知名爱国人士，热心祖国教育事业，生前为故乡宁波的教育事业做出积极贡献。1985年独力捐巨资创建王宽诚教育基金会，其宗旨在于为国家培养高级科技人才，为祖国四个现代化效力。

王宽诚先生在世时聘请海内外知名学者担任基金会考选委员会和学务委员会委员，共商大计，确定采用“送出去”和“请进来”的方针，为国家培育各科专门人才，并为提高国内和港澳高等院校的教学水平，资助学术界人士互访，用以促进中外文化交流。在此方针指导下，1985、1986两年，基金会在国家教委支持下，选派学生85名前往英、美、加拿大和西德、瑞士、澳大利亚各国攻读博士学位，并计划资助国内学者赴港澳讲学，资助港澳学者到国内讲学，资助美国学者来国内讲学。正当基金会事业初具规模，蓬勃发展之时，王宽诚先生一病不起，于1986年年底逝世。这是基金会的重大损失，共事同仁，无不深切怀念，不胜惋惜。

王宽诚教育基金会在新任董事会主席张二铭先生和安子介、方善桂、胡百全、李福树等董事的主持下，继承王宽诚先生为国家培育人才的遗愿，继续努力，除按计划执行外，并开发与英国学术机构合作的新项目。王宽诚教育基金会过去和现在的工作态度一贯以王宽诚先生所倡导的“公正”二字为守则，谅今后基金会亦将秉此行事，奉行不辍。借此王宽诚教育基金会《学术讲座汇编》出版之际，特简明介绍如上。王宽诚教育基金会日常工作繁重，王明远、王明勤、黄贵康、林延新等董事均不辞劳累，做出积极贡献。

钱伟长

一九九五年十二月

前　　言

王宽诚教育基金会是由已故全国政协常委、香港著名工商企业家王宽诚先生(1907～1986）出于爱国热忱，出资一亿美元于1985年在香港注册登记创立的。

1987年，基金会开设“学术讲座”项目，此项目由当时的全国政协常委、现任全国政协副主席、著名科学家、中国科学院院士、上海大学校长、王宽诚教育基金会贷款留学生考选委员会主任委员兼学务委员会主任委员钱伟长教授主持，由钱伟长教授亲自起草设立“学术讲座”的规定，资助国内学者前往香港、澳门讲学，资助美国学者和港澳学者前来国内讲学，用以促进中外学术交流，提高内地及港澳高等院校的教学质量。

本汇编收集的文章，均系各地学者在“学术讲座”活动中的讲稿。文章作者中，有年逾八旬的学术界硕彦，亦有由王宽诚教育基金会考选委员会委员推荐的学者和后起之秀。文章内容有科学技术，有历史文化，有经济专论，有文学，有宗教和中国古籍研究。本汇编涉及的学术领域颇为广泛，而每篇文章都有一定的深度和广度，分期分册以《王宽诚教育基金会学术讲座汇编》的名义出版，并无偿分送港澳和国内外部分高等院校、科研机构和图书馆，以广流传。

王宽诚教育基金会除资助“学术讲座”学者进行学术交流之外，在钱伟长教授主持的项目下，还资助由国内有关高等院校推荐的学者前往欧美亚澳参加国际学术会议，出访的学者均向所出席的会议提交论文，这些论文亦颇有水平，本汇编亦将其收入，以供参考。

王宽诚教育基金会学务委员会

凡　例

（一）编排次序

本书所收集的王宽诚教育基金会学术讲座的讲稿及由王宽诚教育基金会资助学者赴欧美亚澳参加国际学术会议的论文均按照收到文稿日期先后或文稿内容编排刊列，不分类别。

（二）分期分册出版并作简明介绍

因文稿较多，为求便于携带，有利阅读与检索，故分期分册出版，每册约150页至200页不等。为便于读者查考，每篇学术讲座的讲稿均注明作者姓名、学位、职务、讲学日期、地点、访问院校名称。国内及港澳学者到欧、美、澳及亚洲的国家和地区参加国际学术会议的论文均注明学者姓名、参加会议的名称、时间、地点和推荐的单位。上述两类文章均注明由王宽诚教育基金会资助字样。

