



# 第27届中国控制会议论文集

Proceedings of the 27<sup>th</sup> Chinese Control Conference

第二册

Volume 2

主 编 程代展 李 川

副主编 陈 杰 段广仁 黄 捷 贾英民 李少远

赵千川 黄 一 刘智敏



北京航空航天大学出版社  
BEIHANG UNIVERSITY PRESS

TP13-53/2

:27(2)

2008

# 第 27 届中国控制会议论文集

Proceedings of the 27<sup>th</sup> Chinese Control Conference

## 第二册

Volume 2

主 编 程代展 李 川

副 主 编 陈 杰 段广仁 黄 捷 贾英民 李少远

赵千川 黄 一 刘智敏



北京航空航天大学出版社

BEIHANG UNIVERSITY PRESS

## 内容简介

本书共收入1026篇论文。这些论文经过中国自动化学会控制理论专业委员会组织评审,为第27届中国控制会议正式发表论文。论文内容包括系统理论与控制理论,非线性系统及其控制,复杂性与复杂系统理论,分布参数系统,混杂系统与DEDS,大系统,随机系统,稳定性与镇定,建模、辨识与信号处理,最优控制与优化,鲁棒控制与 $H_\infty$ 控制,自适应控制与学习控制,变结构控制,神经网络,模糊系统与模糊控制,模式识别,控制设计方法,遗传算法与演化计算,运动控制,智能机器人,分布式控制系统,信息处理系统,故障诊断,通讯网络系统,CIMS与制造系统,交通系统,生物与生态系统,社会经济系统,工业系统等领域的应用研究成果。

本书可供从事自动控制理论及应用研究的高等院校教师和研究生、科研单位的研究人员以及工业部门的工程技术人员研究参考。

本书进入IEEE会议出版程序,论文可从IEEE Xplore下载。2006年起,CCC论文集被EI(Engineering Index)收录。

## 图书在版编目(CIP)数据

第27届中国控制会议论文集/程代展,李川主编.—北京:北京航空航天大学出版社,2008.7

ISBN 978-7-81124-390-1

I.第… II.①程…②李… III.自动控制理论—学术会议—文集 IV.TP13-53

中国版本图书馆CIP数据核字(2008)第093148号

IEEE Catalog Number: CFP0840A

# 第27届中国控制会议论文集

## Proceedings of the 27<sup>th</sup> Chinese Control Conference

主 编 程代展 李 川

副 主 编 陈 杰 段广仁 黄 捷 贾英民 李少远

赵千川 黄 一 刘智敏

责任编辑 沈 涛 刘 标

北京航空航天大学出版社出版发行

北京市海淀区学院路37号(100083) 发行部电话:010-82317024 传真:010-82328026

<http://www.buaapress.com.cn> E-mail:bhpress@263.net

涿州市新华印刷有限公司印装 各地书店经销

\*

开本:890×1240 1/16 印张:315 字数:10 201千字

2008年7月第1版 2008年7月第1次印刷 印数:600册

ISBN 978-7-81124-390-1 定价:990.00元(全七册)

# 目录 (Contents)

## 系统理论与控制理论 (System Theory and Control Theory)

Parameterized Solution to Generalized Sylvester Matrix Equation .....	Qiao Yupeng, Qi Hongsheng, Cheng Daizhan	2
同伦不动点个数与条件极点配置		
Number of Homotopic Fixed Point and Constrained Pole Disposition .....	孙亮, 于建均, 陈梅莲	7
传递函数矩阵零空间的最小多项式基和亏数		
On the Minimal Polynomial Base for Null Spaces of A Transfer Function Matrix and Its Defect .....	陈万义	11
基于编队技术的多列地铁列车运行规划		
The Operation Scheme of Subway Trains Based Formation Technology .....	路飞, 宋沐民	14
基于自助法的对数正态分布质量控制图		
Quality Control Charts for Log-Normal Distribution Based on Bootstrap Method .....	辛士波, 李元生, 李梅	18
线性规划模型预处理技术		
Presolving Techniques in Linear Programming Model .....	李博, 张国光, 吕香奋	22
确定性代数 Riccati 方程的一个比较定理		
A Comparision Theorem for Deterministic Algebraic Riccati Equation .....	李晶晶, 张维海, 穆玉兵	26
时滞 MIMO 系统的信息融合解耦补偿器设计		
Information Fusion Decoupling Compensator for Time-Delayed MIMO System .....	甄子洋, 王志胜	31
基于 LMI 的统一 D-稳定可靠控制器设计方法		
An Unified LMI Approach to Reliable D-Stabilization Controller Design for Linear Systems .....	肖民卿	36
基于广义采样保持函数的离散时间多输入多输出系统零点的稳定性		
Stability of Zeros of Sampled MIMO Systems with GSHF .....	梁山, 石飞光章, 谢开明	41
分力合成主动振动抑制方法和闭环反馈控制的同时设计		
Concurrent Desing of Component Synthesis Vibration Suppression Method and Feedback Control .....	张建英, 刘瞰	45
基于状态观测器的多面体不确定广义系统鲁棒预测控制		
Observer-Based Robust Predictive Control of Singular Systems with Polytopic Uncertainties .....	杨园华, 刘晓华	50
$L_2$ Gain Control Synthesis of Discrete-Time Switched Systems with Input Time-Delay .....	Xie Dongmei, Chen Xiaoxin, Xu Ning	54
LS-SVM Based Stable Generalized Predictive Control .....	Liu Bin, Jiang Zheng, Fang Kangling	58
Prejudgement and Control Treatment for Blockage of Pressure Screen Mesh in Papermaking Industry .....	Li Minghui, Li Yan	62
基于跟踪微分器的磁浮列车悬浮系统传感器主动容错控制		
Sensor Active Fault Tolerant Control of Maglev Suspension System Based on Tracking-Differentiator .....	李云, 薛松, 龙志强	67
On Invariant Ellipsoid for Linear Systems by Saturated Controls .....	Zhou Bin, Duan Guangren	71
代数比例相似观测器的定义及其数学分析		
Definition of Algebra Proportion Analogical Observer and Its Mathematical Analysis .....	韩忠旭	76

