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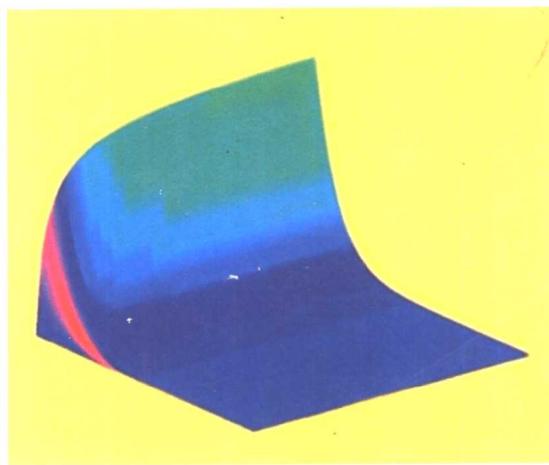
国外高校电子信息类优秀教材

# 现代通信系统

## — 应用 MATLAB

Contemporary Communication Systems Using MATLAB

(英文影印版)



John G. Proakis Masoud Salehi 著

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# 现代通信系统——应用 MATLAB

Contemporary Communication Systems

Using MATLAB

John G. Proakis Masoud Salehi 著

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北京

## 内 容 简 介

本书为国外高校电子信息类优秀教材(英文影印版)之一。

本书介绍了 MATLAB 在现代通信系统中的应用,包括信号与线性系统、随机过程、模拟调制、模/数转换、基带数字传输、限宽信道数字传输、载波调制数字传输、信通容量和编码、扩频通信系统。本书通过使用 MATLAB 这一“动态实验室”帮助读者提高解决问题的能力 and 严谨思维的能力。

本书可作为通信专业本科生教材,也可作为相关专业工程技术人员的参考书。

### Contemporary Communication Systems Using MATLAB

By John G. Proakis, Masoud Salehi

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# 国外高校电子信息类优秀教材(英文影印版)

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# Preface

Many textbooks on the market today treat the basic topics in analog and digital communication systems, including coding and decoding algorithms and modulation and demodulation techniques. By necessity, the focus of most of these textbooks is on the theory that underlies the design and performance analysis of the various building blocks (e.g., coders, decoders, modulators, and demodulators) that constitute the basic elements of a communication system. However, relatively few of the textbooks, especially those written for undergraduate students, include a variety of applications designed to motivate the students.

## Scope

The objective of this book is to serve as a companion or supplement to any of the comprehensive textbooks in communication systems. The book provides a variety of exercises that may be solved on the computer (generally, a personal computer is sufficient) using the popular student edition of MATLAB<sup>®</sup>. The book is intended primarily for senior-level undergraduate students and graduate students in electrical engineering, computer engineering, and computer science. We assume that the student (or user) is familiar with the fundamentals of MATLAB. Those topics are not covered because several tutorial books and manuals on MATLAB are available.

By design, the treatment of the various topics is brief. We provide the motivation and a short introduction to each topic, establish the necessary notation, and then illustrate the basic concepts by means of an example. The primary text and the instructor are expected to provide the required depth of the topics treated. For example, we introduce the matched filter and the correlator and assert that these devices result in the optimum demodulation of signals corrupted by additive white Gaussian noise (AWGN), but we do not provide a proof of this assertion. Such a proof is generally given in most textbooks on communication systems.

## Organization

This book consists of nine chapters. The first two chapters, on signals and linear systems and on random processes, present the basic background that is generally required in the study of communication systems. One chapter covers analog communication techniques, and the remaining six chapters are focused on digital communications.

## **Chapter 1: Signals and Linear Systems**

This chapter provides a review of the basic tools and techniques from linear systems analysis, including both time-domain and frequency-domain characterizations. Frequency-domain analysis techniques are emphasized because these techniques are most frequently used in the treatment of communication systems.

## **Chapter 2: Random Processes**

In this chapter, we illustrate methods for generating random variables and samples of random processes. The topics include the generation of random variables with a specified probability distribution function, the generation of samples of Gaussian and Gauss-Markov processes, and the characterization of stationary random processes in the time domain and the frequency domain.