（三）文字种类

本书为学术性文章汇编，均以学术讲座学者之讲稿原稿或参加国际学术会议学者向会议提交的论文原稿文字为准，即原讲稿或论文是中文的，即以中文刊出，原讲稿或论文是外文的，仍以外文刊出。

编后记

本书前九集出版后，分赠国内外各地图书馆及高等院校和科研机构，引起广泛注意。先后收到国内各有关院校、各省、市、地区图书馆暨港、澳、欧、美各国大学来信，迭有好评。表示愿意今后和我们保持联系，希望继续赠阅。不少单位反映，认为内容很好，专业性强，有一定深度和广度，学术水平很高，对学校的教学与科研有一定的参考价值。

我们受到国内外有关院校及各地图书馆来函鼓励，倍感亲切，益觉力薄，诚恐失误。若有失当之处，尚请海内外广大读者批评指正。

本书内容广泛，科技方面涉及的领域尤多，均赖中国大百科全书出版社上海分社副编审陈荣乐教授总揽编辑、校对及安排印刷任务，对中外文的复校工作尤为认真，辛劳可佩。上海市印刷三厂职工在排印工作中不辞辛苦，为扩大中外文化交流做出积极的贡献。上海大学上下一心多方支持，使本书发行工作得以顺利进行，谨此一并志谢！

王宽诚教育基金会学务委员会

一九九五年十二月

CONTENTS

王宽诚教育基金会
《学术讲座汇编》第10集，1~6(1995).

Non-Kirchhoff-Love Theory For Elastic Circular Plate With Fixed Boundary Subjected To Uniform Loading

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ABSTRACT

The first order approximation theory for circular plate with fixed boundary subjected to uniform loading on the upper surface is given on the bases of general Non-Kirchhoff-Love theory for elastic plates with arbitrary shapes established in the previous papers[1,2]. It consists of a complete set of 6 equilibrium equations and 11 related boundary conditions derived from the generalized variational principle for three dimensional elastic circular plate under the above boundary and loading conditions[3] and the assumption that e_z and e_{rz} can be approximated by polynomials.

I. INTRODUCTION

The axial symmetrical problems of circular plates can be treated as three dimensional axisymmetrical problems of elasticity. In the following we consider a circular plate with constant thickness. A circular-cylinder coordinates (r, θ, z) is set up on the middle surface as shown in Fig. 1.

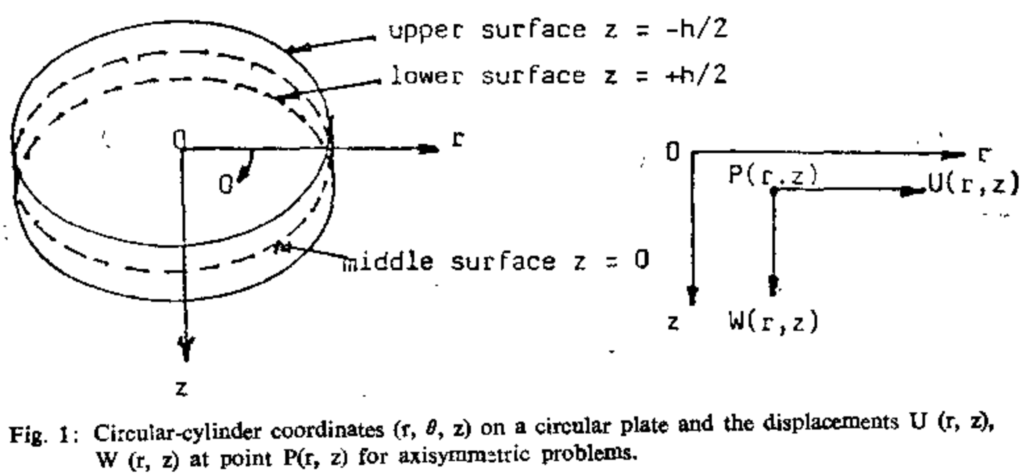

Fig. 1: Circular-cylinder coordinates (r, θ, z) on a circular plate and the displacements U (r, z), W (r, z) at point P(r, z) for axisymmetric problems.