# 基于二维混合模型的输出反馈重复控制设计

## Output Feedback Repetitive Control Design Based on Two-dimensional Hybrid Model

.....	吴敏, 陈诗桓, 周兰, 余锦华	81
广义跳变系统的 $L_2$ - $L_\infty$ 控制		
$L_2/L_\infty$ Control of Singular Jumping Systems .....	蒋鼎国, 张宇林, 徐保国	86
Predictive Fusion Weighted Method Based on Multisensor Time-Delay Network ....	Liu Xikui, Li Yan	91
$H_\infty$ Filter Design for Markovian Jump Linear Systems with Mode-Dependent Time-Varying Delays .....	Zhang Xianming, Xiao Shenping, He Yong	95
Linear Estimation for Discrete-Time Systems with Markov Jump Delays .....	Han Chunyan, Zhang Huanshui	100
Delay-dependent Absolute Stability Criteria for Lur'e Singular Systems with Time-varying Delay .....	Wang Huijiao, Xue Anke, Lu Renquan	105
Further Delay-Range-Dependent Robust Stability Analysis for Time-Delay Systems .....	Yu Jianjiang, Zhang Kanjian, Fei Shumin, Zhao Xianlin	110
Collective Behavior of Swarms with General Nonlinear Attraction and Repulsion Functions .....	Pan Weiyun, Yang Liyan, Zheng Yufan, Yu Hongwang	115
模糊逻辑神经网络控制在交流调速系统中的仿真研究		
Simulation Study on Fuzzy Logic Neural-Network Control in the AC Drive System .....	乔美英, 兰建义	120
无源声探测目标跟踪算法		
The Tracking Algorithm of Passive Acoustic Detect System .....	廖永汉, 郭云飞, 彭冬亮	124
非线性随机时滞系统的时滞相关控制		
Delay-Dependent Control of Nonlinear Stochastic Delayed Systems .....	陈云, 薛安克, 赵晓东, 王俊宏	128
Data Reconciliation by Two-Step Risk Analysis of Modeling .....	Mei Congli, Liu Guohai	132
Delay-dependent Exponential Stability of Uncertain Stochastic Systems with Time-varying Delay and Nonlinearities .....	Yan Huaicheng, Meng Max Q.-H., Zhang Hao, Huang Xinhan	136
System Analysis of An Optimal Noise Shaped Quantizer for Audio-band Digital Amplifier .....	Hu Jwusheng, Chen Keng-Yuan	141
非线性随机时滞系统的指数 $H_\infty$ 保性能控制		
Exponential $H_\infty$ Guaranteed Cost Control for Nonlinear Stochastic Delay Systems .....	陈云, 薛安克, 鲁仁全	146
Parameterization of Full-order PI Observers in Second-order Linear Systems .....	Wang Guosheng, Liu Feng, Lv Qiang, Duan Guangren	152
一类不确定离散广义时滞系统的鲁棒耗散控制		
Robust Dissipative Control for A Class of Discrete-Time Singular Time-Delay Systems with Uncertainties .....	杨丽, 张庆灵	156
$H_\infty$ Control for Discrete-Time Systems with Limited Communication Channel .....	Guo Yafeng, Li Shaoyuan	160
基于线性矩阵不等式方法的鲁棒滤波综述		
LMI-based Robust Filtering: A Survey .....	何潇, 王子栋, 周东华	165
Multivariable Continuous-time Generalized Predictive Control with Multiple Time Delays .....	Wei Huan, Pan Lideng, Li Quanshan	170
高阶不等容环节代数等价观测器转换方法		
Transform Method of the Higher-order Different Capacity Link Algebraic Equivalent Observer .....	李丹, 周传心, 韩忠旭	175

Neural Networks Predictive Control Using AEPsO .....	Hou Zhixiang, Chen Hui, Li Heqing 180
$H_{\infty}$ Filtering of 2-D FM LSS Model with State Delays .....	Peng Dan, Guan Xinping, Li Yumei, Chen Cailian 184
On Numerical Reliability of Pole Assignment Algorithms — A Case Study .....	Duan Guangren 189
Linear Quadratic Regulation for Linear Continuous-time Systems with Distributed Input Delay .....	Liu Shuai, Xie Lihua, Zhang Huanshui 195
外场标定条件下捷联惯导系统误差状态可观测性分析	
Observability Analysis for the Error States of SINS under the Outer Field Conditions .....	杨晓霞, 黄一 200

## 非线性系统及其控制 (Nonlinear System and Control)

Designing New Controller for Fed-batch Cultivation of Recombinant E.coli .....	Mohseni Saleh, Vali Ahmad Reza, Babaeipour Valiollah 207
Analysis and Quenching of Limit Cycles of Electro-Hydraulic Servovalve Control Systems with Friction and Interval Transport Lag .....	Wang Yuan-Jay, Huang Jian-Xie, Zhang Ya-Zhu, Zhung Feng-Kun 212
Three-Phase Four-Leg Active Power Filter Based on Nonlinear Optimal Predictive Control .....	Wang Xiaogang, Xie Yunxiang, Shuai Dingxin 217
Stabilization of Stochastic Nonholonomic Systems with Unknown Stochastic Disturbances .....	Zheng Xiuyun, Wu Yuqiang 223
Output Maneuvering Control of Nonlinear Systems with Uncontrollable Unstable Linearization .....	Wei Chunling, Wang Qiangde 228
Universal Construction of Control Lyapunov Functions for Multi-Input Linear Systems .....	Cai Xiushan 233
A Differential Flatness Approach for Rotorcraft Fault Detection .....	Zhang Nan, Drouin Antoine, Doncescu Andrei, Mora-Camino Félix 237
Differential Flatness Applied to Vehicle Trajectory Tracking .....	Lu W.C., Duan L., Hsiao Fei-Bin, Mora-Camino Félix 242
Global Chaos Synchronization for Modified Chua's Circuit Systems via Linear State Error Feedback Control .....	Chen Yun, Wu Xiaofeng, Gui Zhifang 248
State Feedback Control for Nonlinear Stochastic System with Time-Delay .....	Liu Biyu, Li Meilan 252
一种基于神经网络解耦控制的多电机同步系统	
Multi-motor Synchronous System Based on Neural Network Decoupling Control .....	刘星桥, 陈冲, 刘国海, 赵亮 257
基于有机机制模拟的计算智能方法非线性映射模型	
A Nonlinear Mapping Model of Computational Intelligence Methods Based on Organic Mechanism Simulation .....	柴园园, 贾利民, 张尊栋 261
Stabilization of Discrete-time Networked Control Systems with Nonlinear Perturbation .....	Zhou Lei, Zhang Zhenjuan, Lu Guoping, Xiao Xiaoqing 266
Real Time Nonlinear Model Predictive Control Induced by Local Linear Controller .....	He Yuqing, Han Jianda 271
一类线性脉冲时滞微分系统的振动性	
Oscillations of First Order Linear Delay Differential Systems with Impulses .....	何小亚 276
一种基于敏感度方程的非线性预测控制算法	
Nonlinear Model Predictive Control Algorithm Based on Sensitivities Equations .....	王平, 田学民 280



