No. 12

交叉拱系网状扁壳的计算方法

董石麟

交叉拱系网状扁壳的计算方法*

董石麟

（浙江大学）

〔提要〕 本文对各种交叉拱系组成的网状扁壳进行了分析研究，给出了网状扁壳的位移法及混合法的一般方程式和等代刚度的计算公式。对两组、三组、四组及多组拱系所构成的各种网格形式的网状扁壳，分别作了详细的讨论。指出了加肋扁壳是网状扁壳的一个推广。

一、概 述

网壳结构是平板型网架结构的一种发展，它具有空间工作、受力合理、重量轻、刚度大的明显特点，是大跨结构的最佳形式之一，多年来引起了国内外的关注。对于网格布置比较稠密的网状薄壳，可当作连续体来考虑，利用壳体结构的已有解答或通过一般壳体的分析方法进行计算。文献〔1，2〕曾研究了菱形网格的圆柱面扁壳的计算方法；文献〔3，4，5，6，7〕讨论了正三角形、等腰三角形网格的网状扁壳；文献〔8〕探索了直角形网格的圆柱面扁壳的近似分析法。

本文研究了单层的交叉拱系网状扁壳以及由平面桁架拱系组成的双层网状扁壳；按其网格形式来分，有三角形、等腰三角形、正三角形、直角三角形、矩形、菱形以及直角三角

* 本文刊载在土木工程学报1985年第3期。

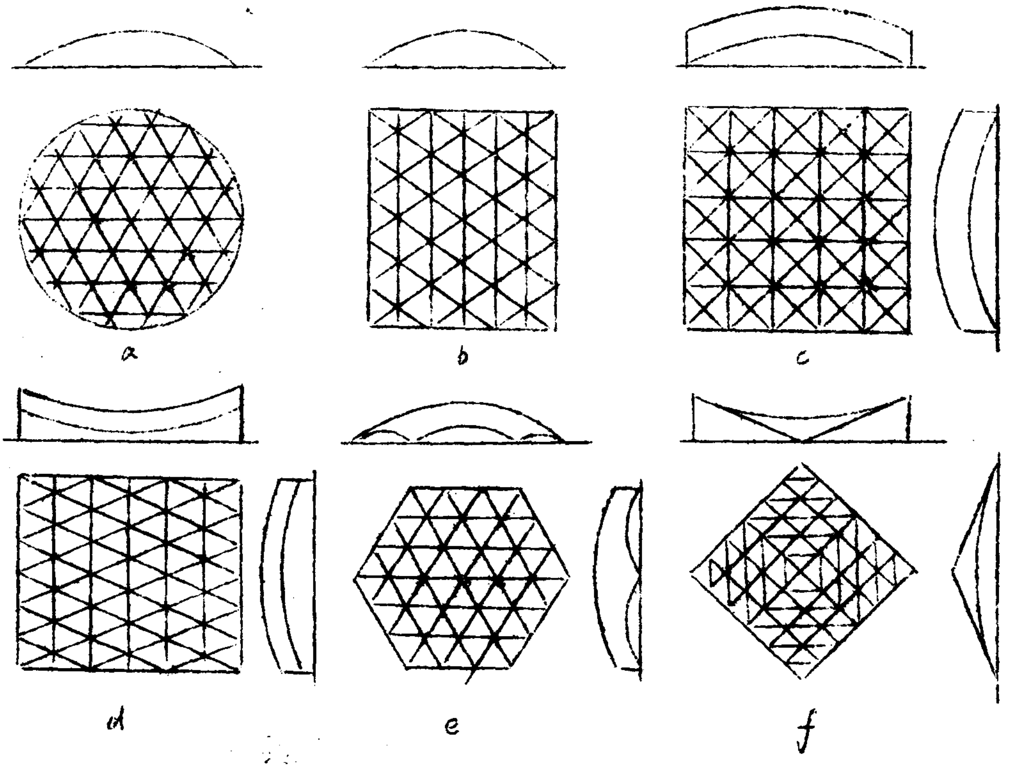
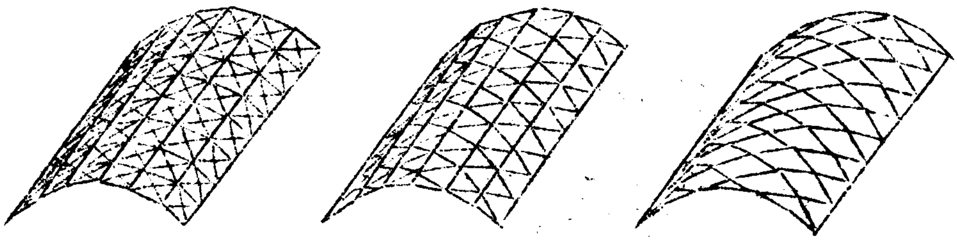


