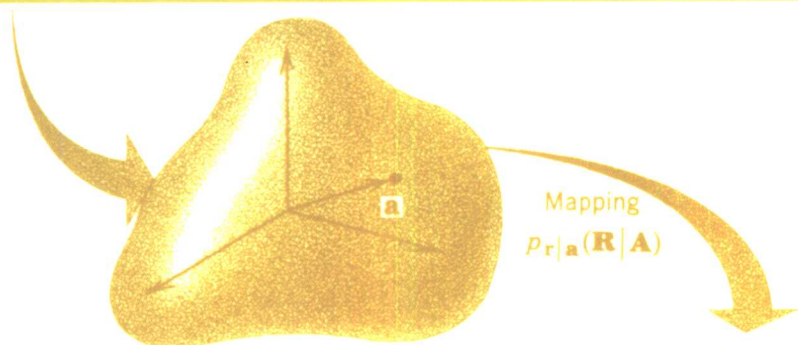


英文版

通信与信息科学教育丛书



# 检测、估值与调制理论

〔美〕Harry L. Van Trees 著

## Detection, Estimation, and Modulation Theory



电子工业出版社

PUBLISHING HOUSE OF ELECTRONICS INDUSTRY  
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北京 · BEIJING

## 内 容 简 介

本书是一本非常经典的详细介绍检测、估值与调制理论的教学参考书,在美国自 1968 年首次出版以来,已重印 28 次。主要内容包括随机过程的表示、检测与估值的分类、信号检测、信号参数的估计、连续波形的估计、线性估计和线性调制理论等。

本书可用做通信与电子、信息与信号处理等专业的高年级本科生、研究生教学参考书。

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## 出版说明

近年来,通信与信息科技发展之快和应用之广,大大超出了人们的预料和专家的预测。从国民经济到社会生活的日益信息化,标志着通信与信息科技的空前发展。

为了满足高等院校师生教改和教学的需求以及广大技术人员学习通信与信息新技术的需要,电子工业出版社约请北京地区的清华大学、北京大学、北京航空航天大学、北京邮电大学、北方交通大学、北京理工大学,南京地区的东南大学、解放军理工大学、南京邮电学院,上海地区的上海交通大学,成都地区的西南交通大学、电子科技大学,西安地区的西安电子科技大学、西安交通大学,天津地区的南开大学,深圳地区的深圳大学,东北地区的哈尔滨工业大学等全国知名高等院校教学第一线上的教授和信息产业部有关科研院所的专家,请他们推荐和反复论证,从国外优秀的英文版图书中精选出版了这套《通信与信息科学教育丛书》(英文版)。

本套丛书可作为高等院校通信、计算机、电子信息等专业的高年级本科生、研究生的教材或教学参考书,也适合广大信息产业技术人员参考。

本套丛书所选取的均是国际上通信与信息科学领域具有代表性的经典著作,它们在全世界许多大学被用做教材或教学参考书。其主要特点是具有较强的先进性、实用性和权威性。丛书内容丰富,深入浅出,层次清楚,理论与应用并重,能够较好地引导读者将现代通信信息与信息科学的原理、技术与应用有机结合。我们希望本套丛书能够进一步推动国内高等院校教学与国际接轨,同时满足广大技术人员及时学习通信与信息科学领域中新知识的需求。

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*To Diane*

and Stephen, Mark, Kathleen, Patricia,  
Eileen, Harry, and Julia

and the next generation—  
Brittany, Erin, Thomas, Elizabeth, Emily,  
Dillon, Bryan, Julia, Robert, Margaret,  
Peter, Emma, Sarah, Harry, Rebecca,  
and Molly

## *Preface for Paperback Edition*

In 1968, Part I of *Detection, Estimation, and Modulation Theory* [VT68] was published. It turned out to be a reasonably successful book that has been widely used by several generations of engineers. There were thirty printings, but the last printing was in 1996. Volumes II and III ([VT71a], [VT71b]) were published in 1971 and focused on specific application areas such as analog modulation, Gaussian signals and noise, and the radar-sonar problem. Volume II had a short life span due to the shift from analog modulation to digital modulation. Volume III is still widely used as a reference and as a supplementary text. In a moment of youthful optimism, I indicated in the Preface to Volume III and in Chapter III-14 that a short monograph on optimum array processing would be published in 1971. The bibliography lists it as a reference, *Optimum Array Processing*, Wiley, 1971, which has been subsequently cited by several authors. After a 30-year delay, *Optimum Array Processing*, Part IV of *Detection, Estimation, and Modulation Theory* will be published this year.