一类非线性切换系统观测器设计的 LMI 方法	
A LMI Approach to Observer Design for A Class of Switched Nonlinear Systems	向峥嵘, 周川, 王荣浩 285
Design of Flight Control Law for Underwater Supercavitating Vehicle	Han Yuntao, Sun Yao, Mo Hongwei, Bai Tao 289
机载 ISAR 距离跟踪建模及其非线性滤波研究	
Modeling Range-Only Tracking in Airborne ISAR and Nonlinear Filter	黄慧敏, 文成林, 徐晓滨 294
Modeling and Analyzing the Spread of Worms Based on Quarantine	Li Tao, Weng Huihui, Zhu Zhengping 299
Robust Synchronization with Error Bound of Horizontal Platform Systems with Parameter Mismatch	Ma Mihua, Cai Jianping, Lin Meili 302
Chaos Synchronization of Master-slave Hyperbolic-type Generalized Lorenz Systems via Linear State Error Feedback Control	Gui Zhifang, Wu Xiaofeng, Chen Yun 307
基于 BUCK-BOOST 变换器的 T-S 模糊建模与控制	
T-S Fuzzy Modeling and Control Based on BUCK-BOOST Converter	陈进军, 纪志成 312
Adaptive Output-Feedback Stabilization for A Class of Uncertain Nonlinear Systems	Shang Fang, Liu Yungang 317
分导飞行器的载荷释放失效在线检测及控制律重构	
On-line Detection of Load Release Failure and Controller Reconfiguration for A Post-Boost Vehicle	李晓云, 姜苍华, 段广仁 323
双馈风力发电机组最大风能捕获控制方法研究	
Control of Capturing the Maximum Wind Energy with DFIG	谢桦, 张德宏 328
Further Discussion on A Block Triangular Observer Form	Wang Yebin, Lynch Alan F. 332
基于观测器的欠驱动船舶的输出反馈镇定	
Observer Based Output Feedback Stabilization of Underactuated Surface Vessel	刘展雷, 马保离 337
导弹拦截的一种非线性自适应制导律	
A Nonlinear Adaptive Guidance Law for Missile Interceptions	梁宏伟, 马保离 342
一种新的提前一步预测控制算法	
A New One-Step-Ahead Predictive Control Algorithm	阎纲, 梁昔明, 龙祖强, 李翔 345
Feedback Linearization and Continuous Sliding Mode Control for A Quadrotor UAV	Zhou Fang, Zhang Zhi, Liang Jun, Wang Jian 349
基于 CMAC 与 PID 复合的高速公路入口匝道控制	
Freeway On-Ramp Control Based on the Composition of PID Controller and CMAC	梁新荣, 范业坤 354
Adaptive Stabilization for A More General Class of High-Order Uncertain Nonlinear Systems	Sun Zongyao, Liu Yungang 358
Changing Supply Function in Superposition Form and Its Applications	Chen Zhiyong 363
吊车-双摆系统的增量式滑模控制	
Incremental Sliding Mode Control for Double-Pendulum-Type Overhead Crane System	董云云, 王中华, 冯志全, 程金 368
Determining Controllability of Snakeboard by Differential Forms	Yang Xiaosong 372
一种基于 Lyapunov 理论的永磁同步电机转矩控制方法	
Torque Control Method of Permanent Magnet Synchronous Motor Based on Lyapunov Theory	周京华, 陈亚爱, 刘坤 375

A Hamiltonian Approach to Dynamics and Stabilization of Spacecraft Multibody Systems	Liu Yunping, Wu Hongtao, Fang Xifeng	378
Nonlinear Control for Output Regulation of Ball and Plate System	Wang Hongrui, Tian Yantao, Fu Siyan, Sui Zhen	382
Parallel Simultaneous Stabilization of Two Nonlinear Port-Controlled Hamiltonian Systems Subject to Actuator Saturation	Wei Airong, Wang Yuzhen	388
面向服务的实时分布嵌入式控制软件实现策略		
Service-oriented Design and Implementation Strategy of Real-time Distributed Embedded Control Software	张晶, 王剑平, 张果, 张云生	393
Control Effort Reduction in Output-Feedback Stabilization for A Class of Stochastic Nonlinear Systems	Duan Na, Xie Xuejun	398
Internal Model Based Optimal Tracking Control for Nonlinear Discrete-time Systems with Sinusoidal Disturbances	Wang Haihong, Sun Liang	403
Robust $H_\infty$ Adaptive Control for A Class of Time-Delay Nonlinear Systems	Bi Weiping, Luo Chengxun, Li Sha, Mu Xuegang	408
Nonlinear Stabilizing of Multi-machine Power Systems Based on Feedback Domination Method	Chang Naichao, Mei Shengwei	412
The Stabilization of Fitzhugh-Nagumo Systems with One Feedback Controller	Yu Xin, Li Yongyong	417
A Fuzzy Logic Based Approach to Compensating for the Dynamic Inverse Error in the Cruise Missile TF/TA System	Wu Tianbao, Yin Hang, Huang Xianlin	420
Robust Dynamic Output Feedback Stabilization for A Class of Nonlinear Systems with State and Input Delay	Li Qiang, Jiao Xiaohong, Yang Jie	425
Model Predictive Control for A Missile with Blended Lateral Jets and Aerodynamic Fins	He Fenghua, Ma Kemao, Yang Baoqing, Yao Yu	430
Global Disturbance Rejection of Lower Triangular Systems with Unknown Exosystem	Liu Lu	435
Guaranteed Cost Control of Linear Time-Delay Systems with Input Constraints: the Razumikhin Functional Approach	Zuo Zhiqiang, Liu Liu, Wang Yijing, Zhao Huimin, Zhang Guoshan	440
Stability of Switched Nonlinear Time-varying Perturbed Systems	Yuan Yanyan, Qin Huashu, Cheng Daizhan	444
基于车辆动力学和非线性观测器的车辆质心侧偏角估计		
Vehicle Side-Slip Angle Estimation Based on Vehicle Dynamics and Nonlinear Observers	郭洪艳, 陈虹, 丁海涛, 毕春光, 赵海艳	449
基于变论域模糊 $H_\infty$ 控制器的迟滞非线性系统的跟踪控制		
Tracking Control of the Nonlinear System with Hysteresis Nonlinearity via the Variable Universe Fuzzy $H_\infty$ Controller	赵加祥, 陈增强, 徐旭林	454
用阶梯信号同步非线性系统		
Synchronization of Nonlinear Systems with Stair-Step Signal	金辉宇, 康宇, 殷保群	459
Output Feedback Attitude Tracking Control for A Rigid Spacecraft with Dynamic Uncertainty	Xian Bin, Huang Mu, Li Dong, Cui Cuijie, Yang Kaiyan	464
基于 Backstepping 的双臂空间机器人关节空间鲁棒控制		
Robust Output Tracking for Uncertained Space Robot System with Dual-Arms Using Backstepping Method	唐晓腾, 陈力	469
A Novel Taylor Series Based Approach for Control Computation in NN-ANARX Structure Based Control of Nonlinear Systems	Belikov J, Vassiljeva K, Petlenkov E, Nõmm S	474



