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第三册

Volume 3

主　编　程代展　吴　敏

副主编　樊晓平　胡德文　黄　一　贾英民　刘智敏

彭　辉　王　龙　张纪峰　郑大钟

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内容简介

本书共收入 874 篇论文。这些论文是经中国自动化学会控制理论专业委员会组织评审，作为第 26 届中国控制会议发表的论文。论文内容包括系统理论与控制理论，模式识别，非线性系统及其控制，控制设计方法，复杂性与复杂系统理论，遗传算法与演化计算，分布参数系统，运动控制，混杂系统与 DEDS，智能机器人，大系统，分布式控制系统，随机系统，信息处理系统，稳定性与镇定，故障诊断，建模、辨识与信号处理，通讯网络系统，最优控制与优化，CIMS 与制造系统，鲁棒控制与 H_∞ 控制，交通系统，自适应控制与学习控制，生物与生态系统，变结构控制，社会经济系统，神经网络，工业系统，模糊系统与模糊控制等领域的应用研究成果。

本书可供从事自动控制理论及其应用研究的高等院校教师和研究生、科研单位的研究人员以及工业部门的工程技术人员参考。

本书进入 IEEE 会议出版程序，论文可从 IEEE Explore 下载。

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稳定性与镇定

Stability and Stabilization

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Stability of Linear Systems with Time Delay: A New Delay Fractioning Approach*

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Abstract: This paper investigates the stability of linear systems with time delay.Both constant and varying delay cases are considered respectively. The criteria of stability is derived based on a new type of Lyapunov-Krasovskii functional and is formulated as feasibility problems of linear matrix inequalities. Numerical examples are provided to illustrate the effectiveness of our main results.

Key Words: Delay Fractioning, LMI, Lyapunov Functional,Time-varying Delay

1 INTRODUCTION

Time delays are frequently encountered in many fields of science and engineering such as communication network, chemical process and economics. In many cases, time delays are often the sources of instability and poor performance[5]. Recently, improved performances have been reported by using Lyapunov-Krasovskii theory and LMIs techniques[1-3,5,6-13]. As we know, one of the difficulties in applying Lyapunov methods is the lack of efficient algorithms for constructing the Lyapunov functional. In general, the use of reduced functionals may result in conservatism. From here, many generalizations have been proposed, involving the various terms[6]:

$$
\begin{aligned}
V_1(x_t) &= x(t)^{\mathrm{T}} P x(t) \\
V_2(x_t) &= \int_{t-h}^{t} x(\theta)^{\mathrm{T}} S x(\theta) \\
V_3(x_t) &= \int_{t-h}^{t} \int_{v}^{t} \dot{x}(\theta)^{\mathrm{T}} R \dot{x}(\theta) \mathrm{d}\theta \mathrm{d}v \\
V_4(x_t) &= x(t)^{\mathrm{T}} \int_{-h}^{0} P(\theta) x(t+\theta) \mathrm{d}\theta \\
V_5(x_t) &= \int_{-h}^{0} \int_{-h}^{0} x(t+\theta)^{\mathrm{T}} P(\theta,\eta) x(t+\eta) \mathrm{d}\theta \mathrm{d}\eta \\
V_6(x_t) &= \int_{-h}^{0} x(\theta)^{\mathrm{T}} S(\theta) x(\theta) \mathrm{d}\theta
\end{aligned}
$$

