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1. *The Piezomagnetic Field Associated with the Mogi Model.*

By Yoichi SASAI,

Earthquake Research Institute.

(Received May 21, 1979)

Summary

An analytical solution can be obtained for the piezomagnetic field accompanying a strain nucleus of the center of dilatation within the semi-infinite elastic medium. In volcanology, the problem is called the Mogi model, which successfully explains surface displacements around a volcano associated with its eruptions. The stress-induced magnetization within a uniformly magnetized crustal layer can be expressed in the form of a linear combination of stress components. The convolution integrals representing the piezomagnetic field are then evaluated by the Fourier transform method. The solution consists of combination of dipoles and quadrupoles at depth. Especially under appropriate conditions of actual volcanoes, the piezomagnetic field accompanying the Mogi model is equivalent to that of a magnetic dipole embedded at the dilating center. The stress-induced magnetization might be responsible for at least a portion of the rapid geomagnetic changes associated with eruptions of the Oshima volcano, Japan, although the dominant long-period variations would be of thermal origin.

Introduction

A simple mechanical model was introduced by MOGI (1958) to interpret crustal deformations before and after volcanic eruptions. He adopted a hydrostatically pumped sphere as a model of the magma reservoir beneath the volcanic body. According to the elasticity theory, the surface displacements accompanying Mogi's model are equivalent to those of an idealized strain nucleus, namely a center of dilatation in the semi-infinite elastic medium. The problem had already been solved by YAMAKAWA (1955), whose results were employed by MOGI (1958). Observable surface deformations were successfully explained in some cases with a hydrostatic pressure amounting to a few kilobars or more (MOGI 1958, FISKE and KINOSHITA 1969). Another force source of magmatic intrusion type was proposed by YOKOYAMA (1971) so as to attain ground deformations more effectively. The distribution of normal stress across the surface of the source sphere can be represented by P_0 of the spherical harmonics in the case of the Mogi model, while that of the Yokoyama model is a $P_1^1(\cos\theta)$ type. As far as ordinary values of mechanical strength are assumed for competent rocks within volcanoes, a fairly large amount of pressure changes are necessary even in the case of the Yokoyama model to interpret observed surface displacements. A stress-induced magnetic change is therefore anticipated, accompanying the volcanic activities.

The piezomagnetic effect of magnetized rocks has been well established during the past decades (NAGATA 1970, STACEY and BANERJEE 1974). The basic concept of the stress-induced volcano-magnetic effect was first proposed by STACEY, BARR and ROBSON (1965), who calculated the piezomagnetic anomaly field due to some particular stress distribution around a magma chamber. Their model calculation seems, however, to be somewhat inappropriate, because the stress field solution in the *infinite* elastic medium was adopted in their work. YUKUTAKE and TACHINAKA (1967) obtained the piezomagnetic field associated with a dilating cylinder at a depth parallel to the surface, in which boundary conditions at the surface are properly taken into account. Yukutake's model is nothing but a two-dimensional version of the Mogi model, which might be usefull in case of the fissure eruption. The piezomagnetic change accompanying the Mogi model itself was calculated by DAVIS (1976). All these model calculations were numerical ones, with elaborate computer work.

An analytical solution is presented here of the piezomagnetic field produced by the Mogi model. The situation is limited to the case of a point force source in the semi-infinite elastic medium with uniform magnetization from surface to a depth of Currie point isotherm. Only the reversible change with respect to the applied stress is considered here (See NAGATA 1970). The method is based on Fourier transforms of the convolution integrals. Direct suggestions were given to the present work by HAGIWARA (1977), who derived gravity change associated with the Mogi model by means of Fourier integral transforms.

The final result will be shown to have a very simple form. It is not intended in this paper to apply the present result to actual field data. A qualitative comparison will be made in the last section between two possible causes of the volcano-magnetic effect, namely (a) stress effect and (b) temperature effect.