## **Chapter 3: Analog Modulation**

The performances of analog modulation and demodulation techniques in the presence and absence of additive noise are treated in this chapter. Systems studied include amplitude modulation (AM), such as double-sideband AM, single-sideband AM, and conventional AM; and angle-modulation schemes, such as frequency modulation (FM) and phase modulation (PM).

## **Chapter 4: Analog-to-Digital Conversion**

In this chapter, we examine various methods used to convert analog source signals into digital sequences in an efficient way. Conversion allows us to transmit or store the signals digitally. We consider both lossy data compression schemes, such as pulse-code modulation (PCM), and lossless data compression, such as Huffman coding.

## **Chapter 5: Baseband Digital Transmission**

In this chapter, we introduce baseband digital modulation and demodulation techniques for transmitting digital information through an AWGN channel. Both binary and nonbinary modulation techniques are considered. The optimum demodulation of these signals is described, and the performance of the demodulator is evaluated.

## **Chapter 6: Digital Transmission Through Bandlimited Channels**

In this chapter, we consider the characterization of bandlimited channels and the problem of designing signal waveforms for such channels. We show that channel distortion results in intersymbol interference (ISI), which causes errors in signal demodulation. Then we treat the design of channel equalizers that compensate for channel distortion.

## **Chapter 7: Digital Transmission via Carrier Modulation**

We discuss four types of carrier-modulated signals that are suitable for transmission through bandpass channels: amplitude-modulated signals, quadrature-amplitude-modulated signals, phase-shift keying, and frequency-shift keying.

## **Chapter 8: Channel Capacity and Coding**

In this chapter, we consider appropriate mathematical models for communication channels and introduce a fundamental quantity, called the channel capacity, that gives the limit on the amount of information that can be transmitted through the channel. In particular, we consider two channel models: the binary symmetric channel (BSC) and the additive white Gaussian noise (AWGN) channel. These channel models are used in treating block and convolutional codes to achieve reliable communication through such channels.

## **Chapter 9: Spread Spectrum Communication Systems**

The basic elements of a spread spectrum digital communication system are treated in this chapter. In particular, direct-sequence (DS) spread spectrum and frequency-hopped (FH) spread spectrum systems are considered in conjunction with phase-shift keying (PSK) and frequency-shift keying (FSK) modulation, respectively. The generation of pseudonoise (PN) sequences for use in spread spectrum systems is also treated.

## **About the Software**

MATLAB files for this book are available at the BookWare Companion Resource Center, online at <http://www.brookscole.com/engineering/ee/bookware.htm>. The files include all the MATLAB files used in the text. In most instances, we have added numerous comments to the MATLAB files to make them easier to understand. It should be noted, however, that in developing the files, our main objective was the clarity of the MATLAB code rather than its efficiency. In cases where the most efficient code would have made the files difficult to follow, we have chosen to use a less efficient but more readable code.

## **The BookWare Companion Series Resource Center**

New to this updated printing is the BookWare Companion Series Resource Center, a central online Web site that supports the entire series. There you will find downloadable MATLAB files for this book. We intend to keep these files current, thus making the most of the advantages of Web delivery. At the Resource Center, you will also find other resources, such as additional information about our series, links to other helpful MATLAB sites, and ideas on teaching technology in the classroom from BookWare authors and other engineering educators.

Over time, we plan to expand this site into a clearinghouse for the exchange of reliable teaching ideas and for ongoing commentary. Do you have an idea for a unique problem or example that you would like us to consider for the next edition of this book? If so, visit our site and click on the Open Manuscript icon to join an ongoing discussion with the authors, peers, students, and the publisher. The Resource Center is located at <http://www.brookscole.com/engineering/ee/bookware.htm>.

*John G. Proakis*  
*Masoud Salehi*

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# Chapter 1

## Signals and Linear Systems

### 1.1 Preview

In this chapter, we review the basic tools and techniques from linear system analysis used in the analysis of communication systems. Linear systems and their characteristics in the time and frequency domains, together with probability and analysis of random signals, are the two fundamental topics that must be understood in the study of communication systems. Most communication channels and many subblocks of transmitters and receivers can be well modeled as linear time-invariant (LTI) systems, and so the well-known tools and techniques from linear system analysis can be employed in their analysis. We emphasize frequency-domain analysis tools, since these are the most frequently used techniques. We start with the Fourier series and transforms; then we cover power and energy concepts, the sampling theorem, and lowpass representation of bandpass signals.