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The stress components are σ_r, σ_θ, σ_z, $\sigma_{zr}=\sigma_{rz}$, $\sigma_{r\theta}=\sigma_{\theta r}$, $\sigma_{z\theta}=\sigma_{\theta z}$, and the strain components are e_r, e_θ, e_z, $e_{zr}=e_{rz}$, $e_{r\theta}=e_{\theta r}$, $e_{z\theta}=e_{\theta z}$. For circular axisymmetric problems, we have

$$\sigma_{\theta r}=\sigma_{r\theta}=0, \quad \sigma_{\theta z}=\sigma_{z\theta}=0, \quad e_{\theta r}=e_{r\theta}=0, \quad e_{\theta z}=e_{z\theta}=0 \tag{1}$$

There are two displacement components: radial displacement U (r, z) and axial displacement W (r, z). These stress, strain and displacement components satisfy the following relations:

(i) Strain-displacement relations:

$$e_r=\frac{\partial U}{\partial r}, \; e_\theta=\frac{U}{r}, \; e_z=\frac{\partial W}{\partial z}, e_{rz}=\frac{1}{2}\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial r}\right), \; e_{\theta z}=e_{r\theta}=0 \tag{2}$$

(ii) Stress-strain relations:

$$\begin{aligned} &Ee_r=\sigma_r-\nu(\sigma_\theta+\sigma_z), && Ee_{rz}=(1+\nu)\sigma_{rz}, \\ &Ee_\theta=\sigma_\theta-\nu(\sigma_r+\sigma_z), && Ee_{z\theta}=(1+\nu)\sigma_{z\theta}=0, \\ &Ee_z=\sigma_z-\nu(\sigma_\theta+\sigma_r), && Ee_{\theta r}=(1+\nu)\sigma_{\theta r}=0. \end{aligned} \tag{3}$$

(iii) Strain-stress relations:

$$\begin{aligned} &\sigma_r=\frac{E_1}{1-\nu_1^2}\left\{e_r+\nu_1(e_\theta+e_z)\right\}, \; \sigma_{rz}=\frac{E_1}{1+\nu_1}e_{rz} \\ &\sigma_\theta=\frac{E_1}{1-\nu_1^2}\left\{e_\theta+\nu_1(e_r+e_z)\right\}, \; \sigma_{\theta z}=\frac{E_1}{1+\nu_1}e_{\theta z}=0 \\ &\sigma_z=\frac{E_1}{1-\nu_1^2}\left\{e_z+\nu_1(e_r+e_\theta)\right\}, \; \sigma_{\theta r}=\frac{E_1}{1+\nu_1}e_{\theta r}=0 \end{aligned} \tag{4}$$

in which E and ν are respectively Young's modulus and Poisson's ratio, and E_1 and ν_1 are reduced Young's modulus and Poisson's ratio for plane strain problems. They satisfy the relations:

$$E_1=\frac{E}{1-\nu^2}, \quad \nu_1=\frac{\nu}{1-\nu}, \quad \frac{E}{1+\nu}=\frac{E_1}{1+\nu_1} \tag{5}$$

(iv) Stress equilibrium equations:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_r)-\frac{1}{r}\sigma_\theta+\frac{\partial}{\partial z}(\sigma_{zr})=0, \; \frac{\partial \sigma_z}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr})=0 \tag{6}$$

The body forces are neglected.