Control of Switched Linear Systems via Saturated Input .....	Ni Wei, Qin Huashu, Cheng Daizhan	479
On Adaptive Stabilization of Nonlinearly Parameterized Discrete-Time Systems .....	Li Chanying, Guo Lei	484

## 复杂性 & 复杂系统理论 (Complexity and Complex System Theory)

Application of An Improved Particle Filter for State Estimation .....	Xiang Li, Liu Yu, Su Baoku	489
A Tracking Control Scheme for Leader Based Multi-agent Consensus for Discrete-time Case .....	Chen Zengqiang, Xiang Linying, Yuan Zhuzhi	494
多个体系统在空间曲线上的一致性问题的研究		
Consensus Problem of Multi-agent Systems Along A Spatial Curve .....	王宝平, 朱建栋, 匡静	499
复杂系统普适性控制方法的研究		
On General Control Methodology for Complex Systems .....	张尊栋, 贾利民, 柴园园	504
Stabilization of Complex Dynamical Networks by $H_\infty$ Control .....	Lu Jianquan, Ho Daniel W. C.	509
Disturbance Rejection and $H_\infty$ Pinning Control of Networked Multi-agent Systems .....	Li Zhongkui, Duan Zhisheng, Huang Lin	514
Quasi-Average-Consensus of Directed Hybrid Swarm Agents .....	Gu Mingqin, Song Yunzhong	519
煤矿灾变的涌现性分析		
Whole Emergence Analysis of Coal Mine Emergent Accidents .....	李琨, 王晓东, 张云生, 苗琦	524
煤矿灾变的涌现性模型		
Whole Emergence Model of Coal Mine Emergent Accidents .....	王晓东, 李琨, 张云生, 苗琦	528
Multi-agent Coordination with Switching Interaction Structures and Heterogeneous Agents .....	Shi Guodong, Hong Yiguang	531
A Criterion of the Consensus Problems in Multi-agent Systems with One Leader and Multi-follower .....	Li Zili, Chen Zengqiang, Yuan Zhuzhi	536
Synchronization of Master-slave Systems with Mixed Time Delays and Mismatched Parameters .....	Yuan Kun, Cao Jinde	540
基于拓扑马蹄理论的系统动态复杂性分析的综述		
Review of Dynamical Complexity Research Based on Topological Horseshoe Theory ....	袁泉, 杨晓松	544
一类二阶不确定 Sylvester 矩阵方程的鲁棒算法及其仿真		
Robust Algorithm to A Class of Perturbed Second-order Sylvester Matrix Equations and Its Simulation .....	段红梅	548
Synchronization Criteria of Complex Dynamical Networks with Impulsive Effects .....	Yang Zhichun	552
Multi-Model Hybrid System and the Stability Analysis .....	Li Shunxiang, Liu Jia	556
A Hypergraph Model for Clustering Scale-Free Network .....	Zheng Yu, Qian Rong	561
How Many Leaders Are Required for Consensus .....	Liu Zhixin	566
Sliding Mode Control for Neutral Delay Systems with Unmatched Uncertainties .....	Niu Yugang, Ho Daniel W. C.	571

## 分布参数系统 (Distributed Parameter Systems)

Stabilization of the Nonlinear Elastodynamic System with A Boundary Dissipation .....	Zhang Zhifei, Yao Pengfei	577
基于混合遗传算法的热传导系统最优控制问题求解		
Optimal Control Solving of Heat Transfer System by Applying A Hybrid Genetic Algorithm .....	赵瑞艳, 李树荣, 张晓东, 苗荣	580

## Numerical Simulation of Bioluminescence Tomography

..... Gong Wei, Li Ruo, Yan Ningning, Zhao Weibo	585
具有周期边界条件的 D-P 方程的分布控制的能稳性	
Stabilization of the Degasperis-Procesi Equation with Periodic Boundary Condition .....	宗西举 589
Identification of One Class of Distributed Parameter Systems Based on Chebyshev Polynomials	
..... Ji Xiaopeng, Ge Long, Wang Zhiquan	593
$\varepsilon$ -insensitizing Boundary Controls for Semilinear Wave Equations .....	Yan Yuqing 597
Linear Quadratic Differential Game Problems with Constrained Open-loop Controls .....	Xu Yashan 602
Order-Reduced Method of Voltage Transfer Function of Transformer Windings Based on Vector Fitting	
..... Zhang Ping, Nie Xinpeng, Wang Youhua, Zhang Yu	605
线性椭圆方程的 Lyapunov 不等式的最优控制方法	
Lyapunov Inequality for Linear Elliptic Equation, An Optimal Control Approach .....	周中成 610
利用最大值原理研究最优控制存在性	
Existence of Optimal Control by Using Maximum Principle .....	楼红卫 614

## 混杂系统与 DEDS (Hybrid Systems and DEDS)

Completeness, Passivity and Stability of Switched Systems .....	Zhao Jun, Hill David J. 618
不确定离散时滞切换系统的二次保成本控制	
Quadratic Guaranteed Cost Control of Uncertain Discrete-time Delay Switched Systems	
..... 李春明, 刘晓华	623
The Asymptotic Stability for General Stochastic Hybrid Systems .....	Liu Haijun, Mu Xiaowu 628
On the Zero-state Observability/Detectability of Switched and Hybrid Hamiltonian Systems with Infinite	
Number of Switching Subsystems .....	Zhu Liying, Wang Yuzhen 632
多智能体的分散混合控制	
Decentralized Hybrid Control of Multi-agent Systems .....	赵育强, 孙振东 637
小型无人直升机的混合系统建模与控制	
Hybrid Systems Model and Control of A Small Unmanned Helicopter	
..... 李坚强, 裴海龙, 刘馨, 贺跃帮, 陈勇	641
On Common Solutions of Riccati Inequalities: for Plannar Case	
..... Sun Hongfei, Zeng Jianping, Chen Bing	646
Destination Registration Elevator Group Control System Modeling and Dispatching Policy in Peak Traffic	
Pattern .....	Xu Yuge, Luo Fei, Yang Hong 651
$H_\infty$ Control of Switched Systems with Time-Delay	
..... Wang Yijing, Yao Zhenxian, Zuo Zhiqiang, Zhao Huimin, Zhang Guoshan	656
随机市场需求下混杂生产系统的生产控制问题	
Production Control Problem of Hybrid Systems under Market Circumstances	
..... 芮执元, 刘军, 赵俊天	661
Hybrid Predictive Modeling and Simulation of the Teniente Converter ....	Schaaf Max, Cipriano Aldo 666
On Switching Policies for Generalized Switched Server Systems .....	Wang Xingxuan 671
An Event-driven Dynamic Load Balancing Strategy for Streaming Media Clustered Server Systems	
..... Jiang Qi, Xi Hongsheng, Yin Baoqun, Xu Chenfeng	678
具有马氏跳跃参数的切削加工系统控制问题研究	
Control of Cutting System with Markovian Jump Parameters .....	董奕凡, 康宇, 奚宏生 683