### 1.2 Fourier Series

The input-output relation of a linear time-invariant system is given by the convolution integral defined by

$$\begin{aligned} y(t) &= x(t) \star h(t) \\ &= \int_{-\infty}^{\infty} h(\tau)x(t - \tau) d\tau \end{aligned} \tag{1.2.1}$$

where  $h(t)$  denotes the impulse response of the system,  $x(t)$  is the input signal, and  $y(t)$  is the output signal. If the input  $x(t)$  is a complex exponential given by

$$x(t) = Ae^{j2\pi f_0 t} \tag{1.2.2}$$

then the output is given by

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} A e^{j2\pi f_0(t-\tau)} h(\tau) d\tau \\ &= A \left[ \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_0\tau} d\tau \right] e^{j2\pi f_0 t} \end{aligned} \quad (1.2.3)$$

In other words, the output is a *complex exponential with the same frequency as the input*. The (complex) amplitude of the output, however, is the (complex) amplitude of the input amplified by

$$\int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_0\tau} d\tau$$

Note that the above quantity is a function of the impulse response of the LTI system,  $h(t)$ , and the frequency of the input signal,  $f_0$ . Therefore, computing the response of LTI systems to exponential inputs is particularly easy. Consequently, it is natural in linear system analysis to look for methods of expanding signals as the sum of complex exponentials. *Fourier series and Fourier transforms are techniques for expanding signals in terms of complex exponentials.*

A Fourier series is the orthogonal expansion of periodic signals with period  $T_0$  when the signal set  $\{e^{j2\pi nt/T_0}\}_{n=-\infty}^{\infty}$  is employed as the basis for the expansion. With this basis, any periodic signal<sup>1</sup>  $x(t)$  with period  $T_0$  can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi nt/T_0} \quad (1.2.4)$$

where the  $x_n$ 's are called the *Fourier series coefficients* of the signal  $x(t)$  and are given by

$$x_n = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) e^{-j2\pi nt/T_0} dt \quad (1.2.5)$$

Here  $\alpha$  is an arbitrary constant chosen in such a way that the computation of the integral is simplified. The frequency  $f_0 = 1/T_0$  is called the *fundamental frequency* of the periodic signal, and the frequency  $f_n = nf_0$  is called the *nth harmonic*. In most cases either  $\alpha = 0$  or  $\alpha = -T_0/2$  is a good choice.

This type of Fourier series is known as the *exponential Fourier series* and can be applied to both real-valued and complex-valued signals  $x(t)$  as long as they are periodic. In general, the Fourier series coefficients  $\{x_n\}$  are complex numbers even when  $x(t)$  is a real-valued signal.

<sup>1</sup>A sufficient condition for the existence of the Fourier series is that  $x(t)$  satisfy the Dirichlet conditions. For details, see [1].

When  $x(t)$  is a *real-valued* periodic signal, we have

$$\begin{aligned} x_{-n} &= \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) e^{j2\pi nt/T_0} dt \\ &= \frac{1}{T_0} \left[ \int_{\alpha}^{\alpha+T_0} x(t) e^{-j2\pi nt/T_0} dt \right]^* \\ &= x_n^* \end{aligned} \quad (1.2.6)$$

From this it is obvious that

$$\begin{cases} |x_n| = |x_{-n}| \\ \angle x_n = -\angle x_{-n} \end{cases} \quad (1.2.7)$$

Thus, the Fourier series coefficients of a real-valued signal have *Hermitian symmetry*; that is, their real part is even and their imaginary part is odd (or, equivalently, their magnitude is even and their phase is odd).