(v) External forces acting on upper $(z=-\frac{1}{2}h)$ and lower $(z=+\frac{1}{2}h)$ surfaces:

$$\sigma_z=-q, \; \sigma_{rz}=0 \text{ (on upper surface } z=-\frac{h}{2}, \; q>0 \text{ for compression)} \tag{7}$$

$$\sigma_z=0, \; \sigma_{rz}=0 \text{ (on lower surface } z=+\frac{h}{2}, \text{ free from loading)}$$

(vi) Boundary conditions on the edge surfaces:

$$U(a, z)=0, \quad W(a, z)=0 \tag{8}$$

The generalized variational principles for three dimensional elastic bodies under the constrains of prescribed displacement boundary and prescribed external force boundary have been studied by the author in [3]. The related functionals may be found by the method of undetermined Lagrange multipliers. For axial symmetric three dimensional bodies, the strain energy density may be written as

$$s=\frac{1}{2}\frac{E_1}{1-\nu_1^2}\left\{(e_r+e_\theta+e_z)^2+2(1-\nu_1)(e_{zr}^2-e_re_z-e_\theta e_z-e_re_\theta)\right\} \tag{9}$$

It is evident that

$$\delta s=\sigma_r\delta e_r+\sigma_\theta\delta e_\theta+\sigma_z\delta e_z+2\sigma_{zr}\delta e_{zr} \tag{10}$$

The functional of the generalized variational principle for the three dimensional elastic circular plate of thickness h and radius a under the constraints of strain-displacement relations (2), strain-stress relations (3), fixed boundary conditions (8) and prescribed surface loading (7) may be written as

$$\Pi=\int_0^a\int_{(h)} s2\pi r\ drdz\ +\int_0^a qW_{(-)}\,2\pi r\ dr-\int_{(h)}2\pi a(U\sigma_r+W\sigma_{rz})_{r=a}dz \tag{11}$$

in which $\int_{(h)}(\ldots)\,dz$ represents the integration across the thickness of plate, i.e.,

$$\int_{(h)}(\ldots)\,dz=\int_{-h/2}^{h/2}(\ldots)\,dz \tag{12}$$

and $W_{(-)}$ represents the displacement of upper surface, i.e., $W_{(-)}=W(r,-h/2)$, and a is the radius of the plate.

The equations and boundary conditions of the present problem are derived from the stationary conditions $\delta\Pi=0$:

$$\delta\Pi=\int_0^a\int_{(h)}\delta s2\pi r\ drdz+\int_0^a q\delta W_{(-)}2\pi r\ dr-\int_{(h)}2\pi a(U\delta\sigma_r+\sigma_r\delta U+\sigma_{rz}\delta W$$
$$+W\delta\sigma_{rz})_{r=a}dz=0 \tag{13}$$

in which δs may be simplified by means of (10) and (2)

$$\delta s=\sigma_r\frac{\partial}{\partial r}\delta U\ +\sigma_\theta\frac{\delta U}{r}+\sigma_z\frac{\partial}{\partial z}\delta W+\sigma_{rz}\left(\frac{\partial}{\partial r}\delta W+\frac{\partial}{\partial z}\ \delta U\right) \tag{14}$$

Through partial integrations, (13) may be reduced to the following expression

$$\delta\Pi=\int_0^a\int_{(h)}\left\{\left[\frac{1}{r}\frac{\partial}{\partial r}\ (r\sigma_1)\ -\frac{1}{r}\sigma_\theta+\frac{\partial}{\partial z}\ \sigma_{rz}\ \right]\delta U+\left[\frac{1}{r}\frac{\partial}{\partial r}\ (r\sigma_{rz})\right.\right.$$
$$\left.\left.+\frac{\partial}{\partial z}\ \sigma_z\ \right]\delta W\right\}2\pi r\ drdz+\int_{(h)}(U\delta\sigma_r+W\delta\sigma_{rz})_{r=a}2\pi a\ dz$$
$$+\int_0^a[\sigma_z^{(+)}W_{(+)}-(\sigma_z^{(-)}+q)\delta W_{(-)}]\ 2\pi r\ dr+\int_0^a(\sigma_{rz}^{(+)}\delta U_{(+)}$$
$$-\sigma_{rz}^{(-)}\delta U_{(-)})2\pi r\ dr \tag{15}$$