for the case of delay independent stability, the Lyapunov Krasovskii candidate that usually applied is $V(x_t) = V_1(x_t) + V_2(x_t)$; As far as delay dependent stability is concerned, most researchers prefer $V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$ as the Lyapunov Krasovskii functionals in spite of these sufficient condition is far from being necessary. $V_4(x_t), V_5(x_t)$ and $V_6(x_t)$ appear in the complete Lyapunov functional, which is known to be necessary and sufficient for delay dependent stability and has the following form: $V(x_t) = V_1(x_t) + 2V_4(x_t) + V_5(x_t) + V_6(x_t)$. But, the general computation of the time-varying matrices in $V_4(x_t), V_5(x_t), V_6(x_t)$ comes up against computational problems and result cannot be applied for robust stability purposes. To solve this problem, Gu[4] proposed a complex discretized Lyapunov functional approach. Recently, Fridman[2,3] combined the discretized scheme proposed by Gu[4] with the descriptor form approach to analysis the stability of the time-varying delay system. The criteria showed significant improvements over the existing results even under very coarse discretization. There exists an interesting compromise between the reduction of the conservatism and the computational effort. Furthermore, most of the existing results obtained using Lyapunov Krasovskii stability theory for the systems with time-varying delays require constraints on the time derivative of the delays.

In this note, inspired by the idea of delay fractioning in[4], new Lyapunov Krasovskii functional are proposed to obtain less conservative and more concise form of stability conditions for a class of time delay systems.

Notation Throughout this note, the superscript T stands for matrix transposition. $\mathbf{R}^n$ denotes the n dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices. For two symmetric matrices, A and B, $A > (\geqslant) B$ means that $A - B$ is (semi-)positive definite. I_n and $0_{m\times n}$ denote that the identity matrix of size n and null matrix of size $n\times m$ respectively. If the context allows it the dimensions of these matrices are often omitted. Integer r denotes the delay fractioning number. h is the constant time delay. h_m is the lower bound of time delay. h_M is the upper bound of time delay. d is maximal difference between h_M and h_m, $(d = h_M - h_m)$. ρ is the upper bound of delay increase rate.

2 PROBLEM STATEMENTS

Consider the following linear system with time delay:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + A_h x(t-h(t)) \\
x(t) &= \phi(t), \forall t \in [-h_M, 0]
\end{aligned} \tag{1}
$$

where $x(t) \in \mathbf{R}^n$ is the state, $\phi(t)$ is a smooth vector-valued initial function defined in the Banach space$[-h_M, 0]$, $A, A_h \in \mathbf{R}^{n\times n}$ are known real constant matrices. $h(t)$ denotes the time delay, for the constant delay

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cases, it satisfy **Case1**, and for the time varying delay case, it is assumed to satisfy either **Case2** or **Case3** as

$$\textbf{Case1}: \quad h(t) = h$$
$$\textbf{Case2}: \quad h_m \leqslant h(t) \leqslant h_M, \dot{h}(t) \leqslant \rho$$
$$\textbf{Case3}: \quad h_m \leqslant h(t) \leqslant h_M$$

The purpose of this paper is to formulate practically computable criteria to check the stability of the system(1) with different delay cases1,2,3 respectively.

3 MAIN RESULTS

3.1 Constant Delay Case

The following theorem gives improved delay-dependent stability criteria for system(1) with constant delay case1.

Theorem1 Assume the time delay satisfy case1, the linear time delay system is asymptotically stable if there exist positive definite matrices $P, S_i, R_i \in \mathbf{R}^{n\times n}$, such that the LMI shown in (2) holds, and the matrices $E_i = [0_{n\times(in)}, I_n, 0_{n\times(r-i)n}]$.

$$\begin{aligned}\Pi = & E_0^{\mathrm{T}}(PA + A^{\mathrm{T}}P)E_0 + E_0^{\mathrm{T}}PA_hE_r \\ & + E_r^{\mathrm{T}}A_h^{\mathrm{T}}PE_0 + \sum_{i=1}^{r}\{(\frac{h}{r})^2(AE_0 + A_hE_r)^{\mathrm{T}}R_i \\ & (AE_0 + A_hE_r) + E_{i-1}^{\mathrm{T}}S_iE_{i-1} - E_i^{\mathrm{T}}S_iE_i \\ & -(E_{i-1} - E_i)^{\mathrm{T}}R_i(E_{i-1} - E_i)\} < 0 \end{aligned} \qquad (2)$$