The Stress Field of the Mogi Model

We take the Cartesian and cylindrical coordinate systems as shown in Fig. 1, where a semi-infinite elastic body occupies $z \geqq 0$. The problem is to determine the stress field at an arbitrary point in the semi-infinite elastic medium when a small sphere at $(0, 0, D)$ suffers hydro-

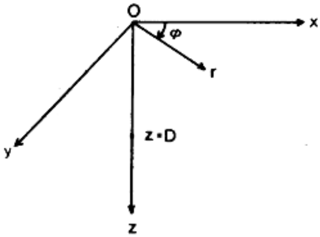


Fig. 1. Coordinates systems.

static pressure ΔP from inside. The displacement field of the present problem was obtained in a rather classical manner by ANDERSON (1936) and independently by YAMAKAWA (1955), whose solution might be available to derive the stress field by differentiation. We deal with the problem here in terms of the Love's strain function Φ, which satisfies the following equation when there is no body force,

$$\nabla^2\nabla^2\Phi = 0 \tag{1}$$

$$\left.\begin{aligned}\nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \\ &= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}\end{aligned}\right\} \tag{2}$$

The displacements are given by

$$2\mu u_r = -\frac{\partial^2\Phi}{\partial r\partial z}\,, \quad 2\mu u_\varphi = -\frac{1}{r}\frac{\partial^2\Phi}{\partial\varphi\partial z}\,, \quad 2\mu u_z = \left[2(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi \tag{3}$$

Six components of the stress tensor in the cylindrical coordinates are derived from the following formulae;

$$\left.\begin{aligned}\sigma_{rr} &= \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{\partial^2}{\partial r^2}\right)\Phi \\ \sigma_{\varphi\varphi} &= \frac{\partial}{\partial z}\left(\nu\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right)\Phi \\ \sigma_{zz} &= \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi \\ \sigma_{r\varphi} &= -\frac{\partial^3}{\partial r\partial\varphi\partial z}\left(\frac{\Phi}{r}\right) \\ \sigma_{\varphi z} &= \frac{1}{r}\frac{\partial}{\partial\varphi}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi \\ \sigma_{zr} &= \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 - \frac{\partial^2}{\partial z^2}\right]\Phi\end{aligned}\right\} \tag{4}$$

$$\nu=\frac{\lambda}{2(\lambda+\mu)}$$

where ν, λ and μ are Poisson's ratio and Lame's constants.

MINDLIN and CHENG (1950) obtained expressions for various types of strain nuclei in the semi-infinite solid in terms of the Galerkin vector stress function. The strain function for a center of dilatation placed at a point $(0, 0, D)$ in the semi-infinite medium is given as follows (MINDLIN and CHENG 1950, See also HAGIWARA 1977);

$$\Phi=C\left\{\log(D-z+R_1)+\frac{\lambda-\mu}{\lambda+\mu}\log(z+D+R_2)-\frac{2z}{R_2}\right\} \tag{5}$$

where

$$\left.\begin{aligned} R_1&=\sqrt{r^2+(D-z)^2}\\ R_2&=\sqrt{r^2+(D+z)^2}\\ r&=\sqrt{x^2+y^2}\end{aligned}\right\} \tag{6}$$

The first term corresponds to a strain nucleus at $(0, 0, D)$ in the infinite medium, while the remainder is the sum of an image nucleus at $(0, 0, -D)$, and the solution of Boussinesq problem for the resultant normal load. The image nucleus cancels the tangential shear stress, and the Boussinesq solution nullifies the normal stress at the surface, respectively. The traction free boundary conditions at the surface are thus satisfied in the solution (5).