Since the variations δU, δW, $\delta U_{r=a}$, $\delta W_{r=a}$, $\delta U_{(+)}$, $\delta U_{(-)}$, $\delta W_{(+)}$, $\delta W_{(-)}$, $(\delta\sigma_r)_{r=a}$, $(\delta\sigma_{rz})_{r=a}$ are all independent to each other, the stationary conditions of variation $\delta\Pi=0$ give the equations of equilibrium (6), the loading conditions (7) and the fixed boundary conditions (8). Thus, expression (11) is proved to be the functional of this problem.

II. APPROXIMATE THEORY OF CIRCULAR PLATE WITH NON-KIRCHHOFF-LOVE ASSUMPTIONS (i.e., e_{rz}, e_z, $\sigma_{rz}\neq 0$)

It has been shown in the previous papers that for Non-Kirchhoff-Love theory of plate,

e_z and e_{rz} may be expressed as polynomial series of z, and the coefficients of each term in the series may be taken as undetermined functions of r. For the first order approximation, we may express e_z and e_{rz} as

$$e_z=A_0+A_1z,\ e_{rz}=(\tfrac{1}{4}h^2-z^2)\,(S_0+S_1z+S_2z^2) \tag{16}$$

in which A_0, A_1, S_1 and S_2 are undetermined functions of r. In this problem, the shearing stress σ_{rz} vanishes on both surfaces $z=\pm h/2$. It should be noted that a disk of radius r cut from the plate is in equilibrium under the loading $\pi r^2 q$ on its surface and the resultant force $2\pi rQ$ of the shearing stress around its boundary. Hence, we have

$$Q=\int_{(h)}\sigma_{rz}dz=\frac{E_1}{1+\nu_1}\int_{(h)}e_{rz}dz=\frac{E_1}{1+\nu_1}\left(\frac{1}{6}h^3S_0+\frac{1}{120}h^5S_2\right)=-\frac{1}{2}qr \tag{17}$$

Eliminating S_0 from (16) and (17) gives

$$e_z=A_0+A_1z,\ e_{rz}=\left(\frac{1}{4}h^2-z^2\right)\left[S_1z-\left(\frac{1}{20}h^2-z^2\right)S_2-\frac{qr}{4(1-\nu_1)D_1}\right] \tag{18}$$

Through integrations of e_z and e_{rz} against z, we obtain the polynomial expressions for $W(r,z)$ and $U(r,z)$:

$$W(r,z)=W(r)+A_0+\frac{1}{2}A_1z^2,$$

$$U(r,z)=u(r)-\frac{dW}{dr}z-\frac{1}{2}\frac{dA_0}{dr}z^2-\frac{1}{6}\frac{dA_1}{dr}z^3+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2S_1$$
$$-\frac{2}{5}\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)zS_2-\frac{qr}{2D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z \tag{19}$$

in which $w(r)$ and $u(r)$ are integration constants to be determined. Thus, we have now the expressions of $U(r,z)$ and $W(r,z)$ in terms of 6 unknown functions $u(r)$, $w(r)$, $A_0(r)$, $A_1(r)$, $S_1(r)$ and $S_2(r)$ to be determined.