In order to obtain less conservative results, we present a new discretization scheme of Lyapunov functional V_2, V_3 as follows

$$\bar{V}_2 = \sum_{r=1}^{r}\int_{t-\frac{i}{r}h}^{t-\frac{i-1}{r}h} x(\theta)^{\mathrm{T}}S_ix(\theta)\mathrm{d}\theta$$

$$\bar{V}_3 = \sum_{r=1}^{r}\int_{t-\frac{i}{r}h}^{t-\frac{i-1}{r}h}\int_{v}^{t} \dot{x}(\theta)^{\mathrm{T}}R_i\dot{x}(\theta)\mathrm{d}\theta\mathrm{d}v$$

Next, we will prove theoem1 through these functionals

Proof choose the following delay fractioning based Lyapunov Krasovskii funcitonal for the system(1)

$$V(x_t) = V_1(x_t) + \bar{V}_2(x_t) + \frac{h}{r}\bar{V}_3(x_t) \qquad (3)$$

Then the derive $V(x_t)$ along the trajectories of system(1) is

$$\dot{V}(x_t) \leqslant z(t)^{\mathrm{T}}\Pi z(t) \qquad (4)$$

where $z(t)^{\mathrm{T}} = [x(t)^{\mathrm{T}}, x(t-\frac{h}{r})^{\mathrm{T}}, \cdots, x(t-h)^{\mathrm{T}}]$, we find that $\dot{V}(x_t) \leqslant -\epsilon\|x(t)\|^2, (\exists\epsilon > 0)$ as long as the LMI (2) holds, which implies that system(1) is asymptotically stable. This completes the proof.

Remark1 The number of variables involve in LMI (2) is $(2r+1)(1+n)n/2$. However, the number of variables in [4](Gouaisbaut and Peaucelle,2006) is $rn(rn+1)/2+n(n+1)$ which is even lower than those of previous results. It is obvious that our result has lower number of variables than the aforementioned results.

Theorem1 only consider the constant case and by proper selection of Lyapunov functional can we make great progress in conservatism and with lower number of variables in LMI. How to extend this to varying delay cases is another important question to discussion.

3.2 Varying Delay and Its Derivative Dependent Case

The following theorem presents the delay and its derivative dependent stability conditions for system (1) with varying delay case2.

Theorem2 Assume the time delay $h(t)$ satisfies Case2, the linear delay system (1) is asymptotically stable if there exist positive definite matrices $P, S_i, R_i \in \mathbf{R}^{n\times n}, X = [X_{ij}], (X_{ij} \in \mathbf{R}^{n\times n}, i, j = 1,2,3), W \in \mathbf{R}^{n\times n}$ such that the LMI shown in (5) holds.

$$\begin{aligned}\Gamma = & d\bar{E}_0^{\mathrm{T}}X_{11}\bar{E}_0 + d\bar{E}_{r+1}^{\mathrm{T}}X_{22}\bar{E}_{r+1} + \sum_{i=1}^{r}\Big\{(\frac{h_m}{r})^2(A\bar{E}_0 \\ & + A_h\bar{E}_{r+1})^{\mathrm{T}}R_i(A\bar{E}_0 + A_h\bar{E}_{r+1}) + \bar{E}_{i-1}^{\mathrm{T}}S_i\bar{E}_{i-1} \\ & -\bar{E}_i^{\mathrm{T}}S_i\bar{E}_i - (\bar{E}_{i-1} - \bar{E}_i)^{\mathrm{T}}R_i(\bar{E}_{i-1} - \bar{E}_i)\Big\} \\ & + d(A\bar{E}_0 + A_h\bar{E}_{r+1})^{\mathrm{T}}X_{33}(A\bar{E}_0 + A_h\bar{E}_{r+1}) + \bar{E}_r^{\mathrm{T}}W \\ & \cdot\bar{E}_r - (1-\rho)\bar{E}_{r+1}^{\mathrm{T}}W\bar{E}_{r+1} + \Omega + \Omega^{\mathrm{T}} < 0 \end{aligned} \qquad (5)$$