A few comments on my career may help explain the long delay. In 1972, MIT loaned me to the Defense Communication Agency in Washington, D.C. where I spent three years as the Chief Scientist and the Associate Director of Technology. At the end of the tour, I decided, for personal reasons, to stay in the Washington, D.C. area. I spent three years as an Assistant Vice-President at COMSAT where my group did the advanced planning for the INTELSAT satellites. In 1978, I became the Chief Scientist of the United States Air Force. In 1979, Dr. Gerald Dinneen, the former Director of Lincoln Laboratories, was serving as Assistant Secretary of Defense for C3I. He asked me to become his Principal Deputy and I spent two years in that position. In 1981, I joined M/A-COM Linkabit. Linkabit is the company that Irwin Jacobs and Andrew Viterbi had started in 1969 and sold to M/A-COM in 1979. I started an Eastern operation which grew to about 200 people in three years. After Irwin and Andy left M/A-COM and started Qualcomm, I was responsible for the government operations in San Diego as well as Washington, D.C. In 1988, M/A-COM sold the division. At that point I decided to return to the academic world.

I joined George Mason University in September of 1988. One of my priorities was to finish the book on optimum array processing. However, I found that I needed to build up a research center in order to attract young research-oriented faculty and

doctoral students. The process took about six years. The Center for Excellence in Command, Control, Communications, and Intelligence has been very successful and has generated over \$300 million in research funding during its existence. During this growth period, I spent some time on array processing but a concentrated effort was not possible. In 1995, I started a serious effort to write the Array Processing book.

Throughout the *Optimum Array Processing* text there are references to Parts I and III of *Detection, Estimation, and Modulation Theory*. The referenced material is available in several other books, but I am most familiar with my own work. Wiley agreed to publish Part I and III in paperback so the material will be readily available. In addition to providing background for Part IV, Part I is still useful as a text for a graduate course in Detection and Estimation Theory. Part III is suitable for a second level graduate course dealing with more specialized topics.

In the 30-year period, there has been a dramatic change in the signal processing area. Advances in computational capability have allowed the implementation of complex algorithms that were only of theoretical interest in the past. In many applications, algorithms can be implemented that reach the theoretical bounds.

The advances in computational capability have also changed how the material is taught. In Parts I and III, there is an emphasis on compact analytical solutions to problems. In Part IV, there is a much greater emphasis on efficient iterative solutions and simulations. All of the material in parts I and III is still relevant. The books use continuous time processes but the transition to discrete time processes is straightforward. Integrals that were difficult to do analytically can be done easily in Matlab®. The various detection and estimation algorithms can be simulated and their performance compared to the theoretical bounds. We still use most of the problems in the text but supplement them with problems that require Matlab® solutions.

We hope that a new generation of students and readers find these reprinted editions to be useful.

HARRY L. VAN TREES

*Fairfax, Virginia*  
*June 2001*

## *Preface*

The area of detection and estimation theory that we shall study in this book represents a combination of the classical techniques of statistical inference and the random process characterization of communication, radar, sonar, and other modern data processing systems. The two major areas of statistical inference are decision theory and estimation theory. In the first case we observe an output that has a random character and decide which of two possible causes produced it. This type of problem was studied in the middle of the eighteenth century by Thomas Bayes [1]. In the estimation theory case the output is related to the value of some parameter of interest, and we try to estimate the value of this parameter. Work in this area was published by Legendre [2] and Gauss [3] in the early nineteenth century. Significant contributions to the classical theory that we use as background were developed by Fisher [4] and Neyman and Pearson [5] more than 30 years ago. In 1941 and 1942 Kolmogoroff [6] and Wiener [7] applied statistical techniques to the solution of the optimum linear filtering problem. Since that time the application of statistical techniques to the synthesis and analysis of all types of systems has grown rapidly. The application of these techniques and the resulting implications are the subject of this book.