The coefficient C should be determined elsewhere, which indicates the intensity of the strain nucleus having a dimension of the moment of force. Consider a small sphere of radius a with its center at $(0, 0, D)$. Taking the spherical coordinates (R, θ, φ) centered at $(0, 0, D)$, the stress components in the spherical coordinates $\sigma_{RR}, \sigma_{\theta\theta}, \sigma_{R\theta}$ are related to those in the cylindrical ones $\sigma_{rr}, \sigma_{zz}, \sigma_{zr}$ as follows;

$$\left.\begin{aligned} \sigma_{RR}&=\frac{1}{R_1^2}\{\sigma_{rr}r^2+\sigma_{zz}(z-D)^2+2\sigma_{zr}r(z-D)\}\\ \sigma_{\theta\theta}&=\frac{1}{R_1^2}\{\sigma_{rr}(z-D)^2+\sigma_{zz}r^2-2\sigma_{zr}r(z-D)\}\\ \sigma_{R\theta}&=\frac{1}{R_1^2}\{(\sigma_r-\sigma_z)r(z-D)+\sigma_{zr}[(z-D)^2-r^2]\}\end{aligned}\right\} \tag{7}$$

Under the assumption that a is small, i.e., $a\ll D$, so that $R_1\ll R_2$, the main contribution to the stress field is that of the first term, $\Phi_1=C\log(D-z+R_1)$. Substituting Φ_1 into equations (4), we have stress components near the small sphere;

$$\left.\begin{aligned} \sigma_{rr} &= -C\,\frac{R_1^2-3r^2}{R_1^5} \\ \sigma_{zz} &= -C\,\frac{R_1^2-3(z-D)^2}{R_1^5} \\ \sigma_{zr} &= -C\,\frac{3r(D-z)}{R_1^5} \end{aligned}\right\} \tag{8}$$

Substituting eqs. (8) into (7) and putting $R_1=a$, we obtain

$$\left.\begin{aligned} \sigma_{RR} &= \frac{2C}{a^3} \\ \sigma_{\theta\theta} &= -\frac{C}{a^3} \\ \sigma_{R\theta} &= 0 \end{aligned}\right\} \tag{9}$$

Now we assume that the normal stress at the spherical surface is balanced with the internal hydrostatic pressure ΔP (the compressive force is taken to be positive). With the aid of the first equation in (9) we obtain

$$C=\frac{a^3\Delta P}{2} \tag{10}$$

The deriving process of the factor C shows that eq. (10) holds under the condition that $a \ll D$. The internal pressure has sometimes been discussed on the basis of eq. (10) by assuming some values of the radius a. ΔP and a are essentially not independent in the Mogi model, so that the factor C itself seems to have a more direct physical meaning. The quantity C has a dimension of the moment of force, indicating the magnitude of the force source within a volcano.

Finally, the stress field components are given as follows by means of eqs. (4);

$$\left.\begin{aligned} \sigma_{rr} &= C\left\{\frac{2}{R_1^3}-\frac{3(z-D)^2}{R_1^5}+\frac{2(2\lambda+3\mu)}{\lambda+\mu}\frac{1}{R_2^3}-\frac{3(z+D)(11z+3D)}{R_2^5}+\frac{30z(z+D)^3}{R_2^7}\right\} \\ \sigma_{\varphi\varphi} &= C\left\{-\frac{1}{R_1^3}+\frac{\lambda-3\mu}{\lambda+\mu}\frac{1}{R_2^3}+\frac{1}{\lambda+\mu}\frac{6(z+D)(\mu z-\lambda D)}{R_2^5}\right\} \\ \sigma_{zz} &= C\left\{-\frac{1}{R_1^3}+\frac{3(z-D)^2}{R_1^5}+\frac{1}{R_2^3}+\frac{3(z+D)(5z-D)}{R_2^5}-\frac{30z(z+D)^3}{R_2^7}\right\} \\ \sigma_{zr} &= C3r\left\{\frac{z-D}{R_1^5}+\frac{3z+D}{R_2^5}-\frac{10z(z+D)^2}{R_2^7}\right\} \\ \sigma_{r\varphi} &= 0 \\ \sigma_{\varphi z} &= 0 \end{aligned}\right\} \tag{11}$$