where $\bar{E}_i = [0_{n\times(in)}, I_n, 0_{n\times(r+1-i)n}], i = 0, 1, \cdots, r+1$

$$\begin{aligned}\Omega = & \bar{E}_0^{\mathrm{T}}P(A\bar{E}_0 + A_h\bar{E}_{r+1}) + d\bar{E}_0^{\mathrm{T}}X_{12}\bar{E}_{r+1} \\ & + (\bar{E}_0^{\mathrm{T}}X_{13} + \bar{E}_{r+1}^{\mathrm{T}}X_{23})(\bar{E}_r - \bar{E}_{r+1})\end{aligned}$$

Proof Choose the following Lyapunov functional

$$V_v(x_t) = V(x_t) + \sum_{j=1}^{4} V_{vj}(x_t) \qquad (6)$$

where $V(x_t)$ is same as (3), and the other four terms are chosen as follows

$$V_v1 = \int_0^t\int_{\delta-h(\delta)}^{\delta-h_m} \ell(\delta,s)^{\mathrm{T}}X\ell(\delta,s)\mathrm{d}\delta\mathrm{d}s \qquad (7)$$

$$V_v2 = \int_{t-h_M}^{t-h_m} (h_M + s - t)\dot{x}^{\mathrm{T}}X_{33}\dot{x}(s)\mathrm{d}s \qquad (8)$$

$$V_v3 = (h_M - h_m)\int_{t-h_m}^{t} \dot{x}(s)^{\mathrm{T}}X_{33}\dot{x}(s)\mathrm{d}s \qquad (9)$$

$$V_v4 = \int_{t-h(t)}^{t-h_m} x(\omega)Wx(\omega)\mathrm{d}\omega \qquad (10)$$

in (7), $\ell(\delta,s)^{\mathrm{T}} = [x(\delta)^{\mathrm{T}}, x(\delta-h(\delta))^{\mathrm{T}}, \dot{x}(s)^{\mathrm{T}}]$

Now, we consider the derivative of $V_v(x_t)$ along the trajectories of system(1), one has

$$\dot{V}_v(x_t) \leqslant \zeta(t)^{\mathrm{T}}\Gamma\zeta(t) \qquad (11)$$

where $\zeta(t)^{\mathrm{T}} = [x(t)^{\mathrm{T}}, x(t-\frac{h_m}{r})^{\mathrm{T}}, \cdots, x(t-h_m)^{\mathrm{T}}, x(t-h(t))^{\mathrm{T}}]$. Therefore, the system(1) with time-varying delay case2 is asymptotically stable when the LMI (5) holds. This completes the proof.

Remark2 The Lyapunov functional terms (7)(8) are chosen as paper[13], but our method is based on delay fractioning technique and the less conservative results can be obtained through ours. Furthermore, our method can deal with the case of $h_m > 0$ besides $h_m = 0$ and [13]only considered the later one.

Tab. 1 Comparison of Maximal Allowable Delay

Methods	ρ is unknown	$\rho = 0.1$	$\rho = 2$	$\rho = 0$
[8]	0.3440			
[11]	0.7218	–	–	–
[6]	–	0.9447	–	1
[10]	0.9999	3.6040	0.9999	4.4721
$h_m = 0$	0.9999	3.6040	0.9999	4.4721
$h_m = 1$	1.576	4.193	1.576	5.143
$h_m = 3$	3.235	4.411	3.235	5.607

3.3 Varying Delay and Its Derivative Independent Case

If the last term $V_4(x_t)$ of (7) is zero, then the delay derivative independent conditions for system (1) would follow.