## 大系统 (Large Scale Systems)

- BMI Approach to the Interconnected Stability and Cooperative Control of Linear Time-delay Systems  
..... Deng Xiaofei, Nian Xiaohong, Cao Li 689
- 随机大系统的信息融合最优分散控制  
Information Fusion Based Optimal Decentralized Control for Stochastic Largescale System  
..... 王志胜, 甄子洋 694
- 关联电力大系统的非线性分散控制  
Nonlinear Dencentralized Control of Interconnected Large-Scale Power Systems ..... 孙妙平, 年晓红 699
- Nonlinear Small-Gain Theorems for Discrete-Time Large-Scale Systems  
..... Jiang Zhongping, Lin Yuandan, Wang Yuan 704
- 基于 CAN 总线的  $\beta$ -甘露聚糖酶发酵控制系统的研究  
Study of  $\beta$ -mannanase Fermentation Control System Based on CAN  
..... 张小玉, 朱小六, 张宇林, 徐保国 709
- 一类非线性大系统的最优化控制方法  
Optimal Control Approach for A Class of Nonlinear Large-Scale Systems ..... 高德欣, 张文武 712
- 不确定相似组合系统的最优分散反馈控制  
Optimal Decentralized Feedback Control of Similar Composite Systems with Uncertainties .... 陈金莉 717
- 先进飞机公共设备管理系统综合技术  
Integration Technology of UMS for the Advanced Aircraft ..... 郭创, 徐浩军, 赵鵬 721
- Robust Decentralized  $H_\infty$  Control for Multi-Channel Continuous-Time Systems with Time-Delays  
..... Chen Ning, Gui Weihua 725
- Robust Decentralized Generalized  $H_2$  Control of Multi-Channel Uncertain Descriptor Systems with Time-Delays  
..... Chen Ning, Gui Weihua 730
- Decentralized Robust Adaptive Control of Uncertain Nonlinear Systems Using Dual High Gain  
..... Krishnamurthy P., Khorrami F. 735

## 随机系统 (Stochastic Systems)

- 一类非线性时变随机系统的能控性  
The Controllability for A Kind of Nonlinear Stochastic Control Systems Which the Coefficient is Time-dependent ..... 刘峰, 董洪斌 741
- Stability and Stabilization of Stochastic Hybrid Systems with Nonlinear Disturbances  
..... Gao Yang, Mu Xiaowu, Zhao Wei 745
- Measurement Feedback  $H^\infty$  Control of Uncertain Nonlinear Stochastic Systems  
..... Ni Yuanhua, Fang Haitao 749
- Robust  $H_2/H_\infty$  Control for Discrete-Time Stochastic Systems with Markovian Jumps and Multiplicative Noise: Finite Horizon Case ..... Hou Ting, Zhang Weihai, Ma Hongji 754
- Performance Assessment of the Output Probability Density Function Control System  
..... Zhang Jinfang, Zhang Jianhua, Hou Guolian 759
- Improved Delay-dependent Robust Stability for Uncertain Stochastic Systems with Time-varying Delay  
..... Zhang Yan, He Yong, Wu Min 764
- Backstepping Controller Design for A Class of Stochastic Nonlinear Systems with Markovian Switching  
..... Wu Zhaojing, Xie Xuejun 769
- 含有动态不确定性非线性随机系统的 H-infinity 控制  
 $H_\infty$  Control of Stochastic Nonlinear Systems with Dynamic Uncertainties ..... 张翼, 季海波, 魏波 774

Robust $H_\infty$ Control of Uncertain Stochastic Nonlinear Systems Driven by Noise of Unknown Covariance .....	Wei Bo, Ji Haibo 779
一类连续随机系统的静态输出反馈 $H_\infty$ 控制	
$H_\infty$ Control for A Class of Stochastic Systems via Static Output Feedback Controller .....	夏建伟 785
Stochastic Finite Horizon $H_\infty$ Control for Nonlinear Discrete-Time Systems .....	Lu Xiao, Zhang Weihai 789
Energy-to-peak Control for Ito Stochastic Differential Systems with Time Delay and Markovian Switching .....	Liu Hongliang, Duan Guangren 794
Markovian 调制的非线性 Ito 时滞微分系统指数稳定性	
Exponential Stability of Nonlinear Ito Differential Systems with Time-Delay and Markovian Switching .....	刘宏亮, 段广仁 798
$L_2$ - $L_\infty$ Filtering for Nonlinear Stochastic Systems .....	Wu Aiguo, Zhang Ying, Duan Guangren 801
A Free-Weighting Matrices Technique for Stochastic Stability Analysis of Uncertain Singular System with Markovian Jumping Parameters .....	Lu Hongqian, Zhou Wuneng, Xu Yuhua, Li Minghao, Jiang Lei 806

**系统理论与控制理论**

**System Theory and  
Control Theory**

# Parameterized Solution to Generalized Sylvester Matrix Equation\*

Qiao Yupeng, Qi Hongsheng, Cheng Daizhan

Key Laboratory of Systems and Control, Chinese Academy of Sciences, Beijing 100190, P.R.China  
E-mail: dcheng@iss.ac.cn

**Abstract:** The paper considers the parameterized solution to generalized Sylvester matrix equation. A set of matrix computation formulas have been developed. Using them, formulas for converting generalized Sylvester matrix mappings into conventional linear mappings are obtained. Based on this equation, an easily computable numerical algorithm for complete parameterized solutions of generalized Sylvester matrix equation is provided. The standard Sylvester matrix equation  $AX - EXF = BY$  and its dual equation  $XA - FXE = YC$  are considered as its special cases and then the corresponding solutions can be produced easily. Some further properties are also investigated. Comparing with existing algorithms, new approach simplified the computation significantly.

**Key Words:** Sylvester matrix equation, Parameterized solution

## 1 INTRODUCTION

Sylvester matrix equation has many applications in control theory. Particularly, when a singular control system is considered, it is used for many design of controls. Say, pole placement, tracking, design of Luenberger observers etc. So it has been studied widely [6, 7, 8, 9, 11, 12] and the references therein for details.

A generalized Sylvester matrix equation considered in this paper is of the following form

$$AX + XB + CXD + EX^T F = GY + MYN + PY^T Q \quad (1)$$

where  $A, C \in M_{n \times n}$ ,  $B, D, N \in M_{p \times p}$ ,  $E, F, P \in M_{n \times p}$ ,  $M, G \in M_{n \times r}$ ,  $Q \in M_{r \times p}$ , with unknowns  $X \in M_{n \times p}$  and  $Y \in M_{r \times p}$ . We use  $M_{m \times n}$  for the set of  $m \times n$  matrices. One can easily check that as the equation is liner with respect to  $X$  and  $Y$ , equation (1) is most general, as the dimension matching condition of matrix product is taking into consideration.