Stress-induced Magnetization

Magnetic properties of compressed rocks have been studied by many rock-magnetitians, and are summarized by NAGATA (1970) and STACEY and BANERJEE (1974). The induced magnetization due to the susceptibility and hard remanent magnetization such as TRM and CRM vary with the uniaxial compression ($\sigma>0$) in such a way as

$$\left.\begin{aligned} J^{/\!/} &= \frac{J_0^{/\!/}}{1+\beta\sigma} \cong J_0^{/\!/}(1-\beta\sigma) \\ J^{\perp} &= \frac{J_0^{\perp}}{1-\frac{1}{2}\beta\sigma} \cong J_0^{\perp}\left(1+\frac{1}{2}\beta\sigma\right) \end{aligned}\right\} \tag{12}$$

where the signs // and $\perp$ indicate the component parallel and perpendicular to the applied stress respectively, while the subscript 0 denotes the value of unstressed state. β is called the stress sensitivity having an order of 10^{-4} bar^{-1}. Empirical relations (12) are presumed to hold for the uniaxial tension ($\sigma<0$) by theoretical considerations.

Eqs. (12) were extended to the general three dimensional case by STACEY, BARR and ROBSON (1965). We may resolve the magnetization J_0 into orthogonal three components (J_1, J_2, J_3) in directions of principal stresses σ_1, σ_2, σ_3 and apply the relations (12) to each component. The stress-induced magnetization in each direction of the principal axis is then represented as

$$\left.\begin{aligned} \Delta J_i \boldsymbol{e}_i &= \beta J_i\left(\frac{\sigma_j+\sigma_k}{2}-\sigma_i\right)\boldsymbol{e}_i \\ (i,\ j,\ k &= 1,\ 2,\ 3. \quad i\neq j\neq k) \end{aligned}\right\} \tag{13}$$

where $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ are the unit vectors in directions of principal stresses σ_1, σ_2, σ_3 at a point considered.

We are now to investigate principal stresses of the Mogi model's stress field. The stress components in eqs. (11) can be transformed into those in the Cartesian coordinates as follows,

$$\mathbf{T}=\begin{bmatrix} \sigma_{rr}\cos^2\varphi+\sigma_{\varphi\varphi}\sin^2\varphi\ , & (\sigma_{rr}-\sigma_{\varphi\varphi})\sin\varphi\cos\varphi, & \sigma_{zr}\cos\varphi \\ (\sigma_{rr}-\sigma_{\varphi\varphi})\sin\varphi\cos\varphi\ , & \sigma_{rr}\sin^2\varphi+\sigma_{\varphi\varphi}\cos^2\varphi\ , & \sigma_{zr}\sin\varphi \\ \sigma_{zr}\cos\varphi\ , & \sigma_{zr}\sin\varphi\ , & \sigma_{zz} \end{bmatrix} \tag{14}$$

Principal stresses are obtained by solving the eigen value equation of the matrix **T**, namely

$$|\mathbf{T}-\sigma\mathbf{I}|=0 \tag{15}$$

where **I** is the unit matrix. The solutions are given in the following:

$$\left.\begin{aligned}\sigma_1&=\frac{\sigma_{rr}+\sigma_{zz}}{2}+\sqrt{\left(\frac{\sigma_{rr}-\sigma_{zz}}{2}\right)^2+\sigma_{zr}^2}\\ \sigma_2&=\sigma_{\varphi\varphi}\\ \sigma_3&=\frac{\sigma_{rr}+\sigma_{zz}}{2}-\sqrt{\left(\frac{\sigma_{rr}-\sigma_{zz}}{2}\right)^2+\sigma_{zr}^2}\end{aligned}\right\} \tag{16}$$

We may introduce an orthonormal matrix **P**, columns of which consist of eigen vectors of **T**. **P** satisfies the matrix equation:

$$\mathbf{P}^{-1}\mathbf{T}\mathbf{P}=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix} \tag{17}$$

,from which we obtain the following expression for **P**.