图1　各种平面图形及曲面形状的网状扁壳

形与菱形相间的网格等；按曲面形式来分，有球面、圆柱面、双曲抛物面等。如图1所示，在圆形、矩形、方形、多边形及菱形平面上，可复盖有交叉拱系组成的网状的球面扁壳、圆柱面壳、双曲扁壳、鞍形扁壳、扭壳等。又如图2所示，同是圆柱面网壳，可由四组、三组、两组拱系组合而成。对所有这些网状扁壳，本文从分析交叉拱系着手，导出了网状扁壳的位移法及混合法的一般基本方程式和等代刚度的计算公式。此外，还讨论了带肋扁壳的情况。文献中所研究的几种网状扁壳均为本文的特例。



a　　　　b　　　　c

图2　三种圆柱面网壳

二、网状扁壳位移法的一般方程式

所讨论的网状扁壳可认为是由几组交叉平面拱系所组成（图3)。拱系的每一榀拱能抵抗包括扭矩在内的拱平面内的各种外荷载，则扁拱系中第 i 组拱的物理方程、平衡方程和几何关系为：

$$N_i = EF_i \varepsilon_i \; ; \qquad M_i = EJ_i X_i \tag{1a}$$

$$\frac{\partial Q_i}{\partial X_i} + k_i N_i + g_i = 0 \; ; \qquad \frac{\partial M_i}{\partial X_i} - Q_i = 0 \tag{1b}$$

$$\varepsilon_i = \frac{\partial u_i}{\partial X_i} - k_i \omega \; ; \qquad x_i = - \frac{\partial^2 \omega}{\partial X_i^2} \tag{1c}$$

$$H_i = GJ_{pi} \omega_i \tag{2a}$$

$$\frac{\partial H_i}{\partial X_i} - m_i = 0 \tag{2b}$$

$$\omega_i = \frac{\partial^2 \omega}{\partial X_i \partial \bar{X}_i} \quad (i = 1, 2, \cdots\cdots) \tag{2c}$$

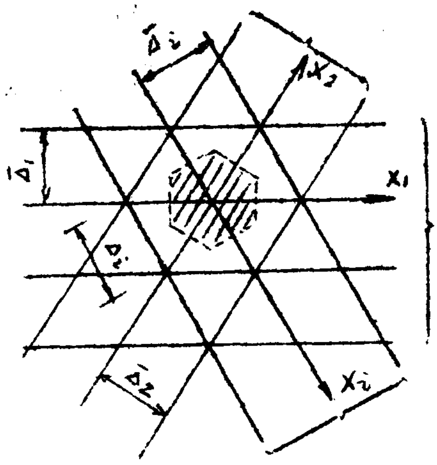


图 3　网状扁壳平面图

式中　u_i，ω为拱的变位；ε_i，X_i，ω_i为轴向应变、弯曲应变及扭转应变；N_i，M_i，Q_i，H_i为轴力、弯矩、切力及扭矩；k_i为拱的曲率；$\bar{x}_i$ 轴与X_i轴正交；g_i，m_i为折算线荷载及线扭矩；EF_i，EJ_i，GJ_{pi}为拱的抗压、抗弯、抗扭刚度；内力与变位的正向如图4所示。对于由平面桁架拱系组成的双层网状扁壳，在忽略横向剪切变形时〔9，10〕，拱截面的特性F_i，J_i，J_{pi}可由上、下弦杆截面积A_{ai}，A_{bi}及桁架高度h确定：

$$F_i = A_{ai} + A_{bi}; \quad J_i = \frac{A_{ai} A_{bi}}{A_{ai} + A_{bi}} h^2; \quad J_{pi} = 0 \qquad (3)$$

(1)　式表示在拱平面内的关系式，(2)　式表示沿拱轴线扭转的关系式，当消去内力与应变，可分别合并为

$$EJ_i \frac{\partial^4 \omega}{\partial x_i^4} + EF_i k_i^2 \omega - EF_i k_i \frac{\partial u_i}{\partial x_i} = g_i \qquad (4)$$

$$-GJ_{pi} \frac{\partial^3 \omega}{\partial x_i^2 \partial \bar{x}_i} = m_i \qquad (5)$$

当网状扁壳受有分布荷载X，Y，q 时，对于任一单元割离体（图3中阴影部分）可建立三个平衡方程式

$$\left.\begin{aligned}&\sum_i \Delta_i \frac{\partial N_i}{\partial X_i}\cos\alpha_i + AX = 0\\ &\sum_i \Delta_i \frac{\partial N_i}{\partial X_i}\sin\alpha_i + AY = 0\\ &\sum_i \Delta_i \left(g_i - \frac{\partial m_i}{\partial \bar{X}_i}\right) = Aq\end{aligned}\right\} \qquad (6)$$