This book and the subsequent volume, *Detection, Estimation, and Modulation Theory, Part II*, are based on notes prepared for a course entitled “Detection, Estimation, and Modulation Theory,” which is taught as a second-level graduate course at M.I.T. My original interest in the material grew out of my research activities in the area of analog modulation theory. A preliminary version of the material that deals with modulation theory was used as a text for a summer course presented at M.I.T. in 1964. It turned out that our viewpoint on modulation theory could best be understood by an audience with a clear understanding of modern detection and estimation theory. At that time there was no suitable text available to cover the material of interest and emphasize the points that I felt were



important, so I started writing notes. It was clear that in order to present the material to graduate students in a reasonable amount of time it would be necessary to develop a unified presentation of the three topics: detection, estimation, and modulation theory, and exploit the fundamental ideas that connected them. As the development proceeded, it grew in size until the material that was originally intended to be background for modulation theory occupies the entire contents of this book. The original material on modulation theory starts at the beginning of the second book. Collectively, the two books provide a unified coverage of the three topics and their application to many important physical problems.

For the last three years I have presented successively revised versions of the material in my course. The audience consists typically of 40 to 50 students who have completed a graduate course in random processes which covered most of the material in Davenport and Root [8]. In general, they have a good understanding of random process theory and a fair amount of practice with the routine manipulation required to solve problems. In addition, many of them are interested in doing research in this general area or closely related areas. This interest provides a great deal of motivation which I exploit by requiring them to develop many of the important ideas as problems. It is for this audience that the book is primarily intended. The appendix contains a detailed outline of the course.

On the other hand, many practicing engineers deal with systems that have been or should have been designed and analyzed with the techniques developed in this book. I have attempted to make the book useful to them. An earlier version was used successfully as a text for an in-plant course for graduate engineers.

From the standpoint of specific background little advanced material is required. A knowledge of elementary probability theory and second moment characterization of random processes is assumed. Some familiarity with matrix theory and linear algebra is helpful but certainly not necessary. The level of mathematical rigor is low, although in most sections the results could be rigorously proved by simply being more careful in our derivations. We have adopted this approach in order not to obscure the important ideas with a lot of detail and to make the material readable for the kind of engineering audience that will find it useful. Fortunately, in almost all cases we can verify that our answers are intuitively logical. It is worthwhile to observe that this ability to check our answers intuitively would be necessary even if our derivations were rigorous, because our ultimate objective is to obtain an answer that corresponds to some physical system of interest. It is easy to find physical problems in which a plausible mathematical model and correct mathematics lead to an unrealistic answer for the original problem.

We have several idiosyncrasies that it might be appropriate to mention. In general, we look at a problem in a fair amount of detail. Many times we look at the same problem in several different ways in order to gain a better understanding of the meaning of the result. Teaching students a number of ways of doing things helps them to be more flexible in their approach to new problems. A second feature is the necessity for the reader to solve problems to understand the material fully. Throughout the course and the book we emphasize the development of an ability to work problems. At the end of each chapter are problems that range from routine manipulations to significant extensions of the material in the text. In many cases they are equivalent to journal articles currently being published. Only by working a fair number of them is it possible to appreciate the significance and generality of the results. Solutions for an individual problem will be supplied on request, and a book containing solutions to about one third of the problems is available to faculty members teaching the course. We are continually generating new problems in conjunction with the course and will send them to anyone who is using the book as a course text. A third issue is the abundance of block diagrams, outlines, and pictures. The diagrams are included because most engineers (including myself) are more at home with these items than with the corresponding equations.

One problem always encountered is the amount of notation needed to cover the large range of subjects. We have tried to choose the notation in a logical manner and to make it mnemonic. All the notation is summarized in the glossary at the end of the book. We have tried to make our list of references as complete as possible and to acknowledge any ideas due to other people.

A number of people have contributed in many ways and it is a pleasure to acknowledge them. Professors W. B. Davenport and W. M. Siebert have provided continual encouragement and technical comments on the various chapters. Professors Estil Hoversten and Donald Snyder of the M.I.T. faculty and Lewis Collins, Arthur Baggeroer, and Michael Austin, three of my doctoral students, have carefully read and criticized the various chapters. Their suggestions have improved the manuscript appreciably. In addition, Baggeroer and Collins contributed a number of the problems in the various chapters and Baggeroer did the programming necessary for many of the graphical results. Lt. David Wright read and criticized Chapter 2. L. A. Frasco and H. D. Goldfein, two of my teaching assistants, worked all of the problems in the book. Dr. Howard Yudkin of Lincoln Laboratory read the entire manuscript and offered a number of important criticisms. In addition, various graduate students taking the course have made suggestions which have been incorporated. Most of the final draft was typed by Miss Aina Sils. Her patience with the innumerable changes is

sincerely appreciated. Several other secretaries, including Mrs. Jarmila Hrbek, Mrs. Joan Bauer, and Miss Camille Tortorici, typed sections of the various drafts.