$$\left.\begin{aligned}\mathbf{P}&=\begin{bmatrix}\dfrac{\sigma_1-\sigma_{zz}}{\Delta}\cos\varphi, & -\sin\varphi, & -\dfrac{\sigma_{zr}}{\Delta}\cos\varphi\\ \dfrac{\sigma_1-\sigma_{zz}}{\Delta}\sin\varphi, & \cos\varphi, & -\dfrac{\sigma_{zr}}{\Delta}\sin\varphi\\ \dfrac{\sigma_{zr}}{\Delta}, & 0, & \dfrac{\sigma_{rr}-\sigma_3}{\Delta}\end{bmatrix}\\ \text{where}\\ \Delta&=\sqrt{(\sigma_1-\sigma_{zz})^2+\sigma_{zr}^2}=\sqrt{(\sigma_3-\sigma_{rr})^2+\sigma_{zr}^2}\end{aligned}\right\} \tag{18}$$

The matrix **P** defines the transformation of the original Cartesian coordinates $\{e_x, e_y, e_z\}$ into another set of Cartesian coordinates $\{e_1, e_2, e_3\}$, whose axes coincide with the principal axes of the stress tensor **T**. Direction cosines of σ_1, σ_2 and σ_3 axis are then given by components of the column vector of **P**. An arbitrary vector $J'=(J_1', J_2', J_3')^t$ in the principal axes coordinates $\{e_1, e_2, e_3\}$ is related in a following way to the vector $J=(J_x, J_y, J_z)^t$, which is the same vector as J' but is described in the original coordinate $\{e_x, e_y, e_z\}$,

$$J'=\mathbf{P}^{-1}J \tag{19}$$

For the convenience of following calculations, we may rewrite eq. (18) and put

$$\mathbf{P}^{-1}=\mathbf{P}^t=\begin{bmatrix}\lambda_1 & \mu_1 & \nu_1\\ \lambda_2 & \mu_2 & \nu_2\\ \lambda_3 & \mu_3 & \nu_3\end{bmatrix}$$

$$=\begin{bmatrix}\dfrac{b}{\sqrt{b^2+c^2}}\cos\varphi, & \dfrac{b}{\sqrt{b^2+c^2}}\sin\varphi, & \dfrac{c}{\sqrt{b^2+c^2}}\\ -\sin\varphi, & \cos\varphi, & 0\end{bmatrix}$$

$$\left.\begin{array}{l}\left[-\dfrac{c}{\sqrt{b^2+c^2}}\cos\varphi\,,\quad -\dfrac{c}{\sqrt{b^2+c^2}}\sin\varphi\,,\quad \dfrac{b}{\sqrt{b^2+c^2}}\right]\\ \text{where}\\ \quad b=\dfrac{\sigma_{rr}-\sigma_{zz}}{2}+\sqrt{\left(\dfrac{\sigma_{rr}-\sigma_{zz}}{2}\right)^2+\sigma_{zr}^2}\\ \quad c=\sigma_{zr}\end{array}\right\} \tag{20}$$

We are in a position to obtain the stress-induced magnetization. Here the horizontal and the vertical magnetization cases will be dealt with separately.

(1) *In the case of Horizontal Magnetization in the x Direction*: we put $\boldsymbol{J}=J_H\boldsymbol{e}_x$ and substitute it into eq. (19);

$$\boldsymbol{J}'_H=\lambda_1 J_H\boldsymbol{e}_1+\lambda_2 J_H\boldsymbol{e}_2+\lambda_3 J_H\boldsymbol{e}_3 \tag{21}$$

Applying the empirical formulae of piezomagnetism (13) to the elementary volume $dV=dxdydz$ at a point (x, y, z), we obtain the increments of magnetization in the principal axis directions as follows,

$$\left.\begin{array}{l} d\boldsymbol{M}_1=\beta T_1\lambda_1 J_H dV\boldsymbol{e}_1\\ d\boldsymbol{M}_2=\beta T_2\lambda_2 J_H dV\boldsymbol{e}_2\\ d\boldsymbol{M}_3=\beta T_3\lambda_3 J_H dV\boldsymbol{e}_3\end{array}\right\} \tag{22}$$

where

$$\left.\begin{array}{l} T_1=\dfrac{\sigma_2+\sigma_3}{2}-\sigma_1\\ T_2=\dfrac{\sigma_1+\sigma_3}{2}-\sigma_2\\ T_3=\dfrac{\sigma_1+\sigma_2}{2}-\sigma_3\end{array}\right\} \tag{23}$$