Corollary1 Assume the time delay $h(t)$ satisfies Case3, the time delay system (1) is asymptotically stable if there exist positive definite matrices $P, S_i, R_i \in \mathbf{R}^{n\times n}, X = [X_{ij}], (X_{ij} \in \mathbf{R}^{n\times n}, i, j = 1, 2, 3)$ such that the LMI shown in (12) holds.

$$\begin{aligned}\bar{\Gamma} = {} & d\bar{E}_0^{\mathrm{T}} X_{11}\bar{E}_0 + d\bar{E}_{r+1}^{\mathrm{T}} X_{22}\bar{E}_{r+1} + \sum_{i=1}^{r}\{(\frac{h_m}{r})^2(A\bar{E}_0 \\ & + A_h\bar{E}_{r+1})^{\mathrm{T}} R_i(A\bar{E}_0 + A_h\bar{E}_{r+1}) + \bar{E}_{i-1}^{\mathrm{T}} S_i\bar{E}_{i-1} \\ & - \bar{E}_i^{\mathrm{T}} S_i\bar{E}_i - (\bar{E}_{i-1} - \bar{E}_i)^{\mathrm{T}} R_i(\bar{E}_{i-1} - \bar{E}_i)\} \\ & + d(A\bar{E}_0 + A_h\bar{E}_{r+1})^{\mathrm{T}} X_{33}(A\bar{E}_0 + A_h\bar{E}_{r+1}) \\ & + \bar{E}_r^{\mathrm{T}} W\bar{E}_r + \Omega + \Omega^{\mathrm{T}} < 0 \end{aligned} \tag{12}$$

Remark3 Most of the existing delay-derivative-dependent conditions for the stability of systems with time-varying delay generally require a constraint of $\rho < 1$. Instead, the conditions provided in corollary1 hold for all $\rho \in \mathbf{R}$.

4 NUMERICAL EXAMPLE

In this section, some examples are used to demonstrate the effectiveness of our main results.

Consider the linear delay system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_h = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

For Case1, the maximal allowable delay obtained by theorem1 is 4.4721 when we take $r = 1$. The result is equivalent to the main classical results of the literature[1,11]. However, if we take $r = 2$, the maximal allowable delay obtained by theorem1 is 5.71, and when take $r = 3$, the maximal allowable delay is 5.96. It shows that the conservatism is reduced as the fractioning number r increased.

For Case2 and Case3, table1 shows the results obtained by theorem2 and corollary1 with $r = 2$.

The results are compared in Tab. 1. It can be seen that less conservative results for both delay-derivative-dependent and delay-derivative-independent cases are obtained by our stability criteria even under very coarse delay fractioning.

5 CONCLUSIONS

This note presents a new Lyapunov-Krasovskii functional based on delay fractioning technique for a class of linear delay systems. Both constant delay case and time-varying delay cases are considered. For the time-varying delay cases, both delay-derivative-dependent and delay-derivative-independent criteria are provided by our new form of Lyapunov functional, and the conservatism can be reduced with the increasing of fractioning number r. Numerical examples have demonstrated the effectiveness of the proposed results.

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BMI Approach to Decentralized and Cooperative Control of Large-Scale System*

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Abstract: Based on the bilinear matrix inequality (BMI) technique, the problems of designing decentralized controllers and cooperative controllers of linear large-scale systems are considered. Necessary and sufficient conditions of the optimal decentralized and cooperative stabilization of the linear large-scale systems are obtained. The problems of designing decentralized and cooperative controllers are formulated into the non-convex optimization problems with BMI constraints. To solve these problems, the alternate optimized algorithms are proposed. Finally, example is given to illustrate the main results. It shows that a large-scale system can be stabilized via cooperative controllers or decentralized controllers and needs not presumer each subsystem be stable.