Several methods have been proposed for solving Sylvester equation and its dual equations [4, 5, 10]. Since equation (1) is linear with respect to  $X$  and  $Y$ , it is very natural to consider converting it into a standard linear algebraic equation. If it can be realized, finding parameterized solution is standard. Indeed, it can be achieved, but we need a set of notations and computation formulas for matrices. We briefly describe them here and refer to [2] for more details.

Let  $A = (a_{ij}) \in M_{m \times n}$ . Its column stacking form is expressed as

$$V_c(A) = (a_{11}, a_{21}, \dots, a_{m1}, \dots, a_{1n}, a_{2n}, \dots, a_{mn})^T \quad (2)$$

Its row stacking form is

$$V_r(A) = (a_{11}, a_{12}, \dots, a_{1n}, \dots, a_{m1}, a_{m2}, \dots, a_{mn})^T \quad (3)$$

Let  $x = (x_i) \in \mathbb{R}^{mn}$ . Then

1.

$$V_c^{-1}(x, m) = \begin{bmatrix} x_1 & x_{m+1} & \cdots & x_{(n-1)m+1} \\ x_2 & x_{m+2} & \cdots & x_{(n-1)m+2} \\ \vdots & \vdots & \ddots & \vdots \\ x_m & x_{2m} & \cdots & x_{nm} \end{bmatrix} \quad (4)$$

2.

$$V_r^{-1}(x, n) = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{(m-1)n+1} & x_{(m-1)n+2} & \cdots & x_{mn} \end{bmatrix} \quad (5)$$

Next, we convert a linear matrix mapping into a conventional linear mapping. Given a mapping  $\rho : X \mapsto \rho(X)$ , where  $X \in M_{n \times p}$ . Say, we use column stacking form. Denote  $x = V_c(X) \in \mathbb{R}^{np}$ ,  $y = V_c(\rho(X))$ , and assume the matrix mapping  $\rho$  is a linear mapping, with its matrix form  $M_\rho^c$ , which means

$$y = V_c(\rho(X)) = M_\rho^c x \quad (6)$$

For various linear matrix mappings we can construct their matrix forms respectively. The followings are some typical ones.

**Theorem 1** Assume  $A \in M_{m \times n}$ ,  $B \in M_{p \times q}$ ,  $C \in M_{m \times p}$ ,  $C \in M_{n \times q}$ , and  $X \in M_{n \times p}$ .

1. If  $\rho : X \mapsto AX$ , then

$$M_\rho^c = I_p \otimes A \quad (7)$$

2. If  $\rho : X \mapsto XB$ , then

$$M_\rho^c = B^T \otimes I_n \quad (8)$$

3. If  $\rho : X \mapsto CX^T$ , then

$$M_\rho^c = (I_n \otimes C)W_{[p,n]} \quad (9)$$

4. If  $\rho : X \mapsto X^T D$ , then

$$M_\rho^c = (D^T \otimes I_p)W_{[p,n]} \quad (10)$$

5. If  $\rho : X \mapsto AXB + CX^T D$ , then

$$M_\rho^c = (B^T \otimes A) + (D^T \otimes C)W_{[p,n]} \quad (11)$$

Note that in the above  $\otimes$  is the Kronecker product, and  $W_{[m,n]}$  is a swap matrix. Let  $A \in M_{m \times n}$ , then

$$\begin{cases} W_{[m,n]} V_r(A) = V_c(A) \\ W_{[n,m]} V_c(A) = V_r(A) \end{cases} \quad (12)$$

The swap property (12) uniquely determin  $W_{[n,m]}$ . We refer to [2] or [1] for details.

\* Work was supported by NNSF 60274012, 60221301, 60334040 of China.

Next, we use row stacking form. That is, for  $\rho : X \mapsto \rho(X)$ , we denote  $x = V_r(X)$ ,  $y = V_r(AX)$ , and express the matrix form of  $\rho$  by  $M_\rho^r$ , which means

$$y = V_r(\rho(X)) = M_\rho^r x \quad (13)$$

Similar to Theorem 1, under row stacking form we have

**Theorem 2** Assume  $A \in M_{m \times n}$ ,  $B \in M_{p \times q}$ ,  $C \in M_{m \times p}$ ,  $D \in M_{n \times q}$ , and  $X \in M_{n \times p}$ .

1. If  $\rho : X \mapsto AX$ , then

$$M_\rho^r = A \otimes I_p \quad (14)$$

2. If  $\rho : X \mapsto XB$ , then

$$M_\rho^r = I_n \otimes B^T \quad (15)$$

3. If  $\rho : X \mapsto CX^T$ , then

$$M_\rho^r = (C \otimes I_n)W_{[n,p]} \quad (16)$$

4. If  $\rho : X \mapsto X^T D$ , then

$$M_\rho^r = (I_p \otimes D^T)W_{[n,p]} \quad (17)$$

5. If  $\rho : X \mapsto AXB + CX^T D$ , then

$$M_\rho^r = (A \otimes B^T) + (C \otimes D^T)W_{[n,p]} \quad (18)$$

## 2 PARAMETERIZED SOLUTIONS

Using Theorem 1 we can convert equation (1) into a system of linear equations as

$$\begin{bmatrix} R_1 & R_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (19)$$

where

$$\begin{aligned} R_1 &= I_p \otimes A + B^T \otimes I_n + D^T \otimes C + (F^T \otimes E)W_{[p,n]}, \\ R_2 &= -I_p \otimes G - N^T \otimes M - (Q^T \otimes P)W_{[p,r]}, \\ x &= V_c(X), \quad y = V_c(Y). \end{aligned}$$

**Proposition 1** Equation (1) has solutions of  $rp$  degree of freedom, if and only if the coefficient matrix of equation (19) has full row rank. That is

$$\text{rank} \begin{bmatrix} R_1 & R_2 \end{bmatrix} = pn \quad (20)$$

Assume (20) holds, then we have  $rp$  linearly independent solutions of (19)

$$\begin{bmatrix} x^1 \\ y^1 \end{bmatrix}, \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}, \dots, \begin{bmatrix} x^{rp} \\ y^{rp} \end{bmatrix} \quad (21)$$

Then the set of  $rp$  linearly independent solutions of (1) are

$$\begin{cases} X^i = V_c^{-1}(x^i, n) \\ Y^i = V_c^{-1}(y^i, r) \end{cases} \quad i = 1, 2, \dots, rp \quad (22)$$

It follows that the parameterized solution is

$$\begin{cases} X = \sum_{i=1}^{rp} \mu_i V_c^{-1}(x^i, n) \\ Y = \sum_{i=1}^{rp} \mu_i V_c^{-1}(y^i, r) \end{cases} \quad (23)$$

where  $\mu = (\mu_1, \dots, \mu_{rp})^T$  are parameters. Each  $\mu \neq 0$  corresponds to a non-zero solution.