With the aid of the following relations derived from eq. (19),

$$\left.\begin{array}{l} \boldsymbol{e}_1=\lambda_1\boldsymbol{e}_x+\mu_1\boldsymbol{e}_y+\nu_1\boldsymbol{e}_z\\ \boldsymbol{e}_2=\lambda_2\boldsymbol{e}_x+\mu_2\boldsymbol{e}_y+\nu_2\boldsymbol{e}_z\\ \boldsymbol{e}_3=\lambda_3\boldsymbol{e}_x+\mu_3\boldsymbol{e}_y+\nu_3\boldsymbol{e}_z\end{array}\right\} \tag{24}$$

we obtain the total increment of magnetization $d\boldsymbol{M}_H$ at a point as;

$$d\boldsymbol{M}_H=d\boldsymbol{M}_1+d\boldsymbol{M}_2+d\boldsymbol{M}_3=\beta J_H(S_{xx}\boldsymbol{e}_x+S_{xy}\boldsymbol{e}_y+S_{xz}\boldsymbol{e}_z)dxdydz \tag{25}$$

where

$$\left.\begin{aligned} S_{xx} &= \lambda_1^2 T_1 + \lambda_2^2 T_2 + \lambda_3^2 T_3 \\ S_{xy} &= \lambda_1\mu_1 T_1 + \lambda_2\mu_2 T_2 + \lambda_3\mu_3 T_3 \\ S_{xz} &= \lambda_1\nu_1 T_1 + \lambda_2\nu_2 T_2 + \lambda_3\nu_3 T_3 \end{aligned}\right\} \tag{26}$$

Eq. (25) shows that $\beta J_H S_{xx}$, *etc.* are the stress-induced magnetization in the x, y and z direction respectively for a given horizontal magnetization.

(II) *In the case of Vertical Magnetization:* putting $\boldsymbol{J}=J_V\boldsymbol{e}_z$ in eq. (19), we obtain

$$\boldsymbol{J}_V' = \nu_1 J_V \boldsymbol{e}_1 + \nu_2 J_V \boldsymbol{e}_2 + \nu_3 J_V \boldsymbol{e}_3 \tag{27}$$

The incremental magnetizations in the principal axis direction are in this case as follows,

$$\left.\begin{aligned} d\boldsymbol{M}_1 &= \beta T_1 \nu_1 J_V dV \boldsymbol{e}_1 \\ d\boldsymbol{M}_2 &= \beta T_2 \nu_2 J_V dV \boldsymbol{e}_2 \\ d\boldsymbol{M}_3 &= \beta T_3 \nu_3 J_V dV \boldsymbol{e}_3 \end{aligned}\right\} \tag{28}$$

The total stress-induced magnetization $d\boldsymbol{M}_V$ in the vertical magnetization case is given by

$$d\boldsymbol{M}_V = \beta J_V (S_{zx}\boldsymbol{e}_x + S_{zy}\boldsymbol{e}_y + S_{zz}\boldsymbol{e}_z) dx dy dz \tag{29}$$

where

$$\left.\begin{aligned} S_{zx} &= \lambda_1\nu_1 T_1 + \lambda_2\nu_2 T_2 + \lambda_3\nu_3 T_3 \\ S_{zy} &= \mu_1\nu_1 T_1 + \mu_2\nu_2 T_2 + \mu_3\nu_3 T_3 \\ S_{zz} &= \nu_1^2 T_1 + \nu_2^2 T_2 + \nu_3^2 T_3 \end{aligned}\right\} \tag{30}$$

$\beta J_V S_{zx}$, *etc.* are the stress-induced magnetization in the x, y and z direction for a given vertical magnetization.