式中　A为单元割离体的面积，亦即拱系的节点所管辖的平面积，Δ_i为其在x_i轴方向的长度，α_i为x_i轴与水平轴x的夹角。如设$\bar{\Delta}_i$为拱间距，则恒有等式

$$\Delta_i \bar{\Delta}_i = A \qquad (7)$$

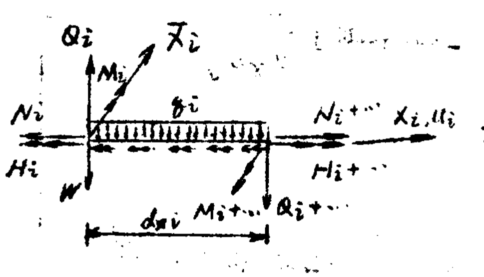


图4　拱的内力和变位

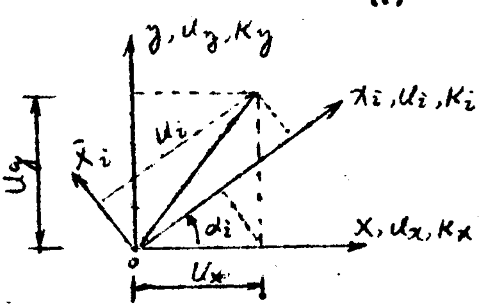


图5　水平变位及曲率

拱系与网状扁壳的水平变位及曲率存在下列简单关系式（图5）：

$$u_i = u_x\cos\alpha_i + u_y\sin\alpha_i \qquad (8)$$

$$k_i = k_x\cos^2\alpha_i + 2t\cos\alpha_i\sin\alpha_i + k_y\sin^2\alpha_i \qquad (9)$$

将式(4)，(5)及（1a），（1c）中的第一式代入式(6)，并注意到(7)，

(8)，(9)三式及下列微分关系式

$$\frac{\partial}{\partial x_1}=\cos\alpha_1\frac{\partial}{\partial x}+\sin\alpha_1\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial \bar{x}_1}=-\sin\alpha_1\frac{\partial}{\partial x}+\cos\alpha_1\frac{\partial}{\partial y}$$

经整理后，便得到以变位 u_x，u_y，ω 表示的网状扁壳的一般方程式

$$\left.\begin{aligned}&L_{11}u_x+L_{12}u_y+L_{13}\omega+X=0\\&L_{21}u_x+L_{22}u_y+L_{23}\omega+Y=0\\&L_{31}u_x+L_{32}u_y+L_{33}\omega=q\end{aligned}\right\}\qquad(10)$$

其中各算符 L_{mn} 分别为

$$L_{11}=\sum_i E\delta_i\cos^2\alpha_i\left(\cos^2\alpha_i\frac{\partial^2}{\partial x^2}+2\cos\alpha_i\sin\alpha_i\frac{\partial^2}{\partial x\partial y}+\sin^2\alpha_i\frac{\partial^2}{\partial y^2}\right)$$

$$L_{12}=L_{21}=\sum_i E\delta_i\cos\alpha_i\sin\alpha_i\left(\cos^2\alpha_i\frac{\partial^2}{\partial x^2}+2\cos\alpha_i\sin\alpha_i\frac{\partial^2}{\partial x\partial y}+\sin^2\alpha_i\frac{\partial^2}{\partial y^2}\right)$$

$$L_{13}=L_{31}=-\sum_i E\delta_i\cos\alpha_i\left(k_x\cos^2\alpha_i+2t\cos\alpha_i\sin\alpha_i+k_y\sin^2\alpha_i\right)\left(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y}\right)$$

$$L_{22}=\sum_i E\delta_i\sin^2\alpha_i\left(\cos^2\alpha_i\frac{\partial^2}{\partial x^2}+2\cos\alpha_i\sin\alpha_i\frac{\partial^2}{\partial x\partial y}+\sin^2\alpha_i\frac{\partial^2}{\partial y^2}\right)$$