The strain components e_z and e_{rz} are given in terms of these functions in (18), other strain components are expressed as

$$e_r=\frac{\partial U}{\partial r}=\frac{du}{dr}-\frac{d^2w}{dr^2}z-\frac{1}{2}\frac{d^2A_0}{dr^2}z^2-\frac{1}{6}\frac{d^2A_1}{dr^2}z^3+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2\frac{dS_1}{dr}$$
$$-\frac{2}{5}\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)z\frac{dS_2}{dr}-\frac{q}{2D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z$$
$$e_\theta=\frac{U}{r}=\frac{u}{r}-\frac{1}{r}\frac{dw}{dr}z-\frac{1}{2}\frac{1}{r}\frac{dA_0}{dr}z^2-\frac{1}{6r}\frac{dA_1}{dr}z^3+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2\frac{S_1}{r}$$
$$-\frac{2}{5}\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)z\frac{S_2}{r}-\frac{q}{2D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z$$
$$e_{\theta z}=e_{\theta r}=0 \tag{20}$$

The stress components in (4) can be rewritten as

$$\sigma_r=\frac{E_1}{1-\nu_1^2}\left\{\frac{du}{dr}+\nu_1\frac{u}{r}-\left(\frac{d^2w}{dr^2}+\nu_1\frac{1}{r}\frac{dw}{dr}\right)z+\nu_1A_0+\nu_1A_1z\right.$$
$$-\frac{1}{2}\left(\frac{d^2A_0}{dr^2}+\frac{\nu_1}{r}\frac{dA_0}{dr}\right)z^2-\frac{1}{6}\left(\frac{d^2A_1}{dr^2}+\nu_1\frac{1}{r}\frac{dA_1}{dr}\right)z^3$$
$$-\frac{(1+\nu_1)q}{2D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2\left(\frac{dS_1}{dr}+\nu_1\frac{S_1}{r}\right)$$

$$-\frac{2}{5}\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)z\left(\frac{dS_2}{dr}+\frac{\nu_1}{r}S_2\right)\Big\}$$

$$\sigma_\theta=\frac{E_1}{1-\nu_1^2}\Big\{\frac{u}{r}+\nu_1\frac{du}{dr}-\left(\frac{1}{r}\frac{dw}{dr}+\nu_1\frac{d^2w}{dr^2}\right)z+\nu_1A_0+\nu_1A_1z$$

$$-\frac{1}{2}\left(\frac{1}{r}\frac{dA_0}{dr}+\nu_1\frac{d^2A_0}{dr^2}\right)z^2-\frac{1}{6}\left(\nu_1\frac{d^2A_1}{dr^2}+\frac{1}{r}\frac{dA_1}{dr}\right)z^3$$

$$-\frac{(1+\nu_1)q}{2D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z+\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2\left(\frac{S_1}{r}+\nu_1\frac{dS_1}{dr}\right)$$

$$-\frac{2}{5}\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)z\left(\frac{S_2}{r}+\nu_1\frac{dS_2}{dr}\right)\Big\}$$

$$\sigma_z=\frac{E_1}{1-\nu_1^2}\Big\{\frac{\nu_1}{r}\frac{d}{dr}(ru)+A_0-\frac{\nu_1}{r}\frac{d}{dr}r\frac{dw}{dr}z+A_1z-\frac{\nu_1}{2r}\frac{d}{dr}r\frac{dA_0}{dr}z^2$$

$$-\frac{\nu_1}{6}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}z^3+\nu_1\left(\frac{1}{4}h^2-\frac{1}{2}z^2\right)z^2\frac{1}{r}\frac{d}{dr}(rS_1)$$

$$-\frac{2}{5}\nu_1\left(\frac{1}{16}h^4-\frac{1}{2}h^2z^2+z^4\right)z\frac{1}{r}\frac{d}{dr}(rS_2)-\frac{\nu_1q}{D_1(1-\nu_1)}\left(\frac{1}{4}h^2-\frac{1}{3}z^2\right)z\Big\}$$

$$\sigma_{rz}=\frac{E_1}{1+\nu_1}\left(\frac{1}{4}h^2-z^2\right)\Big\{zS_1-\left(\frac{1}{20}h^2-z^2\right)S_2-\frac{qr}{4D_1(1-\nu_1)}\Big\} \tag{21}$$

III. DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

Substituting (19) and (21) into (15), the stationary conditions of variation $\delta\Pi=0$ give 6 differential equations and 11 related boundary conditions for the determination of the six unknown functions $w(r)$, $u(r)$, $A_0(r)$, $A_1(r)$, $S_1(r)$ and $S_2(r)$. The six equations are as follows

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(ru)+\nu_1\frac{dA_0}{dr}-\frac{h^2}{24}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_0}{dr}+\frac{7}{480}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_1)=0 \tag{22a}$$

$$-\left[\nu\frac{1}{r}\frac{d}{dr}(ru)+A_0\right]+\frac{1}{24}h^2\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(ru)+\frac{1}{12}\nu_1h^2\frac{1}{r}\frac{d}{dr}r\frac{dA_0}{dr}$$

$$-\frac{1}{320}h^4\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_0}{dr}-\frac{7}{480}h^4\frac{1}{r}\frac{d}{dr}(rS_1)$$

$$+\frac{9}{8960}h^6\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_1)-\frac{h^3q}{24D_1}=0. \tag{22b}$$

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(ru)+\nu_1\frac{dA_0}{dr}-\frac{27}{392}h^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_0}{dr}-\frac{8}{49}(1-\nu_1)h^2S_1$$

$$+\frac{107}{4704}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_1)=0 \tag{22c}$$

$$\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-\nu_1\frac{d}{dr}r\frac{dA_1}{dr}+\frac{1}{40}h^2\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}$$

$$+\frac{1}{175}h^4\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_2)=\frac{qr}{D_1} \tag{23a}$$

$$\frac{1}{40}h^2\left[\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-2\nu_1\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}\right]$$

$$+\frac{1}{1344}h^4\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}-\frac{1}{175}h^4\frac{1}{r}\frac{d}{dr}(rS_2)$$

$$+\frac{1}{12600}h^6\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_2)-\left[\nu_1\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-A_1\right]=\frac{1}{40}\frac{5+3\nu_1}{1-\nu_1}\frac{h^2q}{D_1} \tag{23b}$$

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-\nu_1\frac{dA_1}{dr}+\frac{1}{12}h^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}+\frac{2}{165}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_2)$$

$$-\frac{4}{15}(1-\nu_1)h^2S_2=\frac{qr}{2D_1} \tag{23c}$$

The eleven boundary conditions on $r=a$ are as follows

$$u(a)=0,\quad A_0(a)=0,\quad \frac{dA_0}{dr}=0,\quad S_1(a)=0\quad (r=a)$$

(24a, b, c, d)

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(ru)+\nu_1\frac{dA_0}{dr}-\frac{3}{40}h^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_0}{dr}$$

$$+\frac{27}{1120}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_1)+\frac{1}{10}h^2(1-\nu_1)S_1=0\quad (r=a) \tag{24e}$$

$$\frac{dA_1}{dr}=\frac{qa}{D_1(1-\nu_1)}\quad (r=a) \tag{25a,b}$$

$$S_2(a)=0,\quad \frac{dw}{dr}=-\frac{qah^2}{8D_1(1-\nu_1)},\ w(a)=-\frac{h^2}{105}A_1\ (r=a) \tag{25c, d}$$

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-\nu_1\frac{dA_1}{dr}+\frac{1}{40}h^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}+\frac{1}{175}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_2)$$

$$=\frac{qr}{2D_1}\quad (r=a) \tag{25e}$$

$$\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dw}{dr}-\nu_1\frac{dA_1}{dr}+\frac{5}{168}h^2\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r\frac{dA_1}{dr}-\frac{1}{315}h^4\frac{d}{dr}\frac{1}{r}\frac{d}{dr}(rS_2)$$

$$+\frac{4}{49}h^2(1-\nu_1)S_2=\frac{qr}{2D_1}\quad (r=a) \tag{25f}$$

Equations (22a,b,c) and (23a,b,c) can be solved under the boundary conditions (24a,b,c,d,e) and (25a,b,c,d,e,f). The numerical results will be published elsewhere.

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