As pointed out earlier, the books are an outgrowth of my research interests. This research is a continuing effort, and I shall be glad to send our current work to people working in this area on a regular reciprocal basis. My early work in modulation theory was supported by Lincoln Laboratory as a summer employee and consultant in groups directed by Dr. Herbert Sherman and Dr. Barney Reiffen. My research at M.I.T. was partly supported by the Joint Services and the National Aeronautics and Space Administration under the auspices of the Research Laboratory of Electronics. This support is gratefully acknowledged.

Harry L. Van Trees

Cambridge, Massachusetts  
October, 1967.

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# 1

## *Introduction*

In these two books, we shall study three areas of statistical theory, which we have labeled detection theory, estimation theory, and modulation theory. The goal is to develop these theories in a common mathematical framework and to demonstrate how they can be used to solve a wealth of practical problems in many diverse physical situations.

In this chapter we present three outlines of the material. The first is a topical outline in which we develop a qualitative understanding of the three areas by examining some typical problems of interest. The second is a logical outline in which we explore the various methods of attacking the problems. The third is a chronological outline in which we explain the structure of the books.

### 1.1 TOPICAL OUTLINE

An easy way to explain what is meant by detection theory is to examine several physical situations that lead to detection theory problems.

A simple digital communication system is shown in Fig. 1.1. The source puts out a binary digit every  $T$  seconds. Our object is to transmit this sequence of digits to some other location. The channel available for transmitting the sequence depends on the particular situation. Typically, it could be a telephone line, a radio link, or an acoustical channel. For

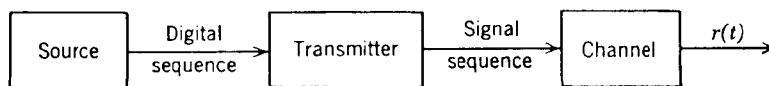


Fig. 1.1 Digital communication system.



## 2 1.1 Topical Outline

purposes of illustration, we shall consider a radio link. In order to transmit the information, we must put it into a form suitable for propagating over the channel. A straightforward method would be to build a device that generates a sine wave,

$$s_1(t) = \sin \omega_1 t, \quad (1)$$

for  $T$  seconds if the source generated a “one” in the preceding interval, and a sine wave of a different frequency,

$$s_0(t) = \sin \omega_0 t, \quad (2)$$

for  $T$  seconds if the source generated a “zero” in the preceding interval. The frequencies are chosen so that the signals  $s_0(t)$  and  $s_1(t)$  will propagate over the particular radio link of concern. The output of the device is fed into an antenna and transmitted over the channel. Typical source and transmitted signal sequences are shown in Fig. 1.2. In the simplest kind of channel the signal sequence arrives at the receiving antenna attenuated but essentially undistorted. To process the received signal we pass it through the antenna and some stages of rf-amplification, in the course of which a thermal noise  $n(t)$  is added to the message sequence. Thus in any  $T$ -second interval we have available a waveform  $r(t)$  in which

$$r(t) = s_1(t) + n(t), \quad 0 \leq t \leq T, \quad (3)$$

if  $s_1(t)$  was transmitted, and

$$r(t) = s_0(t) + n(t), \quad 0 \leq t \leq T, \quad (4)$$

if  $s_0(t)$  was transmitted. We are now faced with the problem of deciding which of the two possible signals was transmitted. We label the device that does this a decision device. It is simply a processor that observes  $r(t)$  and guesses whether  $s_1(t)$  or  $s_0(t)$  was sent according to some set of rules. This is equivalent to guessing what the source output was in the preceding interval. We refer to designing and evaluating the processor as a detection

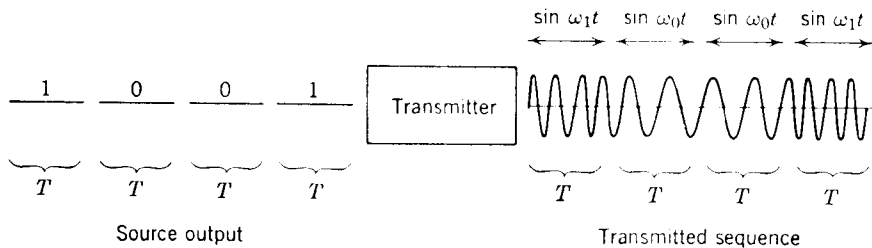


Fig. 1.2 Typical sequences.