$$
\left.
\begin{aligned}
L_{23}=L_{32}=&-\sum_i E\delta_i \sin\alpha_i (k_x\cos^2\alpha_i+2t\cos\alpha_i\sin\alpha_i \\
&+k_y\sin^2\alpha_i)(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y}) \\
L_{33}=&\sum_i D_i(\cos^4\alpha_i\frac{\partial^4}{\partial x^4}+4\cos^3\alpha_i\sin\alpha_i\frac{\partial^4}{\partial x^3\partial y} \\
&+6\cos^2\alpha_i\sin^2\alpha_i\frac{\partial^4}{\partial x^2\partial y^2}+4\cos\alpha_i\sin^3\alpha_i\frac{\partial^4}{\partial x\partial y^3} \\
&+\sin^4\alpha_i\frac{\partial^4}{\partial y^4})+\sum K_i[\cos^2\alpha_i\sin^2\alpha_i\frac{\partial^4}{\partial x^4} \\
&+2(\cos\alpha_i\sin^3\alpha_i-\cos^3\alpha_i\sin\alpha_i)\frac{\partial^4}{\partial x^3\partial y}+(\cos^4\alpha_i \\
&-4\cos^4\alpha_i\sin^2\alpha_i+\sin^4\alpha_i)\frac{\partial^4}{\partial x^2\partial y^2}+2(\cos^3\alpha_i\sin\alpha_i \\
&-\cos\alpha_i\sin^3\alpha_i)\frac{\partial^4}{\partial x\partial y^3}+\cos^2\alpha_i\sin^2\alpha_i\frac{\partial^4}{\partial y^4}] \\
&+\sum E\delta_i(k_x\cos^2\alpha_i+2t\cos\alpha_i\sin\alpha_i+k_y\sin^2\alpha_i)^2
\end{aligned}
\right\} \quad (11)
$$

$E\delta_i$，D_i，K_i 表示 i 方向拱系在其单位宽度上的折算抗压、抗弯、抗扭刚度：

$$E\delta_i=\frac{EF_i}{\bar{\Delta}_i}\,;\quad D_i=\frac{EJ_i}{\bar{\Delta}_i}\,;\quad K_i=\frac{GJ_{pi}}{\bar{\Delta}_i}\,;\qquad (12)$$

计算时，还需要以变位来表示网状扁壳的内力。设有任一截面，其法线 x_n 和 x 轴的夹角为 α_n，则网状扁壳的轴力 T_n、剪力 S_n、弯矩 M_n、扭矩 H_n、横切力 Q_n 及综合横切力 Q_n^* 可用拱系的内力表示如下（图 6）：

$$
\left.
\begin{aligned}
T_n &= \sum_i \frac{M_i}{\bar{\Delta}_i}\cos^2(\alpha_i-\alpha_n) \\
S_n &= \sum_i \frac{N_i}{\bar{\Delta}_i}\cos(\alpha_i-\alpha_n)\sin(\alpha_i-\alpha_n)
\end{aligned}
\right\} \qquad (13)
$$

$$M_n=\sum_i \left[\frac{M_i}{\bar{\Delta}_i}\cos^2(\alpha_i-\alpha_n)-\frac{H_i}{\bar{\Delta}_i}\cos(\alpha_i-\alpha_n)\sin(\alpha_i-\alpha_n)\right]$$

$$H_n=\sum_i \left[\frac{M_i}{\bar{\Delta}_i}\cos(\alpha_i-\alpha_n)\sin(\alpha_i-\alpha_n)+\frac{H_i}{\bar{\Delta}_i}\cos^2(\alpha_i-\alpha_n)\right] \qquad (14)$$

$$Q_n=\sum \frac{Q_i+\dfrac{\partial H_i}{\partial \bar{x}_i}}{\bar{\Delta}_i}\cos(\alpha_i-\alpha_n) \qquad (15)$$

$$Q_n^*=Q_n+\frac{\partial H_n}{\partial \bar{x}_n}$$

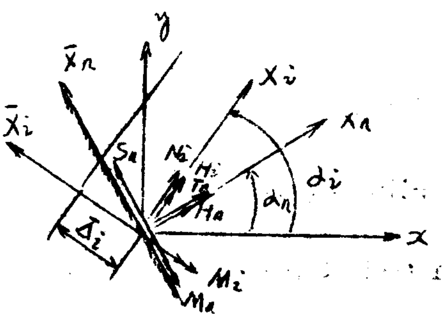


图6　内力关系图

将式（1a），（2a），（1c），（2c）依次代入，并应用式(8)，(9)，便可用变位 u_x，u_y，ω 来表示上述各式；当截面法线 x_n 与 x，y 轴平行时，则有：

$$T_x=\sum_i E\delta_i\cos^2\alpha_i\left(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y}\right)u_x$$

$$+\sin\alpha_i(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y})u_y-(k_x\cos^2\alpha_i$$

$$+2t\cos\alpha_i\sin\alpha_i+k_y\sin^2\alpha_i)\omega]$$

$$T_y=\sum_i E\delta_i\sin^2\alpha_i[\cos\alpha_i(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y})u_x$$

$$+\sin\alpha_i(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y})u_y-(k_x\cos^2\alpha_i$$

$$+2t\cos\alpha_i\sin\alpha_i+k_y\sin^2\alpha_i)\omega] \qquad (16)$$

$$S_{xy}=\sum_i E\delta_i\cos\alpha_i\sin\alpha_i[\cos\alpha_i(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y})u_x$$