Key Words: Decentralized control, Cooperative control, Large-scale system, Optimal algorithm, BMI

1 INTRODUCTION

Large-scale systems with many state variables and complicated structure often appear when it is applied to solve problems such as electric power systems, economic systems, transportation systems, etc[1]. Decentralized control theory is a basic method for large-scale systems. The decentralized control theory has attracted a great amount of interests[2] since 1960s due to its advantages in computation and implementation of control laws. But it is very hard to obtain the necessary conditions and sufficient conditions for decentralized stabilization of large-scale systems.

In traditional study of the stability of large-scale system, it presumes each subsystem is stable or strong stabilized every subsystem via local feedback of subsystem in generally. It composes the large-scale system via decreasing the action of interconnection relatively. But in fact, the cooperation is playing key role to the stability of large-scale systems. Recently, Duan etc. present a series of significant results for cooperative control of linear and nonlinear systems[3]. It shows that unstable subsystems can form a stable large-scale system via appropriate interconnections and cooperation controllers.

LMI (Linear Matrix Inequality) and BMI (Bilinear Matrix Inequality) methods have played a leading role during the last twenty years in linear system theory[4-6]. It is a useful tool to solve a variety of optimization and control problems. On the other hand, some optimization problems are non-convex with BMI constraints, so it is extremely hard to find the globally optimal solutions. Therefore, many researchers have devoted their efforts to develop algorithms to solve these problems[7-8]. In [7], the controller designing problem is directly formulated as an optimization problem with BMI constraints. In this paper, the non-convex optimal problem is solved perfectly by an algorithm.

The paper consists of the following parts. In section 2, the decentralized control and cooperative control problems of large-scale system are described. In section 3 and section 4, necessary and sufficient conditions for decentralized stabilization and cooperative stabilization of large-scale system are presented respectively in terms of BMI. In section 5, an optimization algorithm is proposed to solve the optimization problems. In section 6, example is given to illustrate the main results. Finally, the paper concludes in section 7 with a brief discussion of the results.

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2 PROBLEM FORMULATION

All the results of this paper can be easily extended to the large-scale system composed of N subsystems, for convenient to describe, only the interconnected large-scale system with two subsystems are discussed here.

At first, gives a linear interconnected system

$$\begin{cases} \dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_{11}u_{11}(t) + B_{12}u_{12}(t) \\ \dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + B_{21}u_{21}(t) + B_{22}u_{22}(t) \end{cases} \quad (1)$$

where $x_i(t) \in \mathbf{R}^{n_i}$ (for $i = 1, 2$, $n_1 + n_2 = n$) are state variables, $A_{ij} \in \mathbf{R}^{n_i \times n_j}$, $B_{ij} \in \mathbf{R}^{n_i \times l_j}$ (for $i, j = 1, 2$) are known constant matrices.

$$u_{ii}(t) = K_{ii}x_i(t),\ i = 1, 2 \quad (2)$$

are decentralized controllers and

$$u_{ij}(t) = K_{ij}x_j(t),\ i, j = 1, 2,\ i \neq j \quad (3)$$

are cooperative controllers. where $K_{ij} \in \mathbf{R}^{l_i \times n_i} (i = j), K_{ij} \in \mathbf{R}^{l_j \times n_j} (i \neq j)$, (for $i, j = 1, 2$) are unknown constant matrices to be design.

Define a quadratic performance index

$$J(u_{11}, u_{12}, u_{21}, u_{22}, x_0) = \int_0^\infty [x^{\mathrm{T}}(t)Qx(t) + (\sum_{i=1}^{2}\sum_{j=1}^{2} u_{ij}^{\mathrm{T}}(t)R_{ij}u_{ij}(t)]\mathrm{d}t \quad (4)$$

where: $Q \in \mathbf{R}^{n \times n}$, $R_{ij} \in \mathbf{R}^{l_j \times l_j} (i \neq j)$; $R_{ij} \in \mathbf{R}^{l_i \times l_i} (i = j)$ (for $i = 1, 2$) are given real constant positive definite matrices, $x(0) = x_0$ is the initial value of state x.