Equation (20) is important in finding independent solutions. An easy way to find the solutions is to choose any  $rp$  rows, equivalently, an  $rp \times (r+n)p$  matrix  $\Phi$ , such that

$$\Psi := \begin{bmatrix} R_1 & R_2 \\ \Phi \end{bmatrix}$$

is non-singular. Then the last  $rp$  columns of  $\Psi^{-1}$  form (21), the set of  $rp$  linearly independent solutions of equation (19).

We can also use Theorems 2 to solve (1). First, convert equation (1) into a system of linear equations as

$$\begin{bmatrix} R_3 & R_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (24)$$

where

$$\begin{aligned} R_3 &= A \otimes I_p + I_n \otimes B^T + C \otimes D^T + (E \otimes F^T)W_{[n,p]}, \\ R_4 &= -G \otimes I_p - M \otimes N^T - (P \otimes Q^T)W_{[r,p]}, \\ x &= V_r(X), \quad y = V_r(Y). \end{aligned}$$

Similar argument as for Proposition 1 yields the following corollary

**Corollary 1** Equation (24) has solutions of degree of freedom  $rp$ , if and only if the coefficient matrix of equation (24) has full row rank. That is

$$\text{rank} \begin{bmatrix} R_3 & R_4 \end{bmatrix} = pn \quad (25)$$

Now assume (25) holds, and the linearly independent solutions of (1) have the form of (21). Then the set of  $rp$  linearly independent solutions of (1) are

$$\begin{cases} X^i = V_r^{-1}(x^i, p) \\ Y^i = V_r^{-1}(y^i, p) \end{cases} \quad i = 1, 2, \dots, rp \quad (26)$$

The parameterized solution is

$$\begin{cases} X = \sum_{i=1}^{rp} \mu_i V_r^{-1}(x^i, p) \\ Y = \sum_{i=1}^{rp} \mu_i V_r^{-1}(y^i, p) \end{cases} \quad (27)$$

where  $\mu = (\mu_1, \dots, \mu_{rp})^T$  are parameters.  $\mu \neq 0$  corresponds to non-zero solution.

## 3 APPLICATION ON THE SYLVESTER MATRIX EQUATION AND ITS DUAL EQUATION

In this section we consider the Sylvester matrix equation as a special case of (1). It has the form

$$AX - EXF = BY \quad (28)$$

where  $A, E \in M_{n \times n}$ ,  $B \in M_{n \times r}$ ,  $F \in M_{p \times p}$ , with unknowns  $X \in M_{n \times p}$  and  $Y \in M_{r \times p}$ . The Sylvester matrix equation (28) and its dual equation play an important role in linear system analysis and control design.

A basic assumption for the solution of (28) is the so called  $R$ -controllable.  $(E, A, B)$  is called  $R$ -controllable if

$$\text{rank} \begin{bmatrix} sE - A & B \end{bmatrix} = n, \quad \forall s \in \mathbb{C}, \text{rank}(B) = r \quad (29)$$



The following lemma is proved in [5].

**Lemma 1** If  $E, A, B$  satisfy  $R$ -controllable condition (29) then equation (28) has  $rp$  degree of freedom. In other words, equation (28) has  $rp$  linearly independent solutions.

Recently, in [10] a complete general parametric expression for the solution  $(X, Y)$  is obtained under the assumption that  $(E, A, B)$  is  $R$ -controllable. In this note it can be obtained in a easy way.

For system (28), the equation (19) becomes

$$\begin{bmatrix} I_p \otimes A - F^T \otimes E & -I_p \otimes B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (30)$$

where  $x = V_c(X)$  and  $y = V_c(Y)$ . Then we have the following corollary:

**Corollary 2** Equation (28) has solutions of  $rp$  degree of freedom, if and only if the coefficient matrix of equation (30) has full row rank. That is

$$\text{rank} \begin{bmatrix} I_p \otimes A - F^T \otimes E & -I_p \otimes B \end{bmatrix} = pn \quad (31)$$

Assume  $U$  is a nonsingular matrix such that  $U^{-1}FU := J$  is the Jordan canonical form of  $F$ . We define  $\tilde{X} = XU$  and  $\tilde{Y} = YU$ , then equation (28) can be expressed equivalently as

$$A\tilde{X} - E\tilde{X}J = B\tilde{Y} \quad (32)$$

Correspondingly, (30) becomes

$$\begin{bmatrix} I_p \otimes A - J^T \otimes E & -I_p \otimes B \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = 0 \quad (33)$$

where  $\tilde{x} = V_c(\tilde{X})$  and  $\tilde{y} = V_c(\tilde{Y})$ . Now equation (33) can be written as

$$\Gamma \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = 0 \quad (34)$$

where  $\Gamma$  has block lower triangular form as

$$\Gamma = \begin{bmatrix} A - \lambda_1 E & -B & 0 & \cdots & 0 \\ * & A - \lambda_2 E & -B & \cdots & 0 \\ \vdots & & & & \\ * & * & \cdots & A - \lambda_p E & -B \end{bmatrix}$$

and  $\lambda_i, i = 1, \dots, p$  are eigenvalues of  $F$ . From (34) we have the following

**Proposition 2** Equation (28) has solutions of (minimum) degree of freedom  $rp$ , if and only if

$$\text{rank}(\lambda E - A \ B) = n, \quad \forall \lambda \in \sigma(F) \quad (35)$$

Obviously, Lemma 1 is a particular case of Proposition 2, because (29) assures (35).

Assume (35) holds, then for system (28) the formerly defined  $\Psi$  becomes

$$\Psi := \begin{bmatrix} I_p \otimes A - F^T \otimes E & -I_p \otimes B \\ \Phi \end{bmatrix}$$

then the set of  $rp$  linearly independent solution of equation (28) can be obtained as

$$\begin{cases} X = \sum_{i=1}^{rp} \mu_i V_c^{-1}(x^i, n) \\ Y = \sum_{i=1}^{rp} \mu_i V_c^{-1}(y^i, r) \end{cases}$$

where  $\mu = (\mu_1, \dots, \mu_{rp})^T$  are parameters. Each  $\mu \neq 0$  corresponds to a non-zero solution.

In the design of Luenberger observer, we have to solve the dual equation of equation (28) [10]. Precisely, it is

$$XA - FXE = YC \quad (36)$$

where  $A, E \in M_{n \times n}$ ,  $C \in M_{m \times n}$ ,  $F \in M_{p \times p}$ , with unknowns  $X \in M_{p \times n}$  and  $Y \in M_{p \times m}$ .