$$+\sin\alpha_i(\cos\alpha_i\frac{\partial}{\partial x}+\sin\alpha_i\frac{\partial}{\partial y})u_y-(k_x\cos^2\alpha_i$$

$$+2t\cos\alpha_i\sin\alpha_i+k_y\sin^2\alpha_i)\omega]$$

$$S_{yx}=S_{xy}$$

$$M_x=-\sum_i[\cos^2\alpha_i(D_i\cos^2\alpha_i+K_i\sin^2\alpha_i)\frac{\partial^2\omega}{\partial x^2}$$

$$+\cos^2\alpha_i\sin^2\alpha_i(D_i-K_i)\frac{\partial^2\omega}{\partial y^2}$$

$$+\cos\alpha_i\sin\alpha_i(2D_i\cos^2\alpha_i-K_i\cos2\alpha_i)\frac{\partial^2\omega}{\partial x\partial y}]$$

$$M_y=-\Sigma[\cos^2\alpha_i\sin^2\alpha_i(D_i-K_i)\frac{\partial^2\omega}{\partial x^2}+\sin^2\alpha_i(D_i\sin^2\alpha_i$$

$$+K_i\cos^2\alpha_i)\frac{\partial^2\omega}{\partial y^2}+\cos\alpha_i\sin\alpha_i(2D_i\sin^2\alpha_i$$

$$+K_i\cos2\alpha_i)\frac{\partial^2\omega}{\partial x\partial y}]$$

$$
\begin{aligned}
H_{xy}=-\Sigma\{&\cos^3\alpha_i\,(D_i-K_i)\,\frac{\partial^2\omega}{\partial x^2}+\cos\alpha_i\,\sin\alpha_i\,(D_i\sin^2\alpha_i\\
&+K_i\cos^2\alpha_i)\frac{\partial^2\omega}{\partial y^2}+\cos^2\alpha_i\,(2D_i\sin^2\alpha_i+K_i\cos2\alpha_i)\frac{\partial^2\omega}{\partial x\partial y}\}\\
H_{yx}=-\sum_i\{&\cos\alpha_i\sin\alpha_i\,(D_i\cos^2\alpha_i+K_i\sin^2\alpha_i)\,\frac{\partial^2\omega}{\partial x^2}\\
&+\cos\alpha_i\sin^3\alpha_i\,(D_i-K_i)\frac{\partial^2\omega}{\partial y^2}+\sin^2\alpha_i\,(2D_i\cos^2\alpha_i\\
&-K_i\cos2\alpha_i)\,\frac{\partial^2\omega}{\partial x\partial y}\}
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
Q_x=-\sum_i\{&\cos^2\alpha_i\,[D_i\cos^2\alpha_i+K_i\sin^2\alpha_i]\,\frac{\partial^3\omega}{\partial x^3}\\
&+\cos\alpha_i\sin\alpha_i\,[3D_i\cos^2\alpha_i+K_i\,(\sin^2\alpha_i-2\cos^2\alpha_i)]\frac{\partial^3\omega}{\partial x^2\partial y}\\
&+\cos^2\alpha_i\,[3D_i\sin^2\alpha_i+K_i\,(\cos^2\alpha_i-2\sin^2\alpha_i)]\,\frac{\partial^3\omega}{\partial x\partial y^2}\\
&+\cos\alpha_i\sin\alpha_i\,[D_i\sin^2\alpha_i+K_i\cos^2\alpha_i]\,\frac{\partial^3\omega}{\partial y^3}\}\\
Q_y=-\sum_i\{&\cos\alpha_i\sin\alpha_i\,[D_i\cos^2\alpha_i+K_i\sin^2\alpha_i]\,\frac{\partial^3\omega}{\partial x^3}\\
&+\sin^2\alpha_i\,[3D_i\cos^2\alpha_i+K_i\,(\sin^2\alpha_i-2\cos^2\alpha_i)]\,\frac{\partial^3\omega}{\partial x^2\partial y}\\
&+\cos\alpha_i\sin\alpha_i\,[3D_i\sin^2\alpha_i+K_i\,(\cos^2\alpha_i-2\sin^2\alpha_i)]\frac{\partial^3\omega}{\partial x\partial y^2}\\
&+\sin^2\alpha_i\,[D_i\sin^2\alpha_i+K_i\cos^2\alpha_i]\,\frac{\partial^3\omega}{\partial y^3}\}\\
Q^*_x=-\sum_i\{&\cos^2\alpha_i\,[D_i\cos^2\alpha_i+K_i\sin^2\alpha_i]\,\frac{\partial^3\omega}{\partial x^3}
\end{aligned}
$$