Another form of Fourier series, known as *trigonometric Fourier series*, can be applied only to real, periodic signals and is obtained by defining

$$x_n = \frac{a_n - jb_n}{2} \quad (1.2.8)$$

$$x_{-n} = \frac{a_n + jb_n}{2} \quad (1.2.9)$$

which, after using Euler's relation

$$e^{-j2\pi nt/T_0} = \cos\left(2\pi t \frac{n}{T_0}\right) - j \sin\left(2\pi t \frac{n}{T_0}\right) \quad (1.2.10)$$

results in

$$\begin{aligned} a_n &= \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \cos\left(2\pi t \frac{n}{T_0}\right) dt \\ b_n &= \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \sin\left(2\pi t \frac{n}{T_0}\right) dt \end{aligned} \quad (1.2.11)$$

and, therefore,

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(2\pi t \frac{n}{T_0}\right) + b_n \sin\left(2\pi t \frac{n}{T_0}\right) \quad (1.2.12)$$

Note that for  $n = 0$ , we always have  $b_0 = 0$ , so  $a_0 = 2x_0$ .

By defining

$$\begin{cases} c_n = \sqrt{a_n^2 + b_n^2} \\ \theta_n = -\arctan \frac{b_n}{a_n} \end{cases} \quad (1.2.13)$$

and using the relation

$$a \cos \phi + b \sin \phi = \sqrt{a^2 + b^2} \cos \left( \phi - \arctan \frac{b}{a} \right) \quad (1.2.14)$$

we can write Equation (1.2.12) in the form

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \cos \left( 2\pi t \frac{n}{T_0} + \theta_n \right) \quad (1.2.15)$$

which is the third form of the Fourier series expansion for real and periodic signals. In general, the Fourier series coefficients  $\{x_n\}$  for real-valued signals are related to  $a_n$ ,  $b_n$ ,  $c_n$ , and  $\theta_n$  through

$$\begin{cases} a_n = 2 \operatorname{Re}[x_n] \\ b_n = -2 \operatorname{Im}[x_n] \\ c_n = |x_n| \\ \theta_n = \angle x_n \end{cases} \quad (1.2.16)$$

Plots of  $|x_n|$  and  $\angle x_n$  versus  $n$  or  $nf_0$  are called the *discrete spectra* of  $x(t)$ . The plot of  $|x_n|$  is usually called the *magnitude spectrum*, and the plot of  $\angle x_n$  is referred to as the *phase spectrum*.

If  $x(t)$  is real and even—that is, if  $x(-t) = x(t)$ —then taking  $\alpha = -T_0/2$ , we have

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \sin \left( 2\pi t \frac{n}{T_0} \right) dt \quad (1.2.17)$$

which is zero because the integrand is an odd function of  $t$ . Therefore, for a real and even signal  $x(t)$ , all  $x_n$ 's are real. In this case, the trigonometric Fourier series consists of all cosine functions. Similarly, if  $x(t)$  is real and odd—that is, if  $x(-t) = -x(t)$ —then

$$a_n = \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \cos \left( 2\pi t \frac{n}{T_0} \right) dt \quad (1.2.18)$$

is zero and all  $x_n$ 's are imaginary. In this case, the trigonometric Fourier series consists of all sine functions.

### ILLUSTRATIVE PROBLEM

**Illustrative Problem 1.1 [Fourier series of a rectangular signal train]** Let the periodic signal  $x(t)$ , with period  $T_0$ , be defined by

$$x(t) = A \Pi \left( \frac{t}{2t_0} \right) = \begin{cases} A, & |t| < t_0 \\ \frac{A}{2}, & t = \pm t_0 \\ 0, & \text{otherwise} \end{cases} \quad (1.2.19)$$

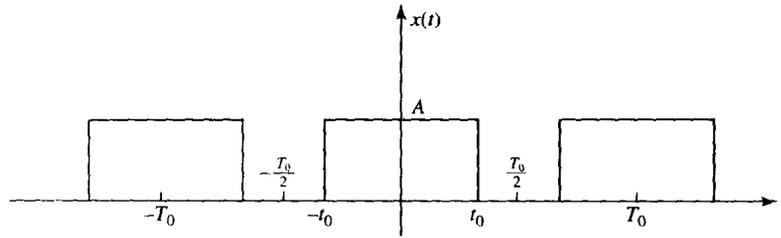


Figure 1.1 The signal  $x(t)$  in Illustrative Problem 1.1

for  $|t| \leq T_0/2$ , where  $t_0 < T_0/2$ . The rectangular signal  $\Pi(t)$  is, as usual, defined by

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ \frac{1}{2}, & t = \pm \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \quad (1.2.20)$$

A plot of  $x(t)$  is shown in Figure 1.1. Assuming  $A = 1$ ,  $T_0 = 4$ , and  $t_0 = 1$ :

1. Determine the Fourier series coefficients of  $x(t)$  in exponential and trigonometric form.
2. Plot the discrete spectrum of  $x(t)$ .