Using Theorems 2, we can convert equation (36) into the form

$$\begin{bmatrix} I_p \otimes A^T - F \otimes E^T & -I_p \otimes C^T \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad (37)$$

where  $x = V_r(X)$  and  $y = V_r(Y)$ .

Define  $\tilde{X} = U^{-1}X$  and  $\tilde{Y} = U^{-1}Y$ . Similar argument as for Proposition 2 yields the following corollary:

**Corollary 3** Equation (36) has solutions of (minimum) degree of freedom  $rp$ , if and only if

$$\text{rank} \begin{bmatrix} \lambda E - A \\ C \end{bmatrix} = n, \quad \forall \lambda \in \sigma(F) \quad (38)$$

Now assume (38) holds, and the linearly independent solutions of (37) have the form of (21). Then the set of  $rp$  linearly independent solutions of (36) are

$$\begin{cases} X^i = V_r^{-1}(x^i, n) \\ Y^i = V_r^{-1}(y^i, m) \end{cases} \quad i = 1, 2, \dots, rp \quad (39)$$

The parameterized solution is

$$\begin{cases} X = \sum_{i=1}^{rp} \mu_i V_r^{-1}(x^i, n) \\ Y = \sum_{i=1}^{rp} \mu_i V_r^{-1}(y^i, m) \end{cases} \quad (40)$$

#### 4 AN ILLUSTRATIVE EXAMPLE

As an application example, we consider a singular linear system [10]

$$\begin{cases} E\dot{x} = Ax + Bu, & x \in \mathbb{R}^n, u \in \mathbb{R}^r \\ y = Cx, & y \in \mathbb{R}^m \end{cases} \quad (41)$$

To assure the uniqueness of the solution, we assume  $(E, A)$  is a normal pair (or the system (41) is normal), if there exists  $s \in \mathbb{C}$  such that

$$\det(sE - A) \neq 0 \quad (42)$$

The system is called  $R$ -observable if

$$\text{rank} \begin{bmatrix} sE - A \\ C \end{bmatrix} = n, \quad \forall s \in \mathbb{C} \quad (43)$$

The Luenberger observer has the following form

$$\begin{cases} \dot{z} = Fz + Gy + Su, & z \in \mathbb{R}^p \\ \omega = Mz + Ny, & \omega \in \mathbb{R}^r \end{cases} \quad (44)$$

The design purpose is finding parameter matrices  $F \in M_{p \times p}$ ,  $G \in M_{p \times m}$ ,  $S \in M_{p \times r}$ ,  $M \in M_{r \times p}$ ,  $N \in M_{r \times m}$ , such that for a certain  $K \in M_{r \times n}$ , any initial  $x(0)$ ,  $z(0)$  and arbitrary input  $u(t)$  we have

$$\lim_{t \rightarrow \infty} (Kx(t) - \omega(t)) = 0 \quad (45)$$

We refer to [3] or [10] for the following result.

**Theorem 3** Assume system (41) is normal and  $R$ -observable. Then system (44) is a  $Kx$  observer, if and only if, there exist matrices  $F, T, G, S, M, N$  satisfying

$$\begin{cases} S = TB \\ TA - FTE = GC \\ K = MTE + NC \\ \text{Re}[\sigma(F)] < 0 \text{ (i.e., } F \text{ is Hurwitz)} \end{cases} \quad (46)$$

Next, we use the same example in [10] to show how convenient our approach is.

**Example 1** Consider system (41). Assume

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

As in [10], we want to design a Luenberger observer to track  $Kx$ , where

$$K = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$F$  can be any stable matrix. Now following [10], we choose

$$F = \begin{bmatrix} 0 & -2 \\ 1 & -2 \end{bmatrix}$$

Consider the second equation of (46) first. Using (37), a straightforward computation shows that it can be write as

$$\begin{bmatrix} -5 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} t \\ g \end{bmatrix} = 0 \quad (47)$$

where  $t = V_r(T)$  and  $g = V_r(G)$ . We can choose a  $\Phi$  and construct the  $(n+r)p \times (n+r)p$  matrix  $\Psi$  as

$$\Psi = \begin{bmatrix} -5 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the last four columns of  $\Psi^{-1}$  form the linearly independent set of solutions of equation (47):

$$\begin{bmatrix} t^1 & t^2 & t^3 & t^4 \\ g^1 & g^2 & g^3 & g^4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 1 \\ -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & -5 & 0 \\ 2 & 0 & 0 & 1 \\ 0 & -3 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Then

$$T_1 = \begin{bmatrix} 0 & 2 & -2 \\ 0 & 1 & 0 \end{bmatrix} \quad T_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad T_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} \quad G_2 = \begin{bmatrix} 2 & 0 \\ -3 & 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} -5 & 0 \\ -1 & 0 \end{bmatrix} \quad G_4 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Using (40), the parameterized solutions are

$$T = \begin{bmatrix} \mu_3 & 2\mu_1 + \mu_4 & -2\mu_1 \\ \mu_2 & \mu_1 & \mu_4 \end{bmatrix}$$

$$G = \begin{bmatrix} 2\mu_2 - 5\mu_3 & 2\mu_1 + \mu_4 \\ -3\mu_2 - \mu_3 & \mu_1 \end{bmatrix}$$

It follows that

$$S = TB = \begin{bmatrix} \mu_3 & 2\mu_1 + \mu_4 \\ \mu_2 & \mu_1 \end{bmatrix}$$

Then we solve the third equation of (46). Denote  $\alpha = V_c(M)$  and  $\beta = V_c(N)$ , we have

$$\begin{bmatrix} E^T T^T \otimes I_3 & C^T \otimes I_2 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = V_c(K) \quad (48)$$

And (48) can be rewrite as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad (49)$$

Its general solution is

$$\begin{bmatrix} 0 & -1 & \alpha_1 & \alpha_2 & 0 & 1 & 1 & 0 \end{bmatrix}^T$$

where  $\alpha_1, \alpha_2 \in \mathbb{R}$  are parameters.

Using (4), we have

$$M = \begin{bmatrix} 0 & \alpha_1 \\ -1 & \alpha_2 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Let  $\mu_1 = \mu_2 = \mu_3 = 0, \mu_4 = 1, \alpha_1 = \alpha_2 = 0$ , we have the particular solution given in [10] as

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \quad N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

## 5 CONCLUSION

This paper considered the parameterized solutions of generalized Sylvester matrix equation (1). Formulas for parameterized numerical solutions were obtained. Comparing it with the methods provided in [10], where the general solutions are parameterized by matrices, the conditions and algorithms developed in this paper are neater and simpler.