$$+\cos\alpha_i\sin\alpha_i\left[4D_i\cos^2\alpha_i+K_i(\sin^2\alpha_i-3\cos^2\alpha_i)\right]\frac{\partial^3\omega}{\partial x^2\partial y}$$

$$+\cos^2\alpha_i\left[5D_i\sin^2\alpha_i+K_i(2\cos^2\alpha_i-3\sin^2\alpha_i)\right]\frac{\partial^3\omega}{\partial x\partial y^2}$$

$$+\cos\alpha_i\sin\alpha_i\left[2D_i\sin^2\alpha_i+2K_i\cos^2\alpha_i\right]\frac{\partial^3\omega}{\partial y^3}\}$$

$$Q^{i}_y=-\sum_i\{\cos\alpha_i\sin\alpha_i\left[2D_i\cos^2\alpha_i+2K_i\sin^2\alpha_i\right]\frac{\partial^3\omega}{\partial x^3}$$

$$+\sin^2\alpha_i\left[5D_i\cos^2\alpha_i+K_i(2\sin^2\alpha_i-3\cos^2\alpha_i)\right]\frac{\partial^3\omega}{\partial x^2\partial y}$$

$$+\cos\alpha_i\sin\alpha_i\left[4D_i\sin^2\alpha_i+K_i(\cos^2\alpha_i-3\sin^2\alpha_i)\right]\frac{\partial^3\omega}{\partial x\partial y^2}$$

$$+\sin^2\alpha_i\left[D_i\sin^2\alpha_i+K_i\cos^2\alpha_i\right]\frac{\partial^3\omega}{\partial y^3}\}$$

(18)

由基本方程式（10）、内力表达式（13），（14），（15）或（16），（17），（18），根据边界条件，原则上已能求解各种网格形式的网状扁壳了。解出变位 u_x，u_y，ω 后，可由式（16），（17），（18）计算网壳的内力。也可由式(1)，(2)计算组成网壳的基本杆件即扁拱的内力。

三、比较网状扁壳与各向异性扁壳

不难看出，如引进一些网状扁壳的物理常数，即折算薄膜刚度和弯曲刚度

$$B_{11}=\sum_i E\delta_i\cos^4\alpha_i$$

$$B_{22}=\sum_i E\delta_i\sin^4\alpha_i$$

$$B_{12}=B_{21}=\sum_i E\delta_i \cos^2\alpha_i \sin^2\alpha_i$$

$$B_{33}=\sum_i E\delta_i \cos^2\alpha_i \sin^2\alpha_i \qquad (19)$$

$$B_{13}=B_{31}=\sum_i E\delta_i \cos^3\alpha_i \sin\alpha_i$$

$$B_{23}=B_{32}=\sum_i E\delta_i \cos\alpha_i \sin^3\alpha_i$$

$$D_{11}=\sum_i [D_i\cos^4\alpha_i+K_i\cos^2\alpha_i\sin^2\alpha_i$$

$$D_{22}=\sum_i [D_i\sin^4\alpha_i+K_i\cos^2\alpha_i\sin^2\alpha_i]$$

$$D_{12}=D_{21}=\sum_i [(D_i-K_i)\cos^2\alpha_i\sin^2\alpha_i]$$

$$D^{(1)}_{33}=\sum_i [D_i\cos^2\alpha_i\sin^2\alpha_i+\frac{K_i}{2}\cos^2\alpha_i(\cos^2\alpha_i-\sin^2\alpha_i)]$$

$$D^{(2)}_{33}=\sum_i [D_i\cos^2\alpha_i\sin^2\alpha_i+\frac{K_i}{2}\sin^2\alpha_i(\sin^2\alpha_i-\cos^2\alpha_i)]$$

$$D_{33}=\frac{D^{(1)}_{33}+D^{(2)}_{33}}{2}=\sum_i [D_i\cos^2\alpha_i\sin^2\alpha_i+$$

$$+\frac{K_i}{4}(\cos^2\alpha_i-\sin^2\alpha_i)^2]$$

$$D^{(1)}_{31}=\sum_i [(D_i-K_i)\cos^3\alpha_i\sin\alpha_i]$$

$$D^{(2)}_{31}=\sum_i [D_i\cos^3\alpha_i\sin\alpha_i+K_i\cos\alpha_i\sin^3\alpha_i]$$

$$D_{13}=D_{31}=\frac{D^{(1)}_{31}+D^{(2)}_{31}}{2}=\sum_i [D_i\cos^3\alpha_i\sin\alpha_i+\frac{K_i}{2}$$