### SOLUTION

1. To derive the Fourier series coefficients in the expansion of  $x(t)$ , we have

$$\begin{aligned} x_n &= \frac{1}{4} \int_{-1}^1 e^{-j2\pi nt/4} dt \\ &= \frac{1}{-2j\pi n} \left[ e^{-j2\pi n/4} - e^{j2\pi n/4} \right] \end{aligned} \quad (1.2.21)$$

$$= \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right) \quad (1.2.22)$$

where  $\operatorname{sinc}(x)$  is defined as

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \quad (1.2.23)$$

A plot of the sinc function is shown in Figure 1.2. Obviously, all the  $x_n$ 's are real (since  $x(t)$  is real and even), so

$$\begin{cases} a_n = \operatorname{sinc}\left(\frac{n}{2}\right) \\ b_n = 0 \\ c_n = \left| \operatorname{sinc}\left(\frac{n}{2}\right) \right| \\ \theta_n = 0, \pi \end{cases} \quad (1.2.24)$$

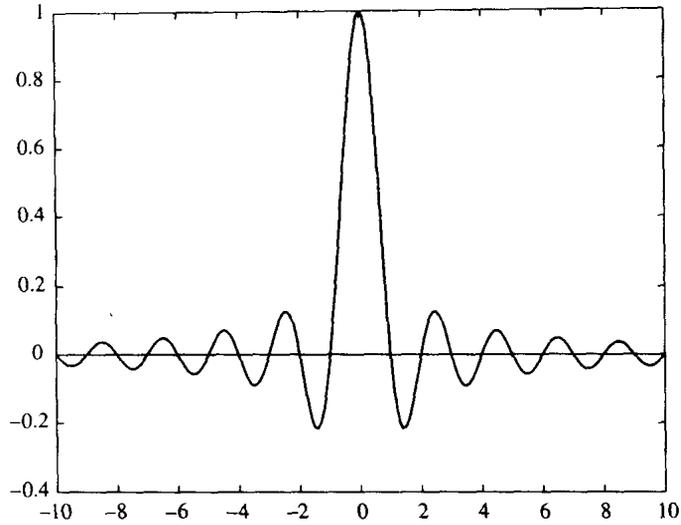


Figure 1.2 The sinc signal

Note that for even  $n$ 's,  $x_n = 0$  (with the exception of  $n = 0$ , where  $a_0 = c_0 = 1$  and  $x_0 = \frac{1}{2}$ ). Using these coefficients, we have

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right) e^{j2\pi nt/4} \\ &= \frac{1}{2} + \sum_{n=1}^{\infty} \operatorname{sinc}\left(\frac{n}{2}\right) \cos\left(2\pi t \frac{n}{4}\right) \end{aligned} \quad (1.2.25)$$

A plot of the Fourier series approximations to this signal over one period for  $n = 0, 1, 3, 5, 7, 9$  is shown in Figure 1.3. Note that as  $n$  increases, the approximation becomes closer to the original signal  $x(t)$ .

- Note that  $x_n$  is always real. Therefore, depending on its sign, the phase is either zero or  $\pi$ . The magnitude of the  $x_n$ 's is  $\frac{1}{2} \left| \operatorname{sinc}\left(\frac{n}{2}\right) \right|$ . The discrete spectrum is shown in Figure 1.4.

The MATLAB script for plotting the discrete spectrum of the signal is given next.

### M-FILE

```
% MATLAB script for Illustrative Problem 1.1.
n=[-20:1:20];
x=abs(sinc(n/2));
stem(n,x);
```