Substituting eqs. (16), (20) and (23) into (26) and (30), we find each component of the stress-induced magnetization in the form of linear combinations of stress components:

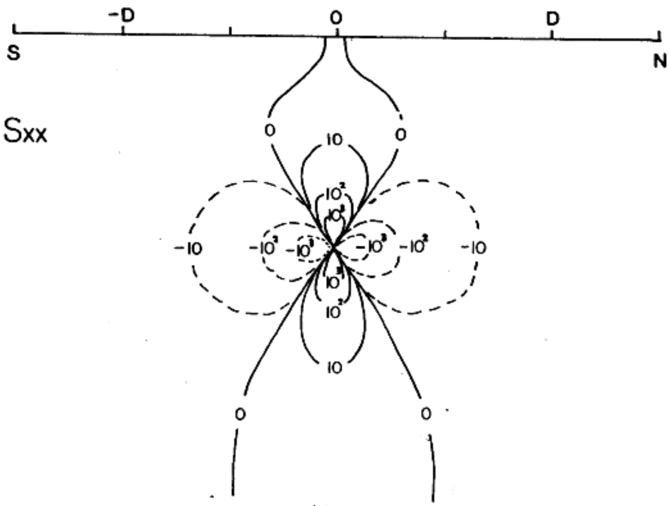
$$\left.\begin{aligned} S_{xx} &= \frac{1}{2}(\sigma_{rr}+\sigma_{zz}-2\sigma_{\varphi\varphi}) - \frac{3}{2}(\sigma_{rr}-\sigma_{\varphi\varphi})\cos^2\varphi \\ S_{xy} &= -\frac{3}{2}(\sigma_{rr}-\sigma_{\varphi\varphi})\sin\varphi\cos\varphi \\ S_{xz} &= -\frac{3}{2}\sigma_{zr}\cos\varphi \\ S_{zx} &= S_{xz} \\ S_{zy} &= -\frac{3}{2}\sigma_{zr}\sin\varphi \end{aligned}\right\} \tag{31}$$

$$S_{zz}=\frac{1}{2}(\sigma_{rr}+\sigma_{\varphi\varphi}-2\sigma_{zz})$$

It should be emphasized that eqs. (31) hold for any axi-symmetric problems with respect to the z axis, because these formulae are deduced solely on the basis of general symmetric properties of the stress tensor and $\sigma_{z\varphi}=\sigma_{r\varphi}=0$.

Finally, the stress-induced magnetization for the Mogi model is given in a concrete form as:

$$S_{xx}=\frac{3}{2}\frac{1}{R_1^3}+\frac{3\lambda+13\mu}{2(\lambda+\mu)}\frac{1}{R_2^3}-3\left(\frac{3\lambda+5\mu}{\lambda+\mu}z+\frac{2\mu}{\lambda+\mu}D\right)\frac{z+D}{R_2^5}$$
$$-\frac{9}{2}\frac{x^2}{r^2}\left\{\frac{1}{R_1^3}-\frac{(z-D)^2}{R_1^5}+\frac{\lambda+3\mu}{\lambda+\mu}\frac{1}{R_2^3}\right.$$
$$\left.-\left(\frac{11\lambda+13\mu}{\lambda+\mu}z+\frac{\lambda+3\mu}{\lambda+\mu}D\right)\frac{z+D}{R_2^5}+\frac{10z(z+D)^3}{R_2^7}\right\}$$
$$S_{xy}=-\frac{9}{2}\frac{xy}{r^2}\left\{\frac{1}{R_1^3}-\frac{(z-D)^2}{R_1^5}+\frac{\lambda+3\mu}{\lambda+\mu}\frac{1}{R_2^3}\right.$$
$$\left.-\left(\frac{11\lambda+13\mu}{\lambda+\mu}z+\frac{\lambda+3\mu}{\lambda+\mu}D\right)\frac{z+D}{R_2^5}+\frac{10z(z+D)^3}{R_2^7}\right\}$$



(a)

Fig. 2(a). Stress-induced magnetization S_{xx} within the N-S meridian plane in unit of kJ_0C/D^3. $\lambda=\mu$ is assumed.