$$\times\cos\alpha_i\sin\alpha_i(\sin^2\alpha_i-\cos^2\alpha_i)]$$

$$D^{(1)}_{32}=\sum_i[D_i\cos\alpha_i\sin^3\alpha_i+K_i\sin^3\alpha_i\sin\alpha_i]$$

$$D^{(2)}_{32}=\sum_i[(D_i-K_i)\cos\alpha_i\sin^3\alpha_i]$$

$$D_{23}=D_{32}=\frac{D^{(1)}_{32}+D^{(2)}_{32}}{2}=\sum_i[D_i\cos\alpha_i\sin^3\alpha+\frac{K_i}{2}$$

$$\times\cos\alpha_i\sin\alpha_i(\cos^2\alpha_i-\sin^2\alpha_i)] \qquad (20)$$

则可使内力表达式（16），（17），（18）简化为

$$\left.\begin{aligned}T_x&=B_{11}\varepsilon_x+B_{12}\varepsilon_y+B_{13}\gamma\\T_y&=B_{21}\varepsilon_x+B_{22}\varepsilon_y+B_{23}\gamma\\S_{xy}&=S_{yx}=S=B_{31}\varepsilon_x+B_{32}\varepsilon_y+B_{33}\gamma\end{aligned}\right\} \qquad (21)$$

$$\left.\begin{aligned}M_x&=D_{11}X_x+D_{12}X_y+D_{13}\tau\\M_y&=D_{21}X_x+D_{22}X_y+D_{23}\tau\\H_{xy}&=D^{(1)}_{31}X_x+D^{(1)}_{32}X_y+D^{(1)}_{33}\tau\\H_{yx}&=D^{(2)}_{31}X_x+D^{(2)}_{32}X_y+D^{(2)}_{33}\tau\end{aligned}\right\} \qquad (22)$$

$$Q_x=D_{11}\frac{\partial X_x}{\partial x}+(2D_{13}+D^{(2)}_{31})\frac{\partial X_x}{\partial y}$$

$$+(D_{12}+2D^{(2)}_{33})\frac{\partial X_y}{\partial x}+D^{(2)}_{32}\frac{\partial X_y}{\partial y}$$

$$Q_y=D_{31}\frac{\partial X_x}{\partial x}+(2D^{(1)}_{33}+D_{21})\frac{\partial X_x}{\partial y}$$

$$+(2D_{23}+D_{32})\frac{\partial X_y}{\partial x}+D_{22}\frac{\partial X_y}{\partial y}$$

$$Q^*_x=D_{11}\frac{\partial X_x}{\partial x}+4D_{13}\frac{\partial X_x}{\partial y}+(D_{12}+4D_{33})\frac{\partial X_y}{\partial x}+2D_{32}\frac{\partial X_y}{\partial y}$$

$$Q_y^*=2D_{31}\frac{\partial X_x}{\partial x}+(D_{21}+4D_{33})\frac{\partial X_x}{\partial y}+4D_{23}\frac{\partial X_y}{\partial x}+D_{22}\frac{\partial X_y}{\partial x}$$

(23)

式中

$$\varepsilon_x=\frac{\partial u_x}{\partial x}-K_x\omega,\quad \varepsilon_y=\frac{\partial u_y}{\partial y}-k_y\omega,$$

$$\gamma=\frac{\partial u_y}{\partial x}+\frac{\partial u_x}{\partial y}-2t\omega; \qquad (24)$$

$$X_x=-\frac{\partial^2\omega}{\partial x^2},\quad X_y=-\frac{\partial^2\omega}{\partial y^2},\quad \tau=-2\frac{\partial^2\omega}{\partial x\partial y} \qquad (25)$$

算符（11）也可简化为：

$$L_{11}=B_{11}\frac{\partial^2}{\partial x^2}+2B_{13}\frac{\partial^2}{\partial x\partial y}+B_{33}\frac{\partial^2}{\partial y^2}$$

$$L_{12}=L_{21}=B_{13}\frac{\partial^2}{\partial x^2}+(B_{12}+B_{33})\frac{\partial^2}{\partial x\partial y}+B_{23}\frac{\partial^2}{\partial y^2}$$

$$L_{13}=L_{31}=-(k_xB_{11}+k_yB_{12}+2tB_{13})\frac{\partial}{\partial x}$$

$$-(k_xB_{13}+k_yB_{23}+2tB_{33})\frac{\partial}{\partial y}$$

$$L_{22}=B_{33}\frac{\partial^2}{\partial x^2}+2B_{23}\frac{\partial^2}{\partial x\partial y}+B_{22}\frac{\partial^2}{\partial y^2}$$

$$L_{23}=L_{32}=-(k_xB_{13}+k_yB_{23}+2tB_{33})\frac{\delta}{\partial x}$$

$$-(k_xB_{12}+k_yB_{22}+2tB_{23})\frac{\partial}{\partial y}$$

$$L_{33}=D_{11}\frac{\partial^4}{\partial x^4}+4D_{13}\frac{\partial^4}{\partial x^3\partial y}+2(D_{12}+2D_{33})$$

$$\cdot\frac{\partial^4}{\partial x^2\partial y^2}+4D_{23}\frac{\partial^4}{\partial x\partial y^3}+D_{22}\frac{\partial^4}{\partial y^4}$$

$$+(k_x^2 B_{11}+2k_x k_y B_{12}+k_y^2 B_{22}+4k_x t B_{13}+4k_y t B_{23}+4t^2 B_{33}) \Big\}$$

(26)

由此可见，当扭曲率 t 为零时，网状扁壳的基本方程式(10)从形式上来说和材料上各向异性双曲扁壳的基本方程式相似〔11〕，但因网状扁壳的 $D_{13} \neq D_{31}^{(1)} \neq D_{31}^{(2)}$，$D_{23} \neq D_{32}^{(1)} \neq D_{32}^{(2)}$，$D_{33} \neq D_{33}^{(1)} \neq D_{33}^{(2)}$，而导致某些内力（$H_{xy}$，$H_{yx}$，$Q_x$，$Q_y$）的表达式与材料上各向异性双曲扁壳的内力表达式不同。

四. 网状扁壳混合法的一般方程式

网状扁壳的一般方程式亦可采用混合法来表示。将式（1a），(4)，(5)，（1c）代入式(6)，并应用式(7)，(8)，(9)，（16），（20），（22），可表达为

$$\left.\begin{aligned}
&\frac{\partial T_x}{\partial x}+\frac{\partial S}{\partial y}+X=0\\
&\frac{\partial S}{\partial x}+\frac{\partial T_y}{\partial y}+Y=0\\
&D_{11}\frac{\partial^4\omega}{\partial x^4}+4D_{13}\frac{\partial^4\omega}{\partial x^3\partial y}+2(D_{12}+2D_{33})\frac{\partial^4\omega}{\partial x^2\partial y^2}\\
&\qquad+4D_{23}\frac{\partial^4}{\partial x\partial y^3}+D_{22}\frac{\partial^4\omega}{\partial y^4}-k_xT_x\\
&\qquad-k_yT_y-2tS=q
\end{aligned}\right\}$$

(27)

引入应力函数 Φ 及切向荷载函数 Ω，使得

$$T_x=\frac{\partial^2\Phi}{\partial y^2}+\Omega\ ,\ T_y=\frac{\partial^2\Phi}{\partial x^2}+\Omega\ ,\quad S=-\frac{\partial^2\Phi}{\partial x\partial y}\tag{28}$$

$$X=-\frac{\partial\Omega}{\partial x}\ ,\quad Y=-\frac{\partial\Omega}{\partial y}\tag{29}$$

则由式（27）得出混合法的平衡方程；而协调方程可由式（24）消去u_x，u_y，并代入式（21），（28）后得出。经整理后一并归纳为

$$\left.\begin{aligned}&L_B\Phi+\nabla_K^2\omega=-\nabla_B^2\Omega\\&L_D\omega-\nabla_K^2\Phi=q+(k_x+k_y)\Omega\end{aligned}\right\}\tag{30}$$

其中

$$\left.\begin{aligned}L_B=&\,b_{22}\frac{\partial^4}{\partial x^4}-2b_{23}\frac{\partial^4}{\partial x^3\partial y}+(2b_{12}+b_{33})\frac{\partial^4}{\partial x^2\partial y^2}\\&-2b_{13}\frac{\partial^4}{\partial x\partial y^3}+b_{11}\frac{\partial^4}{\partial y^4}\\L_D=&\,D_{11}\frac{\partial^4}{\partial x^4}+4D_{13}\frac{\partial^4}{\partial x^3\partial y}+2(D_{12}+2D_{33})\\&\cdot\frac{\partial^4}{\partial x^2\partial y^2}+4D_{23}\frac{\partial^4}{\partial x\partial y^3}+D_{22}\frac{\partial^4}{\partial y^4}\\\nabla_K^2=&\,k_y\frac{\partial^2}{\partial x^2}-2t\frac{\partial^2}{\partial x\partial y}+k_x\frac{\partial^2}{\partial y^2}\\\nabla_B^2=&\,(b_{12}+b_{22})\frac{\partial^2}{\partial x^2}-(b_{13}+b_{33})\frac{\partial^2}{\partial x\partial y}\\&+(b_{11}+b_{12})\frac{\partial^2}{\partial y^2}\end{aligned}\right\}\tag{31}$$

b_{mn}为网状扁壳的折算薄膜柔度，可由折算薄膜刚度B_{mn